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SUMMARY TECHNICAL REPORT
OF THE
NATIONAL DEFENSE RESEARCH COMMITTEE

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SUMMARY TECHNICAL REPORT OF THE
APPLIED MATHEMATICS PANEL, NDRC

VOLUME 2

ANALYTICAL STUDIES IN AERIAL WARFARE

OFFICE OF SCIENTIFIC RESEARCH AND DEVELOPMENT
VANNEVAR BUSH, DIRECTOR

NATIONAL DEFENSE RESEARCH COMMITTEE
JAMES B. CONANT, CHAIRMAN

APPLIED MATHEMATICS PANEL
WARREN WEAVER, CHIEF

WASHINGTON, D. C., 1946

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Richard C. Tolman, *Vice Chairman*

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Frank B. Jewett	Navy Representative[2]
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Maj. Gen. C. C. Williams
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Casper W. Ooms

NOTES ON THE ORGANIZATION OF NDRC

The duties of the National Defense Research Committee were (1) to recommend to the Director of OSRD suitable projects and research programs on the instrumentalities of warfare, together with contract facilities for carrying out these projects and programs, and (2) to administer the technical and scientific work of the contracts. More specifically, NDRC functioned by initiating research projects on requests from the Army or the Navy, or on requests from an allied government transmitted through the Liaison Office of OSRD, or on its own considered initiative as a result of the experience of its members. Proposals prepared by the Division, Panel, or Committee for research contracts for performance of the work involved in such projects were first reviewed by NDRC, and if approved, recommended to the Director of OSRD. Upon approval of a proposal by the Director, a contract permitting maximum flexibility of scientific effort was arranged. The business aspects of the contract, including such matters as materials, clearances, vouchers, patents, priorities, legal matters, and administration of patent matters were handled by the Executive Secretary of OSRD.

Originally NDRC administered its work through five divisions, each headed by one of the NDRC members. These were:

Division A — Armor and Ordnance
Division B — Bombs, Fuels, Gases, & Chemical Problems
Division C — Communication and Transportation
Division D — Detection, Controls, and Instruments
Division E — Patents and Inventions

In a reorganization in the fall of 1942, twenty-three administrative divisions, panels, or committees were created, each with a chief selected on the basis of his outstanding work in the particular field. The NDRC members then became a reviewing and advisory group to the Director of OSRD. The final organization was as follows:

Division 1 — Ballistic Research
Division 2 — Effects of Impact and Explosion
Division 3 — Rocket Ordnance
Division 4 — Ordnance Accessories
Division 5 — New Missiles
Division 6 — Sub-Surface Warfare
Division 7 — Fire Control
Division 8 — Explosives
Division 9 — Chemistry
Division 10 — Absorbents and Aerosols
Division 11 — Chemical Engineering
Division 12 — Transportation
Division 13 — Electrical Communication
Division 14 — Radar
Division 15 — Radio Coordination
Division 16 — Optics and Camouflage
Division 17 — Physics
Division 18 — War Metallurgy
Division 19 — Miscellaneous
Applied Mathematics Panel
Applied Psychology Panel
Committee on Propagation
Tropical Deterioration Administrative Committee

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NDRC FOREWORD

As events of the years preceding 1940 revealed more and more clearly the seriousness of the world situation, many scientists in this country came to realize the need of organizing scientific research for service in a national emergency. Recommendations which they made to the White House were given careful and sympathetic attention, and as a result the National Defense Research Committee [NDRC] was formed by Executive Order of the President in the summer of 1940. The members of NDRC, appointed by the President, were instructed to supplement the work of the Army and the Navy in the development of the instrumentalities of war. A year later, upon the establishment of the Office of Scientific Research and Development [OSRD], NDRC became one of its units.

The Summary Technical Report of NDRC is a conscientious effort on the part of NDRC to summarize and evaluate its work and to present it in a useful and permanent form. It comprises some seventy volumes broken into groups corresponding to the NDRC Divisions, Panels, and Committees.

The Summary Technical Report of each Division, Panel, or Committee is an integral survey of the work of that group. The first volume of each group's report contains a summary of the report, stating the problems presented and the philosophy of attacking them, and summarizing the results of the research, development, and training activities undertaken. Some volumes may be "state of the art" treatises covering subjects to which various research groups have contributed information. Others may contain descriptions of devices developed in the laboratories. A master index of all these divisional, panel, and committee reports which together constitute the Summary Technical Report of NDRC is contained in a separate volume, which also includes the index of a microfilm record of pertinent technical laboratory reports and reference material.

Some of the NDRC-sponsored researches which had been declassified by the end of 1945 were of sufficient popular interest that it was found desirable to report them in the form of monographs, such as the series on radar by Division 14 and the monograph on sampling inspection by the Applied Mathematics Panel. Since the material treated in them is not duplicated in the Summary Technical Report of NDRC, the monographs are an important part of the story of these aspects of NDRC research.

In contrast to the information on radar, which is of widespread interest and much of which is released to the public, the research on subsurface warfare is largely classified and is of general interest to a more restricted group. As a consequence, the report of Division 6 is found almost entirely in its Summary Technical Report, which runs to over twenty volumes. The extent of the work of a division cannot therefore be judged solely by the number of volumes devoted to it in the Summary Technical Report of NDRC: account must be taken of the monographs and available reports published elsewhere.

Perhaps the highest tribute which could have been paid to the role of mathematicians in World War II was the complete lack of astonishment which greeted their contributions. To the Applied Mathematics Panel of NDRC came urgent, varied, and formidable requests from every other group in NDRC and every military service. As expected, these requests were met; and, also as expected, the results were found invaluable in every phase of warfare from defense against enemy attack to the design of new weapons, recommendations for their use, predictions of their usefulness, and analysis of their effects.

To meet such obligations, the Applied Mathematics Panel under the leadership of Warren Weaver, together with members of its staff and of its contractors' staffs, made available the services of a group of men who were not merely able, competent mathematicians but also loyal, devoted Americans cooperating unselfishly in the defense of their country. The Summary Technical Report of the Applied Mathematics Panel, prepared under the direction of the Panel Chief and authorized by him for publication, is a record of their accomplishments and a testimonial to their scientific integrity. They deserve the grateful appreciation of the Nation.

Vannevar Bush, Director
Office of Scientific Research and Development

J. B. Conant, Chairman
National Defense Research Committee

FOREWORD

When the National Defense Research Committee was reorganized at the end of 1942, it was decided to set up a new organization, called the Applied Mathematics Panel [AMP], in order to bring mathematicians as a group more effectively into the work being carried on by scientists in support of the Nation's war effort. At the time of the original appointment of the National Defense Research Committee by President Roosevelt, no mathematicians were included on the Committee, and it was not until the NDRC had been operating for more than a year that the need of a separate division devoted to applied mathematics was recognized. Although many of the operating Divisions of NDRC had set up mathematical groups to handle their own analytical problems, it was intended that the new Applied Mathematics Panel should supplement such groups and should furnish mathematical advice and service to all Divisions of the NDRC, carrying out requested mathematical analyses and remaining available as consultants after the original analyses had been completed. The Panel was organized too late to make possible a fully definitive trial of the success of this type of organization. That mathematics has a fundamental role to play in the science of warfare, I am sure; I have set forth some of the considerations which seem to be relevant and important in the last chapter of Volume 2 of the AMP Summary Technical Report.

The actual development of wartime scientific work proved to be such that the Applied Mathematics Panel has not only been called upon for assistance by NDRC Divisions but has also directly assisted many branches of the Army and Navy. Indeed, at the conclusion of hostilities, when approximately two hundred studies had been undertaken by the Panel, roughly one-half of these represented direct requests from the Armed Services. Furthermore, the consulting activities, growing out of studies originally undertaken to answer specific questions, turned out to be considerably more extensive and significant than was originally anticipated. I think that the importance of this phase of the work cannot be too strongly emphasized. But no account of such general consulting activities is given here, this report being restricted to the formally constituted studies.

The analytical work under AMP studies was carried on by mathematicians associated in groups at various universities and operating under OSRD contracts administered by the Panel. To the men who served as technical representatives of the universities under these contracts, and to the technical aides who assisted the Chief in the administration of the Panel's scientific work, the Panel owes a large measure of whatever success it achieved. These men combined outstanding scientific competence with energy, resourcefulness, and a selfless willingness to devote their own efforts, as well as the efforts of their staffs, to the solution of other people's problems. The general plans for the Panel's activities were based upon the counsel of a group of eminent mathematicians, formally labeled the *Committee Advisory to the Scientific Officer*. This group, meeting every week, and consisting of R. Courant, G. C. Evans, T. C. Fry, L. M. Graves, H. M. Morse, O. Veblen, and S. S. Wilks, had responsibility for the preliminary examination of requests which reached the Panel and for decisions on overall policy. The Chief relied heavily on their advice which, to a large extent, determined the effectiveness of the Panel's activities.

As the work of NDRC developed, the Panel was called upon for assistance by all of NDRC's nineteen Divisions. It is not, therefore, surprising that the scope of the Panel's activities covers a wide range, falling into four broad, though somewhat overlapping, categories:

1. *Mathematical studies based upon certain classical fields* of applied mathematics, such as classical mechanics and the dynamics of rigid bodies, the theory of elasticity and plasticity, fluid dynamics, electrodynamics, and thermodynamics.

2. *Analytical studies in aerial warfare*, including assessment of the performance of sights and antiaircraft fire control equipment; studies relating to the vulnerability of aircraft to plane-to-plane and to antiaircraft fire and the optimal defense of the airplane against these; and analyses of problems arising from the use of rockets in air warfare.

3. *Probability and statistical studies* concerned with the effectiveness of bombing; various aspects of naval warfare, including fire effect analysis and the performance of torpedoes; the design of experiments; sampling inspection; and analyses of many types of data collected by the Armed Services.

4. *Computational services* concerned with the evaluation of integrals; the construction of tables and charts; the development of techniques adapted to the solution of special problems; the nature and capabilities of computing equipment.

The work of the Panel in the first two of these categories is summarized in Volumes 1 and 2 of the AMP Summary Technical Report. Volume 3, together with

two monographs[a] which the Panel has prepared dealing with sampling inspection and techniques of statistical analysis, provides a summary of the work in the third category. The fourth class of activities has been reported in AMP Note 25, *Description of Mathematical Tables Computed under the auspices of the Applied Mathematics Panel, NDRC;* in AMP Note 26, *Report on Numerical Methods Employed by the Mathematical Tables Project;* and in the reports published by the Panel under AMP Study 171, *Survey of Computing Machines.* No attempt has been made to report on work which will shortly be published as articles in scientific journals or on results which are deemed too special to be of continuing interest.

The preparation of this Summary Technical Report was undertaken after the end of World War II, at a time when the members of the Panel's staff and of the contract groups were eager to return to their peacetime careers. Thus the preparation of these three volumes, solely for the purpose of recording for the Services, in easily accessible form, the scientific results of the Panel's activities, was achieved at real personal sacrifice. I am greatly indebted to the authors of the several parts of these volumes and to the Editorial Committee, consisting of Mina Rees, I. S. Sokolnikoff, and S. S. Wilks, for the admirable job they have done in bringing together, under high pressure, a summary of the principal scientific accomplishments of the Panel.

WARREN WEAVER
Chief, Applied Mathematics Panel

[a] *Sampling Inspection and Techniques of Statistical Analysis,* published by the McGraw-Hill Book Co., Inc.

PREFACE

This volume furnishes a summary of those activities of the Applied Mathematics Panel which were concerned with air-to-air, ground-to-air, or air-to-ground warfare exclusive of bombing, the majority of studies having to do with the design and use of fire control equipment. Although much of this work was undertaken at the request of that part of the NDRC which was in charge of research and development in the whole field of fire control, namely, Division 7, many requests were received from other Divisions of NDRC, and many came directly from the Army and Navy and from the Joint Army-Navy-NDRC Airborne Fire Control Committee. Rarely did these requests involve studies which could be made in time to influence design — a situation which arose at least partially from the imperative need to obtain results which could be used in World War II — so that most of the studies were concerned either with the improvement of the theoretical accuracy of equipment by suitable changes in design, or with the best use of existing equipment. Nevertheless, in the attempt to answer specific questions, basic results were derived which should be of continuing interest. In the account here given, it is the basic theory which is emphasized, although a brief account of many specific results is included. Because of the diversity of the sources from which requests were received, and the varying requirements of the requesting agencies, it has been necessary, particularly in Part III (Antiaircraft Analysis), to report on studies which are somewhat disconnected. In both chapters of this part, a general introductory discussion of the nature of the problem is given which will, it is hoped, serve as a background for the more detailed treatment which follows.

By far the most extensive analyses carried on by the Panel in aerial warfare were concerned with air-to-air gunnery, the work in one contract being for several years devoted almost exclusively to this phase of warfare, while several other contracts were concerned with it for shorter periods of time and, in some cases, as incidental aspects of their work. The Panel was fortunate in having as the Director of the Applied Mathematics Group at Columbia (the contract with the most extensive and longest experience in this field) Saunders MacLane, who combined with outstanding mathematical competence the energy and personal effectiveness which kept the Group activities in close touch with Army and Navy needs. The Naval Ordnance Development Award has been conferred on the Group for distinguished service to the research and development of naval ordnance, and in particular for its contribution to the development of gunsights Mark 18 and 23. At Northwestern, where a Panel group headed by Walter Leighton worked in close touch with Division 7 and the Patuxent Naval Air Station, the principal concern was with the development of methods for the experimental assessment of fire control systems for aerial gunnery, with emphasis on camera techniques.

The report on Aerial Gunnery is presented as Part I of this volume, and in it a serious attempt has been made to include an account not only of the Panel's work in the field, but of all available literature regardless of source. The Panel's assigned emphasis on the analytical aspects of the subject is reflected in the emphasis on Panel studies which is bibliographically evident. The reader is referred also to an account of aerial gunnery which has been given by Saunders MacLane[a] in which he surveys rapidly, informally, and in a nontechnical manner the material covered in Part I of this report. Throughout the MacLane paper, critical and specific statements based on wide experience will be found of the way gunnery research was handled during World War II. The present account does not move in that direction. It attempts rather to present a certain approximation to the "state-of-the-art." The body of the report tries to indoctrinate new *technical* workers in aerial gunnery; the introductions and summaries of the several chapters may well interest a more general reader. The field of the account is limited by the omission of (1) engineering details of the various elements of fire control systems, (2) the maintenance of those systems, (3) the training required of aerial gunners, (4) the results of laboratory and airborne experimental programs, and (5) the analysis of gunnery in combat. In the positive direction, the account surveys underlying ballistic, deflection, and aerodynamic theory, and considers the ways in which fire control systems attempt to solve the problem put to them and the errors those systems make in that effort. Although this is a summary report, enough technical detail is supplied to make the report, in large measure, self-contained. In the selection, translation, and grouping of material from many sources, the author has performed a most difficult task with extraordi-

[a] *Aerial Gunnery Problems*, AMG-C Paper 491, Columbia University, August 31, 1945.

nary clarity and effectiveness. Because so many of the papers which are reviewed were written in reply to specific questions which needed quick answers, the job of presenting a unified picture of the work was particularly difficult. The account reflects the unique experience of its author, E. W. Paxson, with Army, Navy, NDRC, and British as well as German activities.

In Part I of this volume, the behavior of weapons and their control mechanisms is studied rather than the tactical employment and strategic consequences of the use of those weapons. In Part IV, a brief report is given of two studies also dealing with air-to-air warfare which came closer to having general tactical or strategic scope than did most other AMP studies. Groups at New Mexico under E. J. Workman, at Mount Wilson Observatory under W. S. Adams, and at Princeton under M. M. Flood carried on the extensive researches reported here which were concerned with the best tactical use of the B-29, while the Statistical Research Group at Columbia, whose director of research was W. Allen Wallis, was responsible for the study of alternative fighter plane armament. In addition to reporting on these two studies, Part IV gives a discussion of a general theory of air warfare and of some of the contributions which mathematics can make to the broad field of national defense. It was P. M. S. Blackett, a British pioneer in the field of operational research, who pointed out that it is the study of how and why weapons perform and how they may be improved that is amenable to the usual approach of the physical sciences, whereas the study of how tactical procedures may be improved and the determination of the costs in resources of war to modify strategic concepts require statistical and variational methods. In Part IV some indication is given by the Chief of the Applied Mathematics Panel of how certain activities of the Panel and of other agencies relate to a scheme for a broader analytical approach to the problems of air warfare and of warfare in general.

Part II is concerned with the sighting methods which are feasible for airborne rockets. Although an important part of the Panel's work in rocketry consisted of conferences on sighting methods and related problems with various groups and in solving problems connected with special sights, no account is here given of the special results obtained except for a brief mention in the introduction to Chapter 9 where bibliographical source material is indicated. The account presented in Part II is concerned only with that part of the Panel's work in the field which is deemed to be of importance for the future. No author is indicated for Part II, since the chapter represents almost wholly a digest prepared in the Panel office of work which originated with Hassler Whitney, who was in charge of the Panel's rocketry program. Whitney served as a member of the Applied Mathematics Group at Columbia; he not only integrated the work carried on at Columbia and at Northwestern for the Panel in the general field of fire control for airborne rockets, but maintained effective and continuous liaison with the work of Division 7 in this field and with the activities at many Army and Navy establishments, particularly the Naval Ordnance Test Station at Inyokern, the Lukas-Harold Corporation, the Dover Army Air Base, Wright Field Armament Laboratory, the Naval Bureau of Ordnance, and the British Air Commission.

The first chapter of Part III is concerned principally with an account of results obtained by the Panel as an outcome of various requests for the analysis of antiaircraft equipment. Most of this work was done either by the Applied Mathematics Group at Columbia or by the Columbia Statistical Research Group. The second chapter of Part III is concerned with fragmentation and damage studies. In this field the basic theory was developed by the British, who also obtained important experimental results concerning the fragmentation characteristics of shells. The Panel used British, Army, Navy, and OSRD reports as the source of its experimental information. Its own contribution was in developing analytical and computing procedures which were feasible in point of time and in applying these procedures to selected examples. The greater part of the work performed for the Panel in this field was carried out by the Statistical Research Group at Columbia where Milton Friedman, Associate Director of the Group, became an expert in the field and served as consultant to the many Army, Navy, and OSRD groups which had frequent occasion to seek his assistance. One major report in this field was prepared by the Applied Mathematics Group at Brown University.

Because so much of the work reported in this volume was concerned with equipment developed by Division 7, the reader interested in this subject will do well to consult the Summary Technical Report of Division 7. For a discussion of the characteristics and present stage of development of airborne radar fire control systems and related problems such as the

coordination of radar and computers, the reader should refer to *MARS*, Volume 2 of the Summary Technical Report of Division 14, Chapters 17 to 22.

It has been the aim of the authors of this volume to present the material in such a way that no prior knowledge of specific technical matters is presupposed on the part of the reader. The reader is expected to have a background in mathematics and physics ordinarily possessed by a person with a bachelor's degree in engineering.

The bibliographies to the various parts of this volume give some indication of the scope of the material which the authors have examined in the writing of this book. They are to be congratulated for having prepared accounts which, in spite of the great diversity in the work surveyed and the brief time available for the preparation of the report, should prove useful to future workers in the field.

MINA REES
Editor

CONTENTS

SUMMARY[a]

In this Summary Technical Report of the Applied Mathematics Panel, a résumé is given of the principal scientific accomplishments of the Panel from its beginning in 1943 until the conclusion of hostilities. The activities here reported cover a wide range, dealing as they do with studies undertaken at the request of each of the nineteen Divisions of NDRC and of many branches of the Army and Navy. For the purpose of this report, that portion of the Panel's work which deals with specific military problems has been divided into three parts: Volume 1, *Mathematical Studies Relating to Military Physical Research;* Volume 2, *Analytical Studies in Aerial Warfare;* and Volume 3, *Probability and Statistical Studies in Warfare Analysis.* In addition to reporting on specific military problems, Volume 1 also indicates directions in which certain of the theories of fluid dynamics have been extended under AMP auspices as an aid in the planning and interpretation of military experiments, and in understanding the operation of enemy weapons. These three volumes contain no account of the new developments in statistical methods which have already been partially reported in a published article[1] and a published book[2] on sequential analysis, nor of certain important new applications of statistical theory which grew out of the Panel's attempt to solve problems presented to it by the Services. These latter are reported in two published monographs, *Sampling Inspection* and *Techniques of Statistical Analysis* (published by McGraw-Hill), which have been prepared under Panel auspices and which form part of the Panel's report of its technical activities.

Most AMP studies were concerned with the improvement of the theoretical accuracy of equipment by suitable changes in design; or with the development of basic theory, particularly in the field of fluid dynamics; or with the best use of existing equipment, particularly in fields like bombing and the barrage use of rockets. Two studies carried out under AMP auspices come closer to having general tactical or strategic scope than do most of the other work. I have myself given an account of these two studies in Part IV of Volume 2, where I have also set forth some incomplete and preliminary ideas of what a general analytical theory of air warfare could and should comprise and some arguments for and against attempting to construct and use such a theory. I have there indicated how certain activities of the Applied Mathematics Panel and of other agencies relate to a scheme for a broad approach to the problems of air warfare and of warfare in general, and I have pointed out some of the contributions which mathematics can make to the field of national defense.

That part of the Panel's work which may be roughly described as classical applied mathematics is presented in Volume 1. Certain phases of this subject were developed under Panel auspices and adapted to problems of military interest, the principal emphasis being on problems of primary concern to the Navy.

In the early stages of the war, certain acoustic equipment employed in submarine detection by echo ranging used a "dome" — a streamlined convex shell filled with water or other liquid, such as oil. The presence of these domes caused interference with the directional pattern sent out from the projector, and in some of the equipment the disturbance was extremely serious. The Panel was asked to study the situation and to suggest changes in the domes which would minimize the disturbances. Practical conclusions were reached regarding desirable materials and design. It was found desirable for practical reasons to use thin shells reinforced by stiffening elements such as ribs and rods rather than to achieve strength by general thickness. Difficulties arising in direction finding due to annoying reflections were also analyzed, and suggestions were made for improving conditions, for example, by corrugations on the inner surface of the side walls of the domes. This dome study was one aspect of the work in wave propagation with which the Panel was concerned. There were others. For example, an investigation was made of the scattering of electromagnetic waves by spherical objects to assist in the analysis of smokes and fogs. A study of somewhat similar mathematical character (but dealing with electromagnetic disturbances rather than actual mechanical waves in a liquid) was undertaken at the request of the Fire Control Division (Division 7, NDRC), which had under development a predictor, the T-28, intended for use with the 40-mm gun. The computing mechanism used by this predictor included a sphere on which were placed electrical windings in such a way that the resulting field was one which corresponded to one simple dipole at the center of the sphere. Although the theoretical way in which the winding should be distributed on the surface of this sphere was well known, it was necessary as a practical matter to

[a] By Warren Weaver.

substitute a winding in which the turns were located in grooves on the sphere. The formulas resulting from the Panel's study of this problem form a basis for practical applications which include ammeters, galvanometers, and direction finders. This mathematical study was of critical importance for the fire control instrument in question, for without it, it was impossible to obtain useful accuracy in the spherical "electromagnetic resolver" which carried out the essential steps in the target predicting process.

The Panel's work in gas dynamics, mechanics, and underwater ballistics is also reported in this first volume. The Panel's work in *gas dynamics* was principally concerned with the theory of explosions in the air and under water, and with certain aspects of jet and rocket theory. New developments were made in the study of shock fronts, associated with violent disturbances of the sort which result from explosions. An interesting and significant aspect of the work was concerned with Mach phenomena which frequently play a practical role in determining the destructive effects of shocks. For example, the advantages of air-bursting large blast bombs were suggested by a consideration of Mach waves. A request from the Bureau of Aeronautics for assistance in the design of nozzles for jet motors to be used for assisted take-off gave rise to an extended study of gas flow in nozzles and supersonic gas jets. As a result, suggestions were made not only for the design of nozzles for jet-assisted take-off, but also for "perfect" exhaust nozzles and compressors (of use in supersonic wind tunnels) and for various instruments to aid in rocket development and experimentation. The jet propulsion studies were related to Army and Navy interest in intermittent jet motors of the V-1 type. Jet propulsion under water was also studied, with results which should prove useful as a guide to experiment in this field where experimentation has thus far not reached the stage where the theoretical results can be fully put to test.

The problems in *mechanics* fall under two general headings: (1) those involving the mechanics of particles and rigid bodies and (2) those involving the mechanics of a continuum. For example, a study in the second category sought possible explanations of the break-up in cylindrical powder grains in the 4½-in. rocket to explain difficulties which were being encountered at the Allegany Ballistics Laboratory, and an experimental program was outlined for the testing of the most probable theories. One of the most interesting of the mechanical studies concerned the so-called spring hammer box used by the U. S. Navy in acoustic mine warfare. The dependence of the operation of this device on various physical parameters (for example, the mass of the hammer) was analyzed with the aid of a simple mechanical model, and of an electrical analog. Another problem of this type studied the dynamics of the gun equilibrator, or balancing system, when an Army gun was mounted on board a ship. The pitching and rolling of the ship naturally introduced special difficulties.

In the section on underwater ballistics, the problems involved are classified according to the various phases in the motion of the projectile: the impact phase, the development of the cavity, and the underwater trajectory. During the impact phase, forces act which are important partly because of their possible effects on the nose structure and mechanism of the projectile, partly because of their influence in determining the projectile's subsequent motion. It is during the impact phase that the greatest deceleration occurs. The theoretical analysis involves, among many other considerations, the direction of entry (vertical or oblique), and the shape of the projectile. Save when the speed of a missile is slow, its entry is accompanied by the formation of a cavity which becomes sealed behind the projectile and accompanies it to a greater or less extent during its underwater motion, influencing that motion in an important way. The underwater trajectory itself presents problems of great complexity. Frequently, slight changes in values of the parameters which determine the motion will cause a complete change in the type of motion. A mathematical discrimination among the several types of motion is made, part of the distinction depending on such things as the position of the center of gravity of the missile, the ratio of its length to its diameter, its density, its radius of gyration, and the manner of its entry. Throughout this treatment, an attempt has been made to integrate into a single report the results which have been obtained by the many agencies concerned with the several phases of the problem and thus to assist the theoretical and experimental studies which must be carried forward in future attempts to understand this difficult array of problems.

Many of the studies reported in Volume 2, as well as those contained in Volume 3, involve probability considerations, a field which is notoriously tricky and within which "common sense" is often quite helpless. For example, what is the optimum mixture of armor-piercing and incendiary ammunition for the rear guns of a bomber? Specifications often designate such

mixtures as five AP to two incendiary (we are neglecting tracers here). Why? The somewhat striking, and by no means obvious, fact is that, given any fixed type of target, it is better to have either *all* AP or *all* incendiary, depending on the nature of the target. The justification for any other intermediate mixture should be based on knowledge of the relative probability of encountering different targets, certain of which would be more vulnerable to AP and others more vulnerable to incendiary. This conclusion was reached as an incidental result of a study which was concerned with alternative fighter-plane armament and which arose out of the enthusiasm of a few persons associated with the Panel for two papers attributable to L. B. C. Cunningham, Chief of the Air Warfare Analysis Section in England, and his associates. Another study concerned with the practical effectiveness of equipment grew out of a request to NDRC from Headquarters, AAF, asking for collaboration with the AAF "in determining the most effective tactical application of the B-29 airplane." The results of this study, obtained on the basis of large-scale experiments in New Mexico and small-scale optical experiments by the Mt. Wilson Observatory staff at Pasadena, were concerned principally with the defensive strength of single B-29's and of squadrons of B-29's against fighter attack and the effectiveness of fighters against B-29's. One indirect result of the optical studies was a set of moving pictures showing the fire power variation of formations as a fighter circles about them. Concerning such pictures the President of the Army Air Forces Board remarked that he "believed these motion pictures gave the best idea to air men as to the relative effect of fire power about a formation yet presented." Certain of these pictures were flown to the Marianas and viewed by General LeMay and by many gunnery officers at the front.

These two studies are reported in the last part of Volume 2. The first three parts of this volume report on special and detailed problems which arise when shots are fired against targets moving in the air or on the ground. The problem of shooting from an aircraft in motion against an enemy aircraft or against a ground target in motion and the problem of shooting from the ground or from a naval craft against an enemy aircraft all involve a number of considerations.

1. Whenever the target is in motion, its position at the instant of firing is different from its position at impact, if impact occurs. For an effective shot, the motion of the target during the time of flight of the bullet or rocket or shell must therefore be predicted, at least approximately. The special character of this problem for the special cases which have come under the Panel's study are discussed for air-to-air warfare in Part I, for rocket fire from the air in Part II, and for ground or ship based antiaircraft fire in Part III.

2. When one's own ship is in motion, the apparent motion of the target is affected.

3. There are oscillations in aim as the gunner attempts to point continuously at the target. These oscillations are greater in air-to-air and in ship-to-air than in ground-to-air gunnery because of the vibrations, rotations, and bumpy motions of one's own ship.

4. There is the effect of gravity on the bullet. In air-to-air gunnery, for the short ranges used in World War II, this was of minor importance, but for rocket fire it introduced very considerable complications.

5. The resistance of the air varies with the altitude. Thus, at 22,000 feet above sea level the air is half as dense as it is at sea level. This will affect the average speed of a bullet, hence its time of flight, and hence the prediction referred to above.

A large part of Volume 2 is devoted to problems connected with so-called flexible gunnery, i.e., with the aiming of those guns, carried on aircraft, which can be pointed in various directions with respect to the aircraft (as contrasted with fixed guns in the wings or nose, which are aimed only by movement of the aircraft). In January 1944, Brigadier General Robert W. Harper, AC/AS (Training), wrote in a letter to Dr. Vannevar Bush, Director of OSRD, that "the problems connected with flexible gunnery are probably the most critical being faced by the Air Forces to-day. It would be difficult to overstate the importance of this work or the urgency of the need; the defense of our bomber formations against fighter interception is a matter which demands increasing coordinated expert attention." This situation arose because of the inadequate training and inadequate deflection rules given to the gunners who had to handle ring sights in bombers. The "relative speed" and "apparent motion" rules currently taught were not thoroughly learned by the gunners and in many cases were by no means adequate when they were properly applied. There were well authenticated cases of gunners who "led" the attacking fighters in a direction exactly opposite to that of the true lead!

The immediate proposal contained in General Harper's letter was that the Applied Mathematics

Panel should recruit and train competent mathematicians who had the "versatility, practicality, and personal adaptability requisite for successful service in the field;" It was planned that these men, after two months' training in this country, would be assigned to the Operations Research Sections in the various theaters to devote their attention to aerial flexible gunnery problems. The Panel was in a position to carry out this program because it had already been drawn into studies of rules for flexible gunnery training and because it had access to many of the ablest young mathematicians in the country. The assignment was completed promptly, and, as a partial result of this undertaking, the Panel found itself even more closely in touch with the Operations Analysis Division of the AAF (with which it had already established cordial working relations) and with the AAF Central School for Flexible Gunnery. Around this interest and the interest of the Army, the Navy, Division 7, and Division 14 in the improvement in the effectiveness of guns as well as gunnery, grew up a very considerable body of knowledge and experience which is reported in Part I of Volume 2. Here an attempt is made to bring together into a single account the state of the art of air-to-air gunnery, not only as that has been affected by the work of the Applied Mathematics Panel, but as it has reflected the activities of agencies in this country and abroad. The topics discussed are:

1. The motion of a projectile from an airborne gun, constituting that branch of exterior ballistics which is called *aeroballistics*.

2. A mathematical *theory of deflection shooting* considered first for the case of a target moving at constant speed on a straight line which lies in a plane with the gun-mount velocity vector; second, for a target which moves in a curved path; and third, for the case where mount and target move in arbitrary space paths.

3. *Pursuit curve theory.* Pursuit curves were important in World War II, since the standard fighter employed a heavy battery of guns so fixed in the aircraft as to fire sensibly in the direction of flight. Thus it was necessary to fly on such a correctly banked turn that a correct and changing aiming allowance was continuously made. This pursuit curve theory is also of importance in the study of guided missiles which continuously change direction under radio, acoustic, or optical guidance unwillingly supplied by the target.

4. The design and characteristics of *own-speed sights* which were introduced as devices designed for use against the special case of pursuit curve attack on a defending bomber. Simple charts which might be used in the air are given, based on optimum rules for determining deflection against an aerodynamic pursuit curve.

5. *Lead computing sights* which do not assume that the fighter is coming in on a pursuit curve but which basically assume that the target's track relative to the gun mount is essentially straight over the time of flight of the bullet. The mechanical sights of the Sperry series are considered in some detail.

6. The basic theory of a *central station fire control system.*

7. The analytical aspects of experimental programs for *testing airborne fire control equipment.* It is recognized that field tests, laboratory tests, and theoretical analyses all have an important place in such a program. Instrumentation for tests, reduction of data, measures of effectiveness, and optimum dispersion are discussed.

8. *New developments*, such as stabilization and the use of radar.

The second part of Volume 2 is devoted largely to a presentation of the results obtained by the Panel in a study intended to determine what sighting methods are feasible for airborne rockets. The essential problems involved in this question have to do with ballistic formulas, attack angle and skid, the effect of wind and target motion, how these various factors affect each proposed sighting method, and how tracking affects and is affected by them.

In Part III of Volume 2 certain special studies of antiaircraft equipment which were made under AMP auspices are discussed, and a report is given of the *flak analysis* and other *fragmentation and damage studies* carried on by the Panel. This report is concerned with some mathematical problems which arise in attempts to estimate the probability of damage to an aircraft or group of aircraft from one or many shots from heavy antiaircraft guns. Related problems arise in air-to-air bombing and in air-to-air or ground-to-air rocket fire, but the major part of the mathematical analysis so far performed has been devoted to problems of flak risk. The emphasis in the discussion is on the description of a method for treating problems of risk, since specific numerical conclusions are likely to become obsolete before further need for them arises, while the techniques by which

the results were obtained will be useful as long as weapons which destroy by means of flying fragments are in use. The original experimental information on which the Panel computations were based came from a variety of sources, principally Army, Navy, OSRD, and British reports. The Panel's chief contribution was the development of computational techniques which could be carried through before the project became obsolete, the selection of pertinent examples, and the applications of the computational techniques to the selected examples. Certain applications of the underlying theory to time-fused and proximity-fused shells, and to proximity-fused rockets are here reported.

Another major field of effort in the work of the Panel is that of *Mathematical Statistics*, reported in Volume 3. A remarkably wide variety of probability and statistical investigations was carried out by the Panel. These investigations ranged from the development of sampling inspection plans in connection with procurement of military material to extensive statistical analyses of combat data. Of the Panel's 194 studies, 53 related to problems in probability and statistical analysis.

The work of the Panel in mathematical statistics can be grouped into the following major categories:

1. *Bombing accuracy* research.

2. *Development of statistical methods* in inspection, research, and development work.

3. Development of new *fire effect tables* and diagrams for the Navy.

4. *Miscellaneous studies* relating to spread angles for *torpedo salvos*, *lead angles for aerial torpedo* attacks against maneuvering ships, *land mine clearance*, performance of *heat-homing devices*, *search problems*, verification of *weather forecasting* for military purposes, procedures for testing *sensitivity of explosives*, distribution of *Japanese balloon landings*, etc.

Of these four main categories of work, category 1 required by far the greatest amount of energy. This activity had its beginning in a fairly small study undertaken for the Armament Laboratory, Wright Field, on the design of a computer for determining the optimum spacing of bombs in a train of bombs dropped from a bomber in attacking a given target under specified conditions. The study was started in 1942 under Division 7, NDRC, and was transferred to the Panel when the Panel was organized. In pursuing this study the group working on it came in contact with individuals in more than a dozen Army, Navy, and NDRC groups interested in bombing accuracy problems. As the war progressed, an increasing number of requests came from these groups for studies of all kinds of accuracy and coverage problems arising in train bombing, area bombing, pattern bombing, guided-missile bombing, incendiary bombing, and so on. By the end of the war the work in this field had grown to the point where the major effort of three Panel research groups was being spent on nineteen studies dealing with probability and statistical aspects of bombing problems.

The methods and results developed in category 2 are of much broader interest than that associated with their wartime applications. During the war, it was recognized by the Services that the statistical techniques which were developed by the Panel for Army and Navy use, on the basis of the new theory of sequential analysis, if made generally available to industry, would improve the quality of products produced for the Services. In March 1945, the Quartermaster General wrote to the War Department liaison officer for NDRC a letter containing the following statement:

> "By making this information available to Quartermaster contractors on an unclassified basis, the material can be widely used by these contractors in their own process control and the more process quality control contractors use, the higher quality the Quartermaster Corps can be assured of obtaining from its contractors. For, by and large, the basic cause of poor quality is the inability of the manufacturer to realize when his process is falling down until he has made a considerable quantity of defective items. . . . With thousands of contractors producing approximately billions of dollars worth of equipment each year, even a 1% reduction in defective merchandise would result in a great saving to the Government. Based on our experience with sequential sampling in the past year, it is the considered opinion of this office that savings of this magnitude can be made through wide dissemination of sequential sampling procedures."

On the basis of this and similar requests, the Panel's work on sequential analysis was declassified, and the reports mentioned above were published. The Quartermaster Corps reported in October 1945 that at least 6,000 separate installations of sequential sampling plans had been made and that in the few months prior to the end of the war new installations were being made at the rate of 500 per month. The maximum number of plans in operation simultaneously was nearly 4,000.

Thus extensive use was made by the Army of sequential analysis as a basis for sampling inspection. It was at the request of several Navy bureaus that

the Panel undertook to assemble a manual setting forth procedures to be used not only with sequential sampling but also with single and double sampling plans. As an extension and expansion of this manual, the Panel undertook the preparation of its monograph, *Sampling Inspection*. The monograph, *Techniques of Statistical Analysis*, presents a variety of statistical methods which have been developed, or adapted from more general methods, for dealing with various statistical problems which have arisen in connection with research and development work.

The work done in category 3 was of highly specialized long-range interest to the Office of the Commander in Chief of the U. S. Fleet. After the work had been carried forward under the direction of the Panel for nearly two years, arrangements were made to transfer and continue the work under a contract, effective June 1, 1945, between the Navy and Princeton University. During the time this work was under the Panel's direction, a series of nine basic reports was submitted to the Navy. None of this work, which was only partially completed under the direction of the Panel, is reported upon in the Panel's Summary Technical Report.

Certain of the studies in category 4 are of such limited interest that it has been considered neither appropriate nor worth while to report upon them here. Accounts are given of the work which relates to torpedoes, land mine clearance, and the performance of heat-homing devices.

An important adjunct of the probability and statistical work of the Panel was a statistical consulting service for various Army, Navy, and NDRC agencies. Although some of this consulting was done in connection with formal AMP studies and projects in such a way that the results are adequately reported in original Panel reports or the Panel's Summary Technical Report, a large fraction of it was informal and the results of it are to be found in reports and memoranda of many agencies, particularly Divisions 2, 5, 8, and 11 of NDRC; Joint Army-Navy Target Group, Army Air Forces Board; Proving Ground Command, Eglin Field, AAF; Operational Analysis Division, Twentieth Air Force, AAF; Combat Analysis Unit, Statistical Control, AAF; Office of the Quartermaster General; Navy Air Intelligence Group; Navy Operational Research Group; and the Guided Missile Committee of the Joint Chiefs of Staff.

Men from several of the Panel's research groups acted as consultants to these various agencies for periods ranging from two months to two years. In my opinion some of the most useful service which the Panel was able to render came about through the work of these men in their capacities as consultants; the effectiveness of this work increased constantly until the end of the war. The work of these men varied widely: assistance in setting up sampling inspection plans for procurement of matériel, helping in the introduction of a quality control system in rocket production, working on designs of experiments for toxic gas bombing, testing controlled missiles, cooperation in the preparation of an incendiary manual, and dozens of other projects.

I cannot leave the topic of mathematical statistics without emphasizing the powerful yet severely practical role which this relatively young branch of applied mathematics has played in the work of the Panel. The tools of the probabilitist and statistician have been applied to an almost unbelievably wide array of problems. Probability analysis played a fundamental part in a priori investigation of various kinds of weapons and tactics studied by the Panel. As the war progressed and these weapons and tactics were tested at the proving ground and tried out in combat, the analysis of the observational data became primarily statistical. The work of the Panel surely indicates that the Army and Navy will do well in their research, development, and testing of weapons and tactics to see to it that the tools of the mathematical statistician are not overlooked.

PART I

ANALYTICAL ASPECTS OF AERIAL GUNNERY

Chapter 1

AEROBALLISTICS

1.1 INTRODUCTION

THE DISCUSSION of the motion of projectiles fired from *airborne* guns constitutes a modern branch of exterior ballistics which may quite properly be called *aeroballistics*. Certain essential points of difference between classical exterior ballistics and aeroballistics are considered below.

1. The ranges employed in aeroballistics have been short compared with the maximum effective range of the projectile. During World War II these ranges were, in general, no greater than 1,000 yd. For such ranges the effects of changes in density, temperature, and wind along the trajectory are negligible. Bullet drift may also be neglected. Since the projectile velocity does not become subsonic, the resistance encountered by the bullet is proportional to the three-halves power of the speed, and ballistic formulas for the trajectory may therefore be written out in closed and compact form. For short ranges, the trajectory is relatively flat, so that consideration of superelevation to allow for gravity drop is of minor importance.

It can be pointed out that a moving target may be hit at long range [210] using a highly arched trajectory, as well as at short range with a flat trajectory. But the difficulty in predicting the position of the target over a long time of flight of the projectile and the inaccuracy in positioning an airborne gun rule out long-range fire. In addition, the remaining velocity would probably be too low to achieve effective damage.

2. The gun platform can move at speeds up to one-fifth of that of the projectile, and the direction of fire may be at any angles of azimuth and elevation with respect to the direction of motion of that platform. The bullet, therefore, has an initial velocity of departure which is the vector resultant of the velocity imparted by the propellant, acting along the bore axis, and of the velocity supplied by the moving gun mount, acting in the instantaneous direction of motion. In aeroballistics, then, the initial speed varies materially whereas in classical ballistics, a mean constant muzzle velocity may be used because mount motion is negligible.

3. The gun-mounting aircraft may operate at any altitude from sea level to 40,000 ft. Consequently the air density at the point of fire has an important effect on the time of flight and the other ballistic quantities. In classical ballistics the point of fire is usually at or near sea level, and variations in standard conditions of pressure and temperature at that point are important because the ranges are long.

4. In classical ballistics the axis of a projectile makes a small angle with the bore axis which is also the direction of departure. The angle between the axis and the departure direction is called the *initial yaw*. In fire from airborne guns the initial yaw is frequently large because of the material angle between the resultant direction of departure and the bore axis of the gun. Aerodynamic and gyroscopic effects [211] are accentuated. The high cross-wind force leads to the phenomenon of windage jump via precession,[140, 143] and the increased drag influences the time of flight slightly.

5. For air-to-air fire, absolute wind — the motion of the air mass with respect to the ground — has no effect since both the aircraft and the projectile are in the same air mass. The effect of *relative* wind is, however, most important. Let a bullet be fired sidewise from an aircraft and kept in view from the gun position. It seems to lag farther and farther behind and *appears* to move in a path that curves rearward. Actually, it is moving in a straight line with respect to the air mass but fails to keep pace with the firing aircraft since it is decelerating because of air resistance.

1.2 TIME OF FLIGHT

1.2.1 Reference Systems

In aeroballistics the bullet is located by one or the other of two reference systems. The first system is fixed in the air mass with origin at the gun's position

at the instant of fire. As indicated in Figure 1 the position of the bullet at any instant is specified by the two Siacci coordinates P and Q. P is measured along the line of departure to a point directly above the bullet, and Q is measured from this point vertically downward to the bullet. The range covered by the bullet in the air mass is approximately P. The orientation of P can be given by an azimuth angle A,

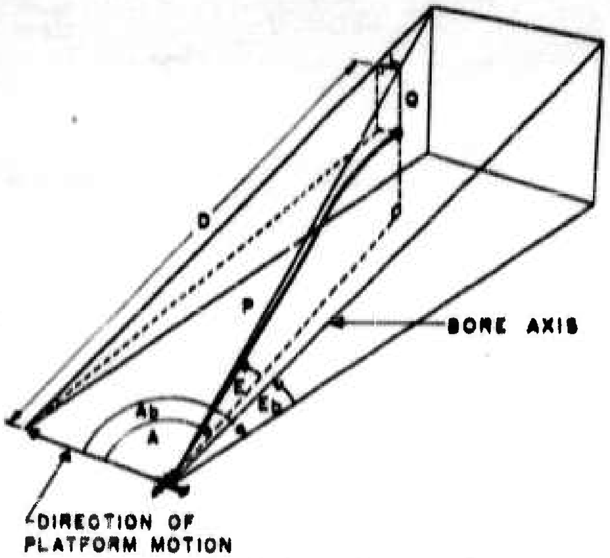

FIGURE 1. Aeroballistic reference systems.

measured in a horizontal plane clockwise from the direction of motion, and by an elevation angle E measured positively upward in a vertical plane. (It is tacitly assumed that the aircraft is flying straight and level.) In the second system, the origin (the point of reference) moves with the velocity of the gun platform at the instant of fire. The bullet's position at any instant is specified by a range D, which is the distance between gun and bullet at that time, and by the lateral and vertical deviations from the bore axis L and Q, called the ballistic deflections. In practice, D is indistinguishable from its projection on the bore axis, and it is this projection (which is the D in the figure) that is called *future range* in ballistic tables. In this latter system, the bore axis is specified by the azimuth and elevation angles A_b and E_b.

1.2.2 Differential Equations of the Trajectory

The basic ballistic equations can be made out most elegantly by the methods of vector analysis.[18a] In Figure 1 the lengths P and Q may be replaced by vectors[a] $\mathbf{P}$ and $\mathbf{Q}$, so that if a vector $\mathbf{R}$ is introduced connecting the origin to the projectile,

$$\mathbf{R} = \mathbf{P} + \mathbf{Q},$$

and

$$\dot{R} = u,$$

where u is the instantaneous speed of the bullet. Assuming that the bullet's axis coincides with the tangent to the trajectory at each instant, i.e., assuming zero yaw, the drag on the bullet caused by air resistance is given by

$$m\frac{\rho_a}{c_5}K_D u^2,$$

where m = mass of the bullet,

ρ_a = air density,

c_5 = ballistic coefficient of the Type 5 projectile ($c_5 = i\frac{m}{d^2}$, where i depends on the shape and d is the diameter),

K_D = drag coefficient (a function of the ratio of u to the speed of sound).

The motion of the bullet is governed, then, by the vector equation

$$\ddot{\mathbf{R}} = -\frac{\rho_a}{c_5}K_D u\dot{\mathbf{R}} + \mathbf{g},$$

where $\mathbf{g}$ is the acceleration of gravity. The components of this equation are the basic equations of the trajectory,

$$\ddot{P} = -\frac{\rho_a}{c_5}K_D u\dot{P} \qquad \ddot{Q} = -\frac{\rho_a}{c_5}K_D u\dot{Q} + g. \tag{1}$$

1.2.3 Solution of the Equations in the Siacci System

To integrate these equations, the Siacci approximation is made as a first step. This approximation replaces u by $\dot{P}$, which is satisfactory for short times of flight. Next, experiments show[10] that K_D is given very closely by $ku^{-1/2}$ for velocities between 1,050 and 2,050 fps. (The expression for the drag then depends on the three-halves power of u.) Using these two ideas, equations (1) may be written

$$\ddot{P} = -\frac{\rho}{c_5}k^*\dot{P}^{3/2} \qquad \ddot{Q} = -\frac{\rho}{c_5}k^*\dot{P}^{1/2}\dot{Q} + g, \tag{2}$$

[a] In the text, a vector is denoted by bold face type and the magnitude of the vector by light-faced italic type. In the figures, a vector is denoted by underlining and deletion of the underline gives the magnitude of the vector.

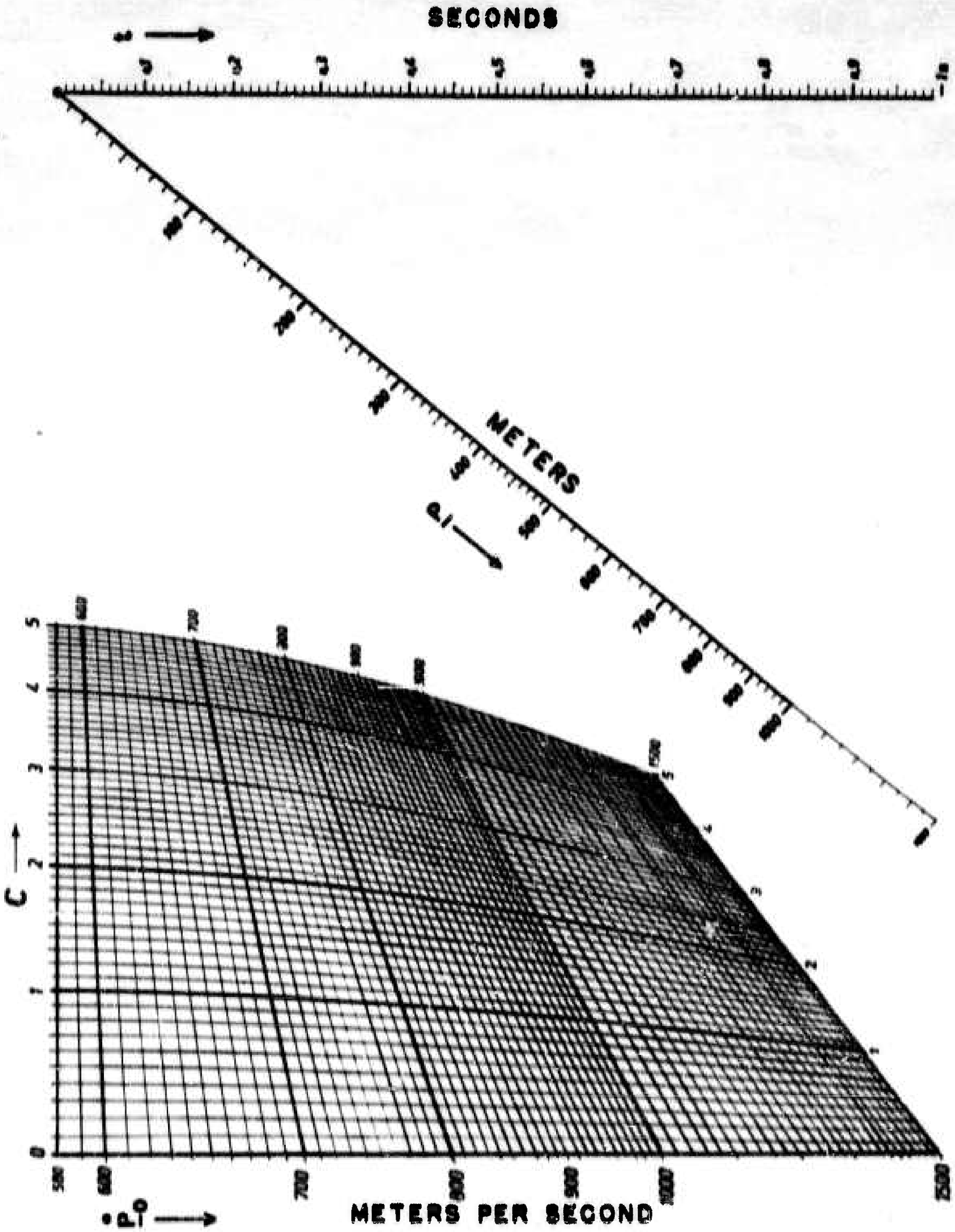

FIGURE 2. Time-of-flight nomogram.

where, now, ρ is the air density relative to a standard ballistic density of 0.07513 lb per cu ft. Equations (2) are immediately integrable and yield

$$\frac{1}{t} = \frac{P_0}{P} - \frac{\rho k^*}{2c_b}\sqrt{P_0} \tag{3}$$

$$Q = \frac{1}{2}gt^2\left[1 - \frac{\rho k^*}{3c_b}\frac{P}{\sqrt{P_0}}\right], \tag{4}$$

where P_0 is the velocity of departure. (Notationally, $\dot{P}_0 = u_0$.) The value of k^* depends on the unit of length employed. If P is in thousands of feet, $k^* = 0.118$; if P is in feet per second, $k^* = 0.00372$; and if P is in yards, $k^* = 0.00646$.

The interpretation of the structure of equation (3) is interesting. In vacuum $\rho = 0$ and as a result the initial velocity persists in the P direction. In air, the second term on the right of equation (3) is a constant, but the reciprocal character of the other two terms of the equation means that the constant has little effect for small t and P. For a sufficiently large value of P the right-hand side becomes zero, but this fact cannot be used to establish maximum range since the speed would fall below the limit for which the three-halves power law is valid. However, this value of P determines a vertical asymptote for the hyperbolic trajectory asserted by equation (3). In fact it is possible to deduce[212] equation (3) by starting with the bilinear form

$$t = \frac{a + bP}{c + dP}$$

and using the Didion-Bernoulli solution of the ballistic problem.[213] The details will not be given. As a final remark, in connection with equation (3), note that the remaining velocity of the projectile at any instant can be obtained[20] by computing dP/dt. This velocity is needed in calculations of impact energy.

The derivation of equation (3) depends explicitly on the three-halves power law. For projectiles of velocity lower than 1,650 fps the more general Siacci procedure must be used.[18d] For example, a frangible projectile has been introduced for training purposes which shatters upon impact.[18] To avoid damage its velocity must be kept low (below 1,500 fps). For the caliber 0.30 Frangible Ball T44 projectile the Siacci functions have been computed,[189] so that trajectories may be deduced.

Figure 2 is a nomogram for the computation of the time of flight t. It uses the continental ballistic coefficient C which is connected to the quantities in equation (3) by the expression

$$C = \frac{0.070\rho}{c_b}.$$

Typical values of c_b are given by Table 1. Despite the apparent accuracy with which the above values of c_b are stated, it should be appreciated that this

TABLE 1. Constants for typical ammunition.

Country	Type	Caliber	c_b	Muzzle velocity (fps)
USA[18]	API M8	0.50 in.	0.439	2,870
USA[18]	AP M2	0.50 in.	0.458	2,700
Germany[19]	API, MG151	20 mm	0.204	2,208
Germany[19]	HE, MG151	20 mm	0.180	2,971
Germany[19]	HEIT, MG131	13 mm	0.243	2,348
Germany[19]	APT, MG17	7.92 mm	0.345	2,587
Japan[19]		20 mm	0.280	1,964
Japan[19]		7.7 mm	0.234	2,420

coefficient varies somewhat from manufacturer to manufacturer and even from lot to lot. As it is determined by experimental firings, it also depends on the conditions of that firing.

1.2.4 Time of Flight in the Relative System

The discussion of time of flight may be concluded by giving formulas appropriate to the moving coordinate system described in Section 1.2.1. This is the natural system to use in airborne gunnery. (The Siacci derivation has the technical advantage of reducing the problem to the classical case with a properly chosen velocity of departure.) In the relative system the bore axis may be specified by the angle θ which it makes with the direction of motion. Evidently

$$\cos\theta = \cos A_b \cos E_b. \tag{5}$$

The speed of departure is given closely by

$$\dot{P}_0 = v_0 + v_a \cos\theta,$$

where v_0 is the muzzle velocity imparted by the propellant and v_a is the *true airspeed* of the firing aircraft. The expression

$$D = P - v_a t \cos\theta$$

is also a good approximation. With the aid of these expressions, equation (3) may be written in the form

$$pv_a \cos\theta\, t^2 - (v_0 - pD)t + D = 0,$$

where

$$p = \frac{\rho k^*}{2c_0}\sqrt{v_0}\sqrt{1 + \frac{v_G}{v_0}\cos\theta}.$$

If only terms of the first order are preserved in the expansion of the smaller of the two roots of the quadratic, the value

$$t = \frac{D}{v_0 - pD}\left[1 + \frac{pDv_G\cos\theta}{(v_0 - pD)^2}\right] \qquad (6)$$

is obtained.[10] More refined work would employ two terms of the expansion and would correct D by $Q \sin E_b$ for fire at high elevation.

1.3 BALLISTIC DEFLECTIONS

1.3.1 Angle Subtended by Gravity Drop

Figure 3 shows the trajectory relative to the moving gun. The displacement of the trajectory from the extended bore axis is called the ballistic deflection. This total deflection is decomposed into a lateral deflection W measured as a great circle arc in the plane

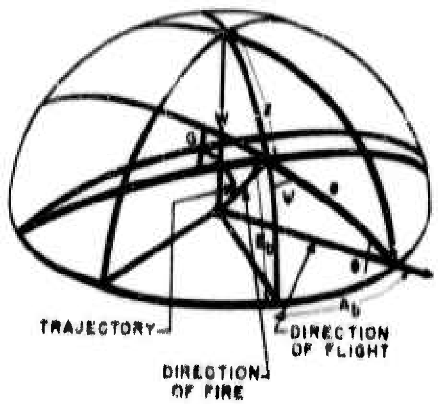

FIGURE 3. Ballistic deflections.

determined by the bore axis and the direction of motion, and into a vertical deflection G measured in a vertical plane through the projectile. By Figure 1 it is seen that the angle subtended by G at the gun is

$$G = 1000\frac{Q\cos E_b}{D},$$

in milliradians.[b] Substituting Q from equation (4) (and investigating numerically the approximations for P and P_0 from Section 1.2.4) leads to the value

$$G = \frac{1}{2}gt^2\left(\frac{1000}{D} - \frac{11\rho k^*}{c_b}\right)\cos E_b, \qquad (7)$$

which is highly accurate for the ranges and speeds of tactical importance.

1.3.2 Trail Angle

An expression for the angle W subtended at the gun by the lateral ballistic deflection may be deduced. For *beam* fire, in the absence of gravity, a gun mount in uniform motion is ahead of the bullet in time t by the distance

$$v_G\left(t - \frac{P}{P_0}\right).$$

This is the distance L in Figure 1. The expression for this distance arises upon noting that in the absence of air resistance the time of flight would be P/P_0 and the gun mount would remain abeam of the bullet.[10a] To obtain the angle subtended by W for nonbeam fire, L must be foreshortened by $\sin\theta$, where θ is defined by equation (5), and divided by D. These operations give

$$W = 1000\frac{v_G}{D}\left(t - \frac{P}{P_0}\right)\sin\theta,$$

in milliradians. With the aid of equation (3) and the approximation $P/P_0 = D/v_0$, W can be brought to the form

$$W = 500\frac{k^*\rho v_G D\sin\theta}{c_0 \cdot E}, \qquad (8)$$

where

$$E = \sqrt{v_0}\left(v_0\frac{\sqrt{v_0}}{\sqrt{P_0}} - \frac{\rho k^* D\sqrt{v_0}}{2c_0}\right).$$

In practice, a constant value is assigned to E by supposing that for all-around fire, $v_0 = P_0$, and by taking an average ρ of, say, 0.5 and an average D of, say, 500 yd. In other words, the main effect of the variables ρ, v_G, and D shows in the numerator of equation (8).

1.3.3 Interpretation and Calculation of Deflections

The trajectory displacement angles W and G will always represent corrections to the major aiming allowance (which allows for target motion *relative* to the gun mount). In fact, failing any relative target

[b] 1,000 milliradians = 1 radian. In aerial gunnery the approximations $\tan\alpha = \alpha$ and $\sin\alpha = \alpha$ are usually made without comment because of the small size of deflection angles. (Mils differ slightly from milliradians. 6,400 mils = 360°. Hence 1,019 mils = 1,000 milliradians.)

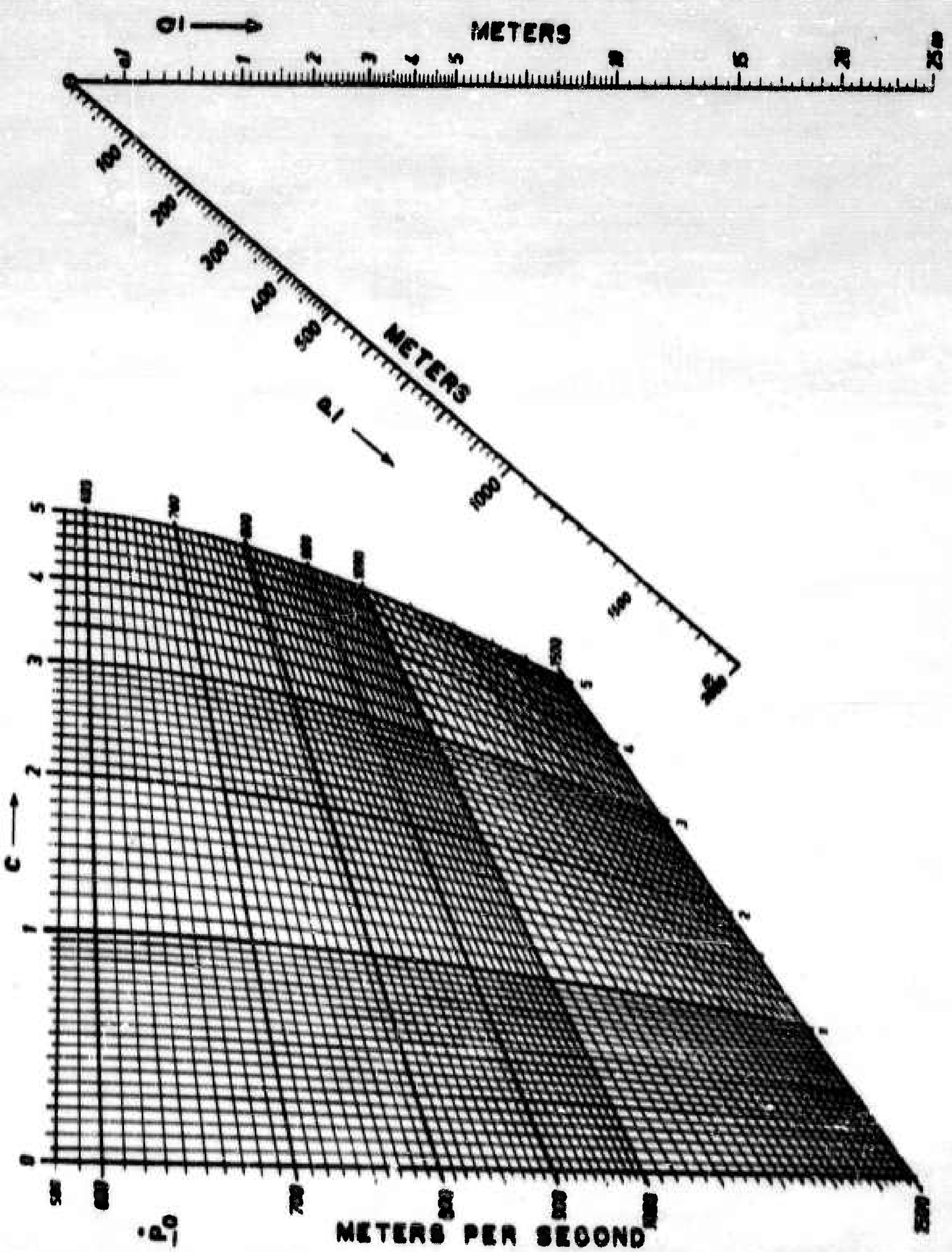

FIGURE 4. Nomogram for trajectory.

motion, they are themselves the aiming allowances or leads. For, if a target is pacing a bomber, i.e., maintaining the same course and speed but not necessarily the same altitude, then the target does not move as seen from behind the gun. Nevertheless, the bore axis must lead the target by the proper values of $\bar{W}$ and $\bar{G}$ to assure a hit.

In some cases, such as in experimental studies of gunsights, the values of the ballistic corrections must frequently be obtained on a mass-production basis. It is then desirable to construct special charts, called *dafographs*,[110] for each of which altitude, speed, muzzle velocity, and ballistic coefficient are fixed. These are specialized tools. More generally, complete trajectories can be constructed with the aid of a nomogram such as that given in Figure 4, and $\bar{W}$ and $\bar{G}$ can be calculated by the methods used in the above derivations. If such nomograms are carefully constructed to a suitable scale, they may well replace tables.[148] The advantage lies in the flexibility — c_0 and v_0 are not restricted to special values. The disadvantage is that auxiliary calculations must be made.

1.4 THE MOTION IN THE SMALL OF THE PROJECTILE

1.4.1 Reasons for the Discussion

The treatment of the trajectory in Sections 1.2 and 1.3 was macroscopic in that it described the general character of the motion, while neglecting certain details. It was assumed that as a projectile left the muzzle of an airborne gun it immediately took up the resultant direction dictated by mount velocity and propellant velocity and then proceeded on a smooth, softly arching curve through the air mass. This description and the resulting formulas have proved quite adequate for precise investigations. Nevertheless, the actual projectile has certain aerodynamic and gyroscopic properties which were neglected above but which may be briefly exposed. The two reasons for such an exposition are: one must be certain that the mean trajectory treatment above neglects no important effects; and in certain new applications such behavior in the small may become significant. As an example of the first point, windage jump has been calculated and found to be so insignificant for the caliber 0.50 projectile that the effect has been dismissed from practical considerations. In support of the second point suppose that a large caliber gun fires upward at a low velocity — chosen low to minimize recoil.[214] The initial yaw is very large (45°) and the question of its damping is important if contact fuses are to function.

1.4.2 General Features of the Motion and Damping of Yaw

The microscopic motion of a bullet involves three effects (1) a vibration of the center of mass with respect to a mean trajectory, (2) a precessional motion of the axis with respect to the center of mass, and (3) a drift of the center of mass to the right of the vertical plane of departure. All three motions occur because of the yaw, which is the angle at any instant between the axis of the bullet and the direction of motion of the center of mass of the bullet. More specifically, the first motion is caused by a cross-wind force which is analogous to the lift on an airplane wing. The second motion is attributed to the drag which is, again, similar to the drag on an airplane wing. The third motion is due to gravity and the curvature of the trajectory. Yaw, cross-wind force, and drag are shown in Figure 5.

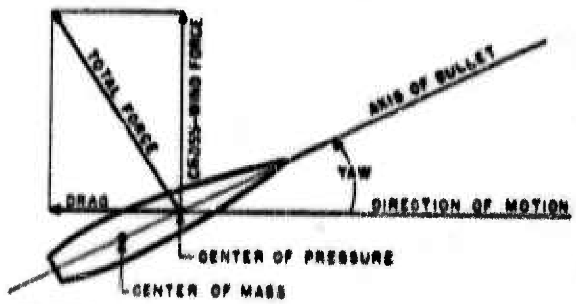

FIGURE 5. Forces acting on a projectile. (Courtesy of John Wiley and Sons.)

Before explaining the motions described above, the nature of the bullet as a potent gyroscope may be discussed. This potency is attributable to the high rate of spin, which is found by dividing the muzzle velocity by the twist (number of feet of barrel for one turn of the rifling). (For the caliber 0.50 AP M2 the spin is 2,100 revolutions per sec.) Over the short ranges of aeroballistics it may be supposed that the spin rate does not decrease but keeps its muzzle value. Figure 5 shows that the bullet resembles a top with a point of support at the center of mass and with the weight of the top replaced by the drag. By analogy with the top, a joint precessional-nutational motion of the axis is to be expected. This behavior is

conveniently pictorialized by giving the pattern traced by the tip of the bullet as an epicyclic motion associated with two circles (as shown in Figure 6).

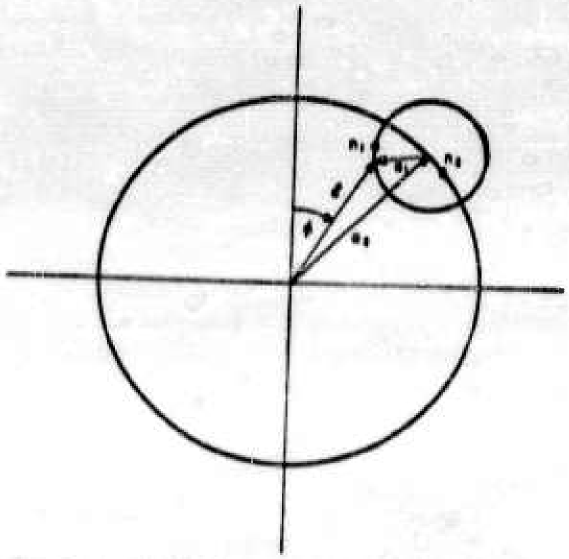

Figure 6. Motion of tip of bullet. (Aberdeen Proving Ground diagram.)

With the notation of that figure, the instantaneous yaw is given by

$$\delta^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos (n_1 - n_2)t,$$

where a_1 is the amplitude of the nutation (of rate n_1), and a_2 is the amplitude of the precession (of rate n_2). Both circles decrease in radius during the time of flight. This damping of the yaw is equivalent to the self-erection of a top. The minimum yaw is $a_1 - a_2$, and, since experiments show that — for calibers up to 37 mm at least — if the minimum yaw is initially zero it remains zero, it follows that the precessional and nutational damping rates are equal, considering that the maximum yaw, $a_1 + a_2$, was not zero. The damping factor, which multiplies the amplitude, may be written in the form

$$(1 - S^{-1})^{-\frac{1}{2}} e^{-\rho_a \alpha t},$$

where S is a stability factor,[14a] depending essentially on the moments of inertia and the spin of the projectile, ρ_a is the air density, and α is a constant to be determined experimentally.

1.4.3 Windage Jump

It is clear from the above discussion that the plane of the yaw angle rotates. (This is the plane containing the axis of the bullet and the tangent to the trajectory.) Since the cross-wind force always acts radially outward in this plane it may be expected that the center of mass would be pulled around in a helical path. It is not so evident that this helical path will suffer an overall angular displacement (windage jump). The following discussion of the phenomenon is heuristic rather than rigorous.

Point an airborne gun horizontally to starboard and fire. Then the initial yaw δ_0 is as indicated in Figure 7. Suppose, contrary to fact,[a] that the nutational amplitude is zero, that there is no damping of the precessional amplitude, that the effect of gravity may be neglected, and that the bullet does not slow down. By aerodynamic analogy, the cross-wind force may be taken proportional to the yaw, which is equivalent to the angle of attack of a wing. Relative to the line of departure (Figure 7) measure x horizontally perpendicular to that line and positive to the right, and measure y vertically perpendicular to that line and positive downward. The coordinates of the

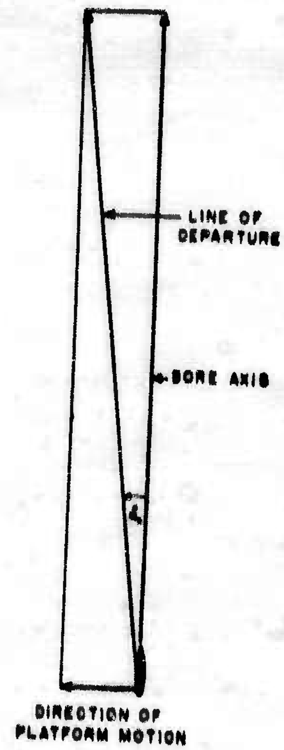

Figure 7. Initial yaw.

[a] The essential physical behavior is not altered by the assumptions.

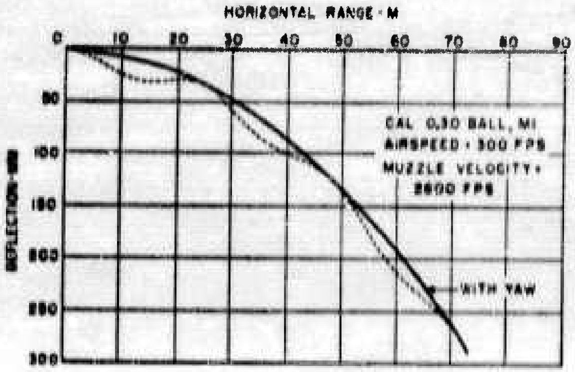

FIGURE 8. Horizontal projection of trajectory near origin. (Aberdeen Proving Ground diagram.)

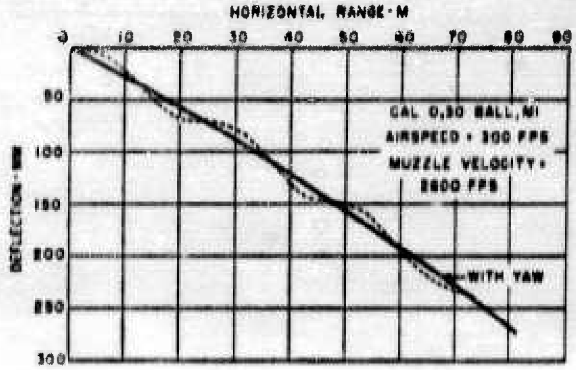

FIGURE 9. Vertical projection of trajectory near origin. (Aberdeen Proving Ground diagram.)

center of mass of the bullet are then (x,y) with respect to the departure line. If the precessional period is n^{-1}, the cross-wind force has as instantaneous components in the x and y directions $k\delta \cos nt$ and $k\delta \sin nt$ respectively. The equations of motion of the center of mass of the bullet are

$$m\frac{d^2x}{dt^2} = k\delta \cos nt$$

$$m\frac{d^2y}{dt^2} = k\delta \sin nt.$$

The quantities k, δ, and n are constant by assumption. Hence an integration that uses the initial values $x = y = \dot{x} = \dot{y} = 0$ gives

$$x = \frac{k\delta}{mn^2}(1 - \cos nt) \qquad y = \frac{k\delta}{mn}\left(t - \frac{1}{n}\sin nt\right).$$

From the form of x, it is evident that the horizontal projection of the helix [d] which approximates the motion is *tangent* to the line of departure and lies to the right of this line as viewed from the gun. More significantly, the form of y shows that the whole trajectory has suffered a downward displacement through an angle of $k\delta/mnP_0$ radians. This is the windage jump. The componential motions are drawn after exact calculations in Figures 8 and 9.

Evident modification of the above analysis for port, upward, and downward fire gives the mnemonic: if the thumb and first two fingers of the right hand are extended at right angles to each other, and if the forefinger is pointed in the direction of mount motion and the second finger in the direction of fire, then the thumb gives the direction of trajectory displacement.

It is clear that *windage jump* is rather a misnomer. The phenomenon is due entirely to initial yaw and the displacement is not a jump. (In particular the random initial yaw of bullets at the muzzle of a ground gun due to bore clearance leads to random displacement and so builds up a dispersion pattern.[222])

The windage jump, as remarked earlier, is small. It may be calculated by the formula [22a]

$$J = \frac{b}{v_0}\frac{v_a}{P_0}\sin\theta, \qquad (9)$$

where b depends on the physical constants of the projectile. Typically: b/v_0 is 17 milliradians for caliber 0.50 AP M2, and is 21 milliradians for caliber 0.50 AP M8.

1.4.4 Drift

To conclude the discussion of the three motions listed in Section 1.4.2, *drift* will be considered here only in qualitative terms. The phenomenon is due to gravity. Initially the center of gravity starts to move downward under gravity while the axis of the bullet remains horizontal. The initial yaw attributed only to this is therefore in a vertical plane and is small. The bullet precesses to the right and down by the usual law of gyroscopic precession. Since the trajectory is curved, its tangent slowly rotates downward. For the small yaw under consideration the precession can be just slow enough to keep pace with the tangent's downward rotation. That is, the bullet keeps pointing to the right of the trajectory. Consequently the cross-wind force is always perpendicular to a vertical plane and is directed to the right. Therefore the bullet drifts slowly to the right under this force. (For caliber 0.50 this drift amounts to about 7 in. in 1,000 yd and so may be neglected.)

[d] The diameter of the helix for small calibers is of the order of ½ in.

1.5 DISPERSION

1.5.1 Philosophy of Rapid Fire Weapons

With but few large caliber exceptions, in aeroballistics a burst (a rapid and continuous sequence of bullets) is directed at a target. The successive bullets will not follow each other in exactly the same path and as a result a dispersed pattern is built up at the target. The directed burst or pattern principle is basic in the philosophy of airborne fire control with small caliber ammunition. The expectation is that if a large number of bullets are placed rapidly on and near a target there will be a high probability that at least one will hit the small sub-area vulnerable to the caliber being used. That this is really a pollination technique is evident from operational statistics indicating that anywhere from 17,000 rounds (B-24, Africa, early World War II) to 2,500 rounds (B-29, Pacific, late World War II) were expended per fighter shot down. In contrast, with large caliber projectiles, elaborately and individually directed, successive hits can be scored on ships angularly smaller than aircraft targets (U. S. Navy, Coral Sea).

1.5.2 Nature and Statistical Description of the Pattern

In aerial gunnery only a certain part of the total dispersion can properly be attributed to ballistics. The situation is somewhat as follows: In tracking and firing at an airborne target an instantaneous mean point of impact fails to stay on the target since it is being carried about by aim wander, deflection errors, and instrumental errors. Aeroballistics may discuss only this mean point of impact and, even then, suffer an enlargement on its concept since a gun mount and a gunner will be introduced. This is justified because of the influence of these factors on the direction of bullet departure. To proceed systematically consider a sequence of three ground experiments.

1. Suppose rounds are fired under precisely the same conditions of exact and *individual* aim. Because of initial yaw at the muzzle, manufacturing variations in powder charge, bullet shape and weight, and position of the charge, a certain pattern will be generated. But it is a remarkably tight pattern. For example, in caliber 0.50 fire through a 45-in. Mann barrel a cone of angular diameter 0.25 milliradian will contain 75 per cent of the rounds.

2. If a *burst* is fired, instead of individual rounds as above, with the mount held rigid, elastic vibrations of the barrel will occur. The vibration is not simply an up-and-down whip. Instead a forced vibration occurs and each bullet sensibly leaves in phase with the motion of the barrel. Typically, the 75 per cent cone is increased to perhaps 2.5 milliradians.

3. If the mount is not held rigid and if a human gunner is asked to hold on a fixed target, the combined mount vibration and gunner error will lead to a 75 per cent cone of diameter from 5 to 25 milliradians, depending on the installation under test.[104, 108, 101]

It is with this third type of dispersion that one normally deals in calculations. In any study, its magnitude must be carefully determined for the given system even to the extent of noting the effect of such variables as aircraft, installation, gunner, age of the gun, length of burst.

When the bullet holes of any burst are collected on a flat target normal to the line of fire, they have a center of gravity which is called the *mean point of impact* [MPI]. It may be noted that the MPI may be *systematically* displaced from the first bullet due to gun jump. Furthermore, with excessively long bursts the heating of the barrel may also cause a steady shift of the MPI. But, neglecting such effects, it is common to consider each round as independent of any other so that the dispersion pattern may be adequately described by a two-dimensional normal or Gaussian distribution with the MPI as mean. In practice the variances in two orthogonal directions will differ, leading to an elliptic distribution. But since the orientation of this elliptical pattern depends in a complicated way on the direction of fire, because of different vibrational responses of the mounting, it has become customary to replace it by a circular distribution (with the two directions statistically independent). If the MPI is taken as the origin, the fraction of a large number of rounds that will lie in the cell $r\,d\theta\,dr$ is

$$p(r,\theta) = \frac{1}{2\pi\sigma^2} e^{-r^2/2\sigma^2} \tag{10}$$

where σ is the standard deviation.[88] Cells of equal probability with their centroids are shown in Figure 10. Since the evidence is that the bullets of a burst define a right circular cone, it is usual to take r and σ in milliradians. In military literature the pattern is specified by giving the diameter of the circle that contains either 50 or 75 per cent of the rounds.

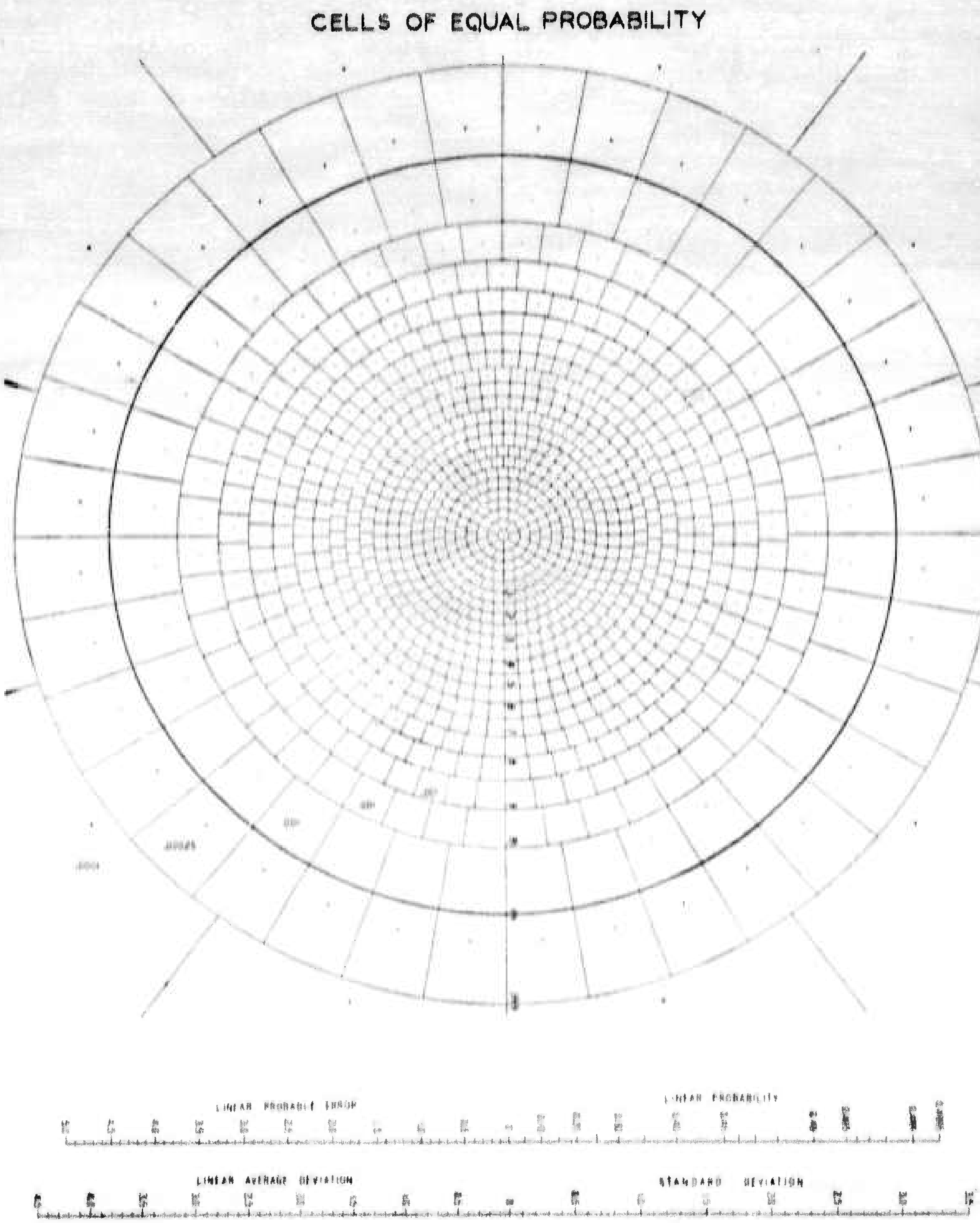

FIGURE 10. Cells of equal probability.

From equation (10) it follows that the diameter of the circle containing h per cent of the rounds is given by

$$d = \sigma\sqrt{8 \ln \frac{100}{100 - h}}\,. \qquad (11)$$

(For example: if $h = 50$, $d/\sigma = 2.354$; and if $h = 75$, $d/\sigma = 3.330$.) The most elegant specification of the pattern size is through σ, since σ is the radius of that *annulus* of width dr that contains the maximum fraction of the total number of rounds composing the pattern.

1.5.3 Change in Pattern Size under Trail Gradients and Forward Fire

Aeroballistic effects can modify the pattern size. Suppose that a turret is firing a burst while it is tracking at a uniform angular rate. Then the pattern being built up at the target, i.e., the pattern relative to the gun, is distorted by the change in lateral ballistic deflection due to the change in θ.[11] This *trail gradient* is

$$\frac{dW}{d\theta} = W_{90°}\cos\theta,$$

where $W_{90°}$ is the lateral ballistic deflection for beam fire, other conditions remaining unchanged. An analysis in the vertical direction gives the same distortion. It is concluded that

$$\sigma_{air} = \sigma\left(1 - \frac{W_{90°}\cos\theta}{1000}\right).$$

A different type of distortion occurs in forward fire from a fighter.[38] A straightforward vector combination of muzzle velocity and fighter velocity gives the expression

$$\sigma_{air} = \sigma\frac{v_0}{v_0 + v_F}\,.$$

This is illustrated by Figure 11. A change as great

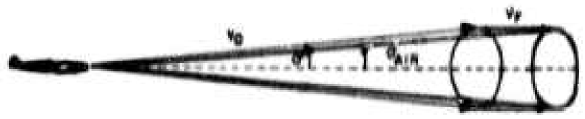

FIGURE 11. Contraction of pattern in forward fire.

as 26 per cent can occur, and it should, therefore, be taken into account.

A final point of interest is the aeroballistic effect on the dispersion pattern caused by the variation in muzzle velocity. This velocity will vary materially (100 fps) not only because of manufacturing variations but also because of the position of the powder in the cartridge, the temperature, the age of barrel, and the length of burst.[50] Since the aiming allowance for target motion is approximately proportional to muzzle velocity (see Chapter 2), if all the rounds of a burst have a mean muzzle velocity different from that assumed by the fire control system, a systematic displacement of the MPI from the target will occur. But consider now the effect of the variation in muzzle velocity around the mean during the burst. The low-valued bullets will underlead and the high-valued ones overlead. The dispersion is increased proportionately along the track of the target, which can be regarded as fortunate, since deflection errors such as the systematic one described above have their greatest value along the track.

1.5.4 Harmonization

Because of dispersion it is a reasonable idea, in view of the flat trajectories in hand, to attempt to remove the necessity of making superelevation allowances for gravity drop in the aiming problem. If the guns are set at some fixed slight angle with respect to the line of sight, then with the usual condition of guns below the gunner's eyes the trajectory of that *ideal* bullet, which will hit the MPI, will arch up, cross the line of sight near the guns, and cross it again much later. With a dispersion pattern there is, then, an arched cone which supplies a beaten zone hugging the line of sight quite closely over ranges of tactical importance. In determining the required angle of elevation it is necessary to consider: parallax (the displacement in the aircraft of the guns from the eye of the gunner), gun climb during a burst, gravity drop, range to be beaten, and direction of fire.[10] Direction of fire is important because nose fire to a given future range implies a smaller time than does tail fire to that same range. In remote fire control systems as in the B-29, the physical difficulties in carrying out a harmonization scheme determined by these considerations are material.[49]

1.6 SUMMARY

Section 1.1 defines aeroballistics indirectly by contrasting certain points connected with fire from airborne guns with items in classical exterior ballistics.

Section 1.2 derives formulas for the time of flight of a projectile as a function of direction of fire, range, altitude, airspeed of the gun mount, ballistic coefficient, and muzzle velocity. Two coordinate systems are used: one fixed in the air mass and the other translating with the gun mount.

Section 1.3 works in the coordinate system relative to the gun mount and deduces various expressions for the angular displacements of the projectile from the bore axis. These displacements are the lateral and vertical ballistic deflections.

Section 1.4 concerns itself with the microscopic motion of a bullet with respect to the mean trajectory of the previous sections. The phenomena of damping of yaw, windage jump, and drift are discussed in elementary terms.

Section 1.5, dealing with dispersion, indicates the relation between patterns determined by ground firings and patterns arising under air firing. The pattern is described by a circular Gaussian distribution. Harmonization of guns and line of sight is mentioned very briefly.

Chapter 2

DEFLECTION THEORY

2.1 INTRODUCTION

2.1.1 Definition of Deflection

In air warfare both the gun platform and the target move at high speeds, often at speeds up to one-fifth of that of the projectile being used. The high speed of the mount, taken on by the bullet, results in an appreciable angle between the bore axis and the direction of departure of the projectile, and the high speed of the target means that it will move a considerable distance during the time of flight of the missile. In addition, a bullet has its own ballistic deflections. For these three reasons, in order to obtain a hit, the bore axis must have, in general, an angular displacement from the line connecting gun and target at the moment of fire. This angle between bore axis and gun-target line is called the *deflection* or the *lead* (even if it is a lag) or the *aiming allowance*. The theory of aerial gunnery is dependent on a precise mathematical account of deflection shooting. Such an account is at hand,[18] and it is the purpose of this chapter to review compactly this systematic theory.

2.1.2 Three Problems of Deflection Theory

The problem put to deflection theory is threefold. Formulas describing perfect shooting must be derived to serve not only further studies but also to supply a norm against which inevitable compromises and approximations may be held. Formulas must be put in a form amenable to a computation which is both rapid and accurate and which has as inputs legitimately available data. Finally, the formulas must be in a form to indicate suitable mechanization. As a combined example of the last two points, legitimate data are those measurable by the gun platform at the instant of fire and accordingly available to a suggested mechanization (fire control system).

2.1.3 Conditions for Validity of Theory

In putting the theory of this chapter to work for any situation other than air-to-air fire with present ammunition at present day ranges (up to 1,000 yd) caution must be observed for three reasons. It is assumed that gun platform, target, and projectile are all in the same air mass which is in nonaccelerated motion. Thus, for a ground target or a ground gun, further consideration must be given to the effect of the motion of the air mass with respect to the ground. In the second instance the detailed consequences of various formulas will usually employ a three-halves power resistance law for the bullet. Chapter 1 has indicated the range of bullet speeds under which this law is appropriate. When these conditions are violated the more exact Siacci treatment of the trajectory must be used.[18] Finally, in most of this treatment it is assumed that bullets travel in straight lines with respect to the air mass. This means that gravity drop, windage jump, and drift are considered small enough so that they may be accounted for terminally by linear superposition.

2.1.4 The Perfect Bullet

In general, in aerial gunnery, each bullet of a burst requires a personal deflection different from that of its fellows because of the continuously changing conditions of fire. This chapter deals with an individual and perfect bullet under general conditions of fire. Considerations of dispersion and of probabilities of hits are necessarily deferred. The target is a point.

2.2 NONACCELERATED TARGET COPLANAR WITH MOUNT VELOCITY

2.2.1 Basic Formula

The plane determined at any instant by the position of the target and by the mount velocity is called

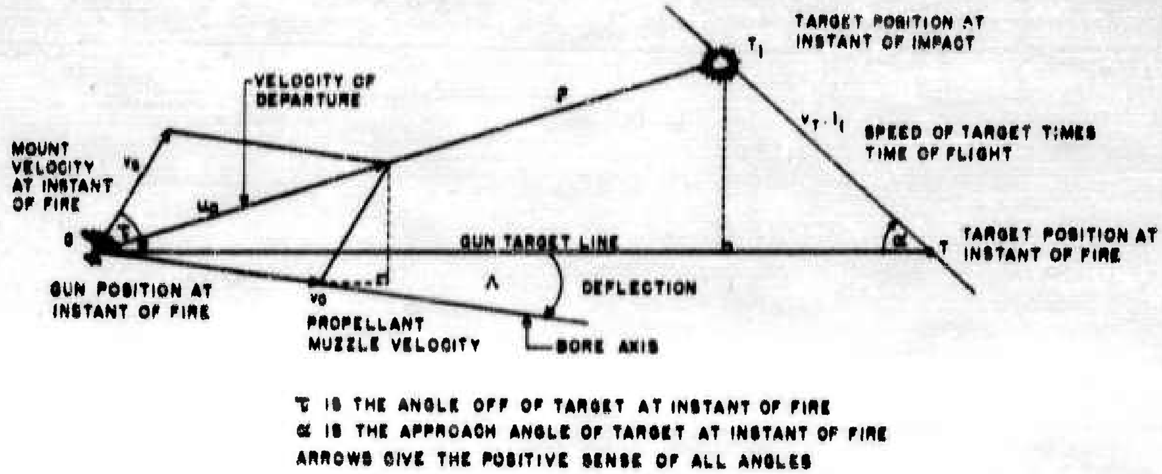

FIGURE 1. Nonaccelerated target coplanar with mount velocity.

the *plane of action*. In individual duels between a fighter and a bomber this plane of action remains sensibly the same during the engagement. (See Chapter 3.) For the purposes of this section it is necessary to assume only that the target moves with constant velocity in the plane of action of the instant of fire, since it is clear that a bullet has no relation to the motion of the mount subsequent to its departure from that mount. The situation contemplated in the air mass is drawn in Figure 1 which is suitably annotated. With the aid of the dotted line constructions, one readily obtains the formula

$$\sin A = \frac{v_G}{v_0}\sin\tau - q\frac{v_T}{v_0}\sin\alpha, \qquad (1)$$

where the *slowdown factor* q is the ratio of the initial speed of the bullet, u_0, to the average speed, $\bar{u}$, over the air range P.

To justify the name given to q, note that $t_f = qt_0$, where t_f is the actual time of flight to future position and t_0 is the time of flight in vacuum to that position. Hence $l = q - 1$ is the proportion by which the time of flight has been increased because of air resistance opposing the bullet's motion.

2.2.2 Own-Speed Lead, Fighter Lead, and a Classic Theorem

Two obvious special cases of equation (1) are important. If either v_T or α is zero,

$$\sin A = \frac{v_G}{v_0}\sin\tau. \qquad (2)$$

In this case A is called the *own-speed allowance*. It is significant that the muzzle velocity of the bullet does not appear in equation (2). Only the direction of departure is important. Tactically speaking, either a target fixed with respect to the air mass or a target approaching or receding along the gun-target line is under fire. This, then, is the allowance to make in strafing a ground target, in the absence of ground wind, or the deflection to take against an attacking fixed gun fighter, to a first approximation. (See Chapter 4.)

As a second case consider that a first approximation fighter, whose guns must fire exactly in the direction of flight is at the point G. Then the angle off, τ, must always be equal to the required deflection, A. Consequently, equation (1) becomes, when the sign situation is thought through,

$$\sin A = -q\frac{v_T}{v_0 + v_G}\sin\alpha,$$

or

$$\sin A = -\frac{v_T}{\bar{u}}\sin\alpha. \qquad (3)$$

The minus sign indicates that *lead* (gun pointing ahead of the target) is taken negative. *Lag* (gun pointing behind the target) is given the plus sign since most emphasis has been placed on the study of fire from a bomber against attacking fighters.

A comparison of equations (2) and (3) supplies a classical, but only approximate, theorem. Place a fighter at T in Figure 1, so that there are now two gun mounts. For the fighter the approach angle, α,

is the same as the angle off, τ. And the speed of the fighter's target, v_T, is the speed v_G of the bomber. Finally, the average velocity $\bar{v}$ of the fighter's bullet is approximately equal to the propellant muzzle velocity, v_0 of the bomber's bullet, since the slowdown of the fighter's bullet is compensated for by its augmented velocity of departure. Consequently, *roughly, in a duel between a fighter and a bomber, the leads taken are equal in size and opposite in sense.* The refinements are explored in Chapters 3 and 4, and the theorem is put to practical use in Section 7.2.4.

2.2.3 Tracking Rate Formulation

Formula (1) is not adapted to mechanization. It requires an estimate of approach angle which cannot be made accurately from the gun platform by existing devices. It also uses the unknown future range in determining the slowdown factor. Finally it is easier mechanically to measure the angle between mount velocity and the bore axis of the gun rather than the angle τ. It seems reasonable, in meeting these three objections, to work in a reference system translating with the mount's velocity at the instant of fire. In this relative reference system the platform velocity is given to the target in a reverse sense. When this is done, it is evident that the angular rate ω of the gun-target line is

$$\omega = \frac{1}{r}(v_G \sin \tau - v_T \sin \alpha),$$

where r is the *present range* or range at the instant of fire. With this formula α can be removed from equation (1), thus meeting the first objection above. The result is

$$\sin A = q\frac{r\omega}{v_0} - l\frac{v_G}{v_0}\sin \tau,$$

where $l = q - 1$. To meet the third objection, take $\gamma = \tau + A$, where γ is the angle between mount velocity and the bore axis. Then

$$\sin A = t_m\omega - b \sin \gamma, \tag{4}$$

where

$$t_m = \frac{t}{v_0 - b \cos \gamma} \tag{5}$$

and

$$b = \frac{b v_G \cos A}{v_0 - b v_G \cos \gamma}. \tag{6}$$

The second objection, conc[illegible] the unknown futu[illegible] range, must be dealt with [illegible] [illegible] [illegible] of t_m appears in Section 5.3.5.) A prefatory discussion of the important formula (4) is, however, in order at this place.

2.2.4 Kinematic and Ballistic Decomposition

To proceed obliquely with this discussion of formula (4), the analogue of Figure 1 is shown in Figure 2 which uses the reference system relative to the gun mount. In that figure, v_R is the relative target

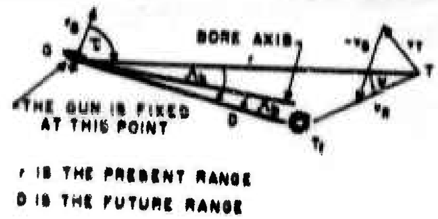

FIGURE 2. Deflections relative to the gun mount.[2]

speed, which is the vector sum of v_T and $-v_G$, and ψ is the approach angle of the relative path. In the absence of lateral ballistic deflection (trail), the gun points at T_f, and the corresponding aiming allowance A_k is called the *kinematic deflection*. But the trajectory curves rearward when viewed from behind the gun. Hence the ballistic correction A_b must be made, gravity being neglected at this point. In this system, the total deflection is expressed, then, in the form[18]

$$A = A_k - A_b. \tag{7}$$

It will be appreciated that this deflection must be the same as the deflection given by equation (4). A direct identification of equation (7) with equation (4) can be made. From Figure 2 it follows that

$$\sin A_k = \frac{v_R t_f}{D}\sin\psi.$$

But $r\omega = v_R \sin\psi$ and $D/t_f = \bar{v}$. Hence

$$\sin A_k = \frac{r\omega}{\bar{v}},$$

where $\bar{v}$ is the average speed of the bullet over the future range D. [illegible] Figure [illegible] in which auxiliary lengths x and y have been introduced, it is easy to show that

$$q^2\bar{v}^2 = (v_0 - b v_G \cos\gamma)^2 + b^2 v_G^2 \sin^2\gamma,$$

and

$$\sin A_b = \frac{b v_G \sin\gamma}{q\bar{v}}.$$

But $(bv_G \sin\gamma)^2$ is of negligible size compared with $(v_0 - bv_G \cos\gamma)^2$. Hence q^D may be replaced by $(v_0 - bv_G \cos\gamma)$; and simple substitution now identifies equations (7) and (4) by

$$A_k = t_m\omega, \quad A_b = b \sin\gamma,$$

provided $\sin A_k$ and $\sin A_b$ are replaced by A_k and A_b measured in radians, and $\cos A$ is taken to be unity in equation (6).

2.2.5 Time-of-Flight Multiplier

The quantity t_m of equation (5) should be examined closely. It has the dimensions of time but since it is present range divided by mean velocity over future range it has no direct physical meaning. It is called *the time-of-flight multiplier*. We may write

$$t_m = \frac{r}{\bar{v}} = \frac{r}{\bar{v}_p}\frac{\bar{v}_p}{\bar{v}} = \frac{\bar{v}_p}{\bar{v}}t_p,$$

where $\bar{v}_p$ is mean velocity over present range and t_p is the time of flight over present range. If the relative motion path is incoming, i.e., $D < r$, it follows that $\bar{v}_p/\bar{v} < 1$ and $t_p\omega$ is slightly too large a kinematic deflection. The reverse is true if the relative motion path is receding.[10] In a mechanization designed for all-around use, the ballistic computer would simply translate present range into present time of flight. But if the tactical circumstances were such that targets were always closing, a correction factor should be applied. This is a first and simple example of a basic principle in fire control theory. The time-of-flight multiplier need not fit any particular ballistic table but should be chosen as a function of range in such a way as to optimize performance over an expected set of tactical situations.[11]

2.3 ACCELERATED TARGET COPLANAR WITH MOUNT VELOCITY

2.3.1 Derivation of Acceleration Correction

For the tactically important case of an accelerated target, which means a curved path, a change in speed, or both, the formulas of the previous section must be modified by the addition of a correction term. If the *chord* of the target's path segment from T to T_f makes an angle $\bar{\alpha}$ with the gun-target line, and if the average speed of the target over this segment is $\bar{v}_T$, then equation (1) may apply and is written[a]

$$\sin A = \frac{v_G}{v_0}\sin\tau - q\frac{\bar{v}_T}{v_0}\sin\bar{\alpha},$$

where $\bar{v}_T = TT_f/t_f$. Conceptually, the actual target has been replaced by an equivalent one, as far as impact is concerned, whose approach angle is the average approach angle of the actual target. An estimate of $\bar{v}_T$ is given by (see Section 2.4.2)

$$\bar{v}_T = v_T + \tfrac{1}{2}t_f \cdot \dot{v}_T,$$

where v_T and $\dot{v}_T$ are evaluated at the moment of fire. If one assumes that the change in path curvature is not radical, over the times of flight contemplated

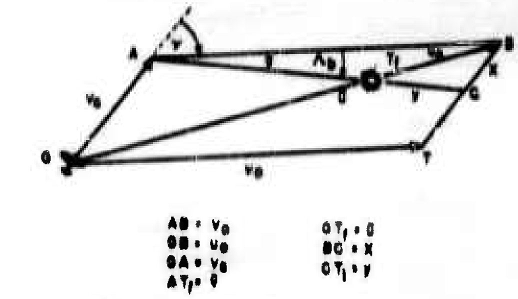

FIGURE 3. The firing parallelogram.

— and this is a valid assumption — the target may be taken to move in a circle at constant speed $\bar{v}_T$ and to turn through an angle $2(\bar{\alpha} - \alpha)$ during t_f. If the angular rate of change of the target is ω_T, the angle through which it turns is also $\omega_T \cdot t_f$ and so

$$\bar{\alpha} = \alpha + \tfrac{1}{2}\omega_T \cdot t_f.$$

Consequently the deflection formula becomes

$$\sin A = \frac{v_G}{v_0}\sin\tau - q\frac{v_T + \frac{1}{2}\dot{v}_T \cdot t_f}{v_0}\sin(\alpha + \tfrac{1}{2}\omega_T \cdot t_f),$$

or

$$\sin A = \frac{v_G}{v_0}\sin\tau - q\frac{v_T}{v_0}\sin\alpha - \tfrac{1}{2}qt_f p, \qquad (8a)$$

where

$$p = \frac{\dot{v}_T}{v_0}\sin(\alpha + \tfrac{1}{2}\omega_T \cdot t_f) + \frac{2v_T}{v_0}\left[\frac{\sin(\alpha + \frac{1}{2}\omega_T \cdot t_f) - \sin\alpha}{t_f}\right],$$

and p is approximated by

$$p = \frac{\dot{v}_T}{v_0}\sin\alpha + \frac{v_T}{v_0}\omega_T\cos\alpha. \qquad (8b)$$

[a] To follow this discussion by figure, modify Figure 1 by drawing an arc through T and T_f and placing bars over α and v_T.

If it is assumed that the gun mount is moving at uniform velocity, then $\dot v_G = 0$. Since $\dot\tau = \omega$, it follows from the physically evident rate relations

$$\begin{aligned} r\omega &= v_G \sin\tau - v_T \sin\alpha \\ -\dot r &= v_G \cos\tau + v_T \cos\alpha \\ \dot\alpha &= \omega_T - \omega, \end{aligned} \tag{9}$$

that

$$p = -\frac{\dot M}{r v_0}, \tag{10}$$

where M is the *angular momentum* $r^2\omega$ of a target of unit mass. [Formula (10) may be verified by calculating the derivative of $r^2\omega$.] With the aid of formula (10) and the first equation of (9), equation (7) can be written

$$\sin A = h\frac{qr\omega}{v_0} - t\frac{v_G}{v_0}\sin\tau \tag{11}$$

where

$$h = 1 + \frac{t_f \cdot \dot M}{2M}.$$

Finally, as in the derivation of equation (4), one may take

$$\sin A = ht_m\omega - b\sin\gamma \tag{12}$$

where t_m, the time-of-flight multiplier, and b, the ballistic deflection factor for beam fire, are exactly as in equations (5) and (6). Conceptually, equation (12) is quite important. It shows that deflection formulas for an accelerated target can be obtained from those for a target in uniform motion by multiplying the time-of-flight multiplier by a (nonconstant) factor h. In the design of eye-shooting systems and own-speed sights, and in the calibration of lead computing sights this is decisive.

2.3.2 Kinematic and Ballistic Decomposition

To parallel the discussion of Section 2.2.4, the situation here may be considered in the reference system relative to the mount. This treatment emphasizes the role played by the angular momentum of the target on its relative path. From Figure 4, it is evident that the area A swept out by the gun-target line during the time of flight of the bullet is

$$A = \frac{1}{2}\int_0^{t_f} r^2\dot\tau\, dt = \frac{1}{2}\int_0^{t_f} r^2\omega\, dt.$$

Put, as above, $M = r^2\omega$ and expand M in a Taylor's series about $t = 0$. Then

$$\begin{aligned} A &= \frac{1}{2}\int_0^{t_f}(M + \dot M t + \cdots)\,dt \\ &= \tfrac{1}{2}(Mt_f + \tfrac{1}{2}\dot M t_f + \cdots), \end{aligned}$$

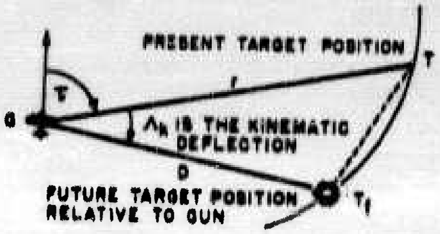

FIGURE 4. Accelerated target, relative to gun mount.

where, now, $M, \dot M, \cdots$ are evaluated at $t = 0$. Since t_f is small, terms of the third and higher orders may be neglected. An independent evaluation of the area A is given by the area of the triangle GTT_f, since t_f is small and the path curvature cannot be large for aerodynamic reasons. The area of the triangle is

$$A = \tfrac{1}{2}rD\sin A_k.$$

When this is equated to the previous expression, one finds that

$$\sin A_k = ht_m\omega, \tag{13}$$

where the letters have the same meaning as in the earlier discussion. Finally, since the argument based on Figure 3 uses only the impact point and is not concerned with the target's meanderings in reaching this point, it applies immediately here. The identification of the air mass coordinate formula and the kinematic-ballistic decomposition is complete.

2.4 THE GENERAL THEORY OF DEFLECTION

2.4.1 Extended Conditions

A general treatment must permit the gun mount and the target to move in arbitrary space paths. It must also allow the bullet to move in a vertical plane rather than in a straight line. For the first requirement it may be said that the paths are smooth in the analytic sense and curve gradually in the tactical sense. By implication, one will not be concerned with derivatives of the third and higher order. Under the second requirement gravity drop is permitted. Although deviations normal to the vertical plane that contains the direction of departure of the bullet are not permitted here, the methods of the section may

be generalized should missiles of the future require consideration of such behavior. For the three-dimensional situation, the evident tools are vector algebra and vector calculus.[10b,21a] The methods are particularly applicable to support fire situations and to aerodynamic lead pursuit curves (Chapter 3). Finally, since the methods are vectorial, specialization to various coordinate systems corresponding to particular decompositions of deflection is readily accomplished.

2.4.2 Derivation of General Formula by Vector Methods

In the air mass the trajectory is described by a vector[b] **R** connected to the Siacci vectors **P** and **Q** (see Section 1.2.2) by

$$\mathbf{R} = \mathbf{P} + \mathbf{Q}.$$

Let $\mathbf{r}$ be the gun-target vector, let $\mathbf{v}_G$ and $\mathbf{v}_T$ be the velocities of mount and target, and let $\mathbf{v}_0$ be the propellant muzzle velocity (in the direction of the bore axis). All vectors are functions of time measured from the instant of fire. In the air mass reference system, one writes triangularly

$$\mathbf{r} + \int_0^{t_f} \mathbf{v}_T dt = \mathbf{R}. \tag{14}$$

Holding this relation in reserve for the moment, it is trivial to observe that the bore axis is placed in lead position by rotation from the gun-target line through the appropriate lead angle. But this rotation suggests the representation of deflections by vectors perpendicular to the bore axis and to the gun-target line and hence implies cross products. By definition

$$\boldsymbol{\lambda}_0 = \frac{\mathbf{r} \times \mathbf{v}_0}{rv_0} \qquad \lambda_0 = \sin A$$

$$\boldsymbol{\lambda}_G = \frac{\mathbf{v}_G \times \mathbf{r}}{rv_0} \qquad \lambda_G = \frac{v_G}{v_0} \sin \tau \tag{15}$$

$$\boldsymbol{\lambda}_T = \frac{\mathbf{v}_T \times \mathbf{r}}{rv_0} \qquad \lambda_T = \frac{v_T}{v_0} \sin \alpha.$$

Now $\mathbf{u}_0 = \mathbf{v}_0 + \mathbf{v}_G$, and $\mathbf{u}_0 = u_0\mathbf{P}/P$. Hence from equation (14)

$$\frac{\mathbf{r}}{t_f} + \bar{\mathbf{v}}_T = \frac{1}{q}(\mathbf{v}_0 + \mathbf{v}_G) + \frac{\mathbf{Q}}{t_f},$$

where $q = u_0/\bar{u}$, $\bar{u} = P/t_f$, and $t_f\bar{\mathbf{v}}_T = \int_0^{t_f} \mathbf{v}_T dt$. Upon multiplication by q there arises

$$\mathbf{v}_0 = -\mathbf{v}_G + q\bar{\mathbf{v}}_T - \mathbf{w} + \frac{u_0}{P}\mathbf{r}, \tag{16}$$

where $\mathbf{w} = u_0\mathbf{Q}/P$. If the cross product of equation (16) by $\mathbf{r}/r$ on the left is taken it follows that

$$\boldsymbol{\lambda}_0 = \boldsymbol{\lambda}_G - q\bar{\boldsymbol{\lambda}}_T + \boldsymbol{\lambda}_w, \tag{17}$$

where the list in equation (15) is now extended by

$$\bar{\boldsymbol{\lambda}}_T = \frac{\bar{\mathbf{v}}_T \times \mathbf{r}}{rv_0} \qquad \bar{\lambda}_T = \frac{\bar{v}_T}{v_0} \sin \bar{\alpha}$$

$$\boldsymbol{\lambda}_w = \frac{\boldsymbol{\omega} \times \mathbf{r}}{rv_0} \qquad \lambda_w = \frac{\omega}{v_0} \sin Z, \tag{18}$$

and where $\bar{\alpha}$ is the average approach angle over t_f and Z is the angle between $\mathbf{r}$ and the true vertical. Formula (17) makes precise one's intuition about the form of the total lead and clearly reduces to equation (1) for the two-dimensional case without gravity.

To obtain the appropriate generalization of equation (11), consider $\bar{\mathbf{v}}_T$ in detail. A good approximation to $\bar{\mathbf{v}}_T$ is given by the expression

$$\bar{\mathbf{v}}_T = \mathbf{v}_T + \tfrac{1}{2}t_f\dot{\mathbf{v}}_T,$$

where $\mathbf{v}_T$ and $\dot{\mathbf{v}}_T$ are evaluated at $t = 0$. This arises by taking three terms in the series expansion of the vector giving target position with respect to origin of the air mass coordinates, differentiating, and integrating from 0 to t_f.

Turn now to the reference system relative to the gun. The velocity of the target in this system is $\mathbf{v}_R = \mathbf{v}_T - \mathbf{v}_G$. The angular momentum of unit mass at the target position is

$$\mathbf{M} = \mathbf{r} \times \dot{\mathbf{r}} = \mathbf{r} \times \mathbf{v}_R,$$

and its derivative is given by

$$\dot{\mathbf{M}} = \mathbf{r} \times \ddot{\mathbf{r}} = \mathbf{r} \times \dot{\mathbf{v}}_R.$$

The angular velocity $\boldsymbol{\omega}$ of the gun-target line is connected to $\mathbf{M}$ by

$$\mathbf{M} = r^2\boldsymbol{\omega}.$$

Using these relations one finds that

$$\mathbf{v}_T \times \mathbf{r} = -\mathbf{M} + rv_0\boldsymbol{\lambda}_G - \tfrac{1}{2}t_f\dot{\mathbf{M}} + rv_0\tfrac{1}{2}t_f\boldsymbol{\lambda}_G',$$

where

$$\boldsymbol{\lambda}_G' = \frac{\dot{\mathbf{v}}_G \times \mathbf{r}}{rv_0}.$$

[b] The *dot product* of vector **A** by **B** is the scalar $\mathbf{A} \cdot \mathbf{B} = AB \cos \theta$ where θ is the angle between **A** and **B**. The *cross product* of **A** by **B** is written $\mathbf{A} \times \mathbf{B}$ and is a vector perpendicular to the plane of **A** and **B**, with a positive sense according to a right hand system (**A**, **B**, $\mathbf{A} \times \mathbf{B}$). The magnitude of $\mathbf{A} \times \mathbf{B}$ is $AB \sin \theta$.

Finally, substitution in equation (17) yields the vector analogue of equation (11)

$$\boldsymbol{\lambda}_0 = \left[\frac{\mathbf{M}}{M} + \tfrac{1}{2}t_f\frac{\dot{\mathbf{M}}}{M}\right]\frac{qr\omega}{v_0} - l\boldsymbol{\lambda}_G - q\tfrac{1}{2}t_f\boldsymbol{\lambda}_G' + \boldsymbol{\lambda}_B. \qquad (19)$$

(The term involving $\boldsymbol{\lambda}_G'$ means that accelerated mount motion is permitted.) All quantities in equation (19), except t_f, are evaluated at the instant of fire. Furthermore, the effects of acceleration are explicitly exhibited in amount and direction. Equation (19) can be regarded as an expression of suitable generality for all fire control applications contemplated in this account. The technical ease with which it is obtained is noteworthy.

2.4.3 Special Coordinate Systems

In the applications, formulas such as (19) must be expressed in particular coordinate systems. The four most common systems are[13a] (1) azimuth and elevation, (2) sight elevation and traverse, (3) gun elevation and traverse, and (4) parallel and perpendicular to the plane of action.

In the first system a fuselage axis and a wing-span axis determine an azimuth plane. The elevation plane is perpendicular to the azimuth plane. The gear system of the usual turret not only illustrates this system but shows that the azimuth plane does not necessarily contain the aircraft's velocity vector (the aircraft may be flying nose high or nose low) and that zenith in the system is not necessarily the zenith with respect to the earth (the aircraft may be diving and turning). If $\mathbf{I}$ is a unit vector in the forward direction of the fuselage axis, if $\mathbf{K}$ is a unit vector directed outward along the starboard wing, if $\mathbf{J}$ is a unit vector directed upward with respect to the aircraft, and if $\mathbf{r}_1$ and $\mathbf{v}_{01}$ are unit vectors in the directions of gun-target line and bore axis respectively, then

$$\mathbf{r}_1 = (\mathbf{I}\cos A + \mathbf{K}\sin A)\cos E + \mathbf{J}\sin E$$
$$\mathbf{v}_{01} = (\mathbf{I}\cos A_0 + \mathbf{K}\sin A_0)\cos E_0 + \mathbf{J}\sin E_0,$$

where A and A_0 are azimuths in the system, and E and E_0 are elevations (of gun-target line and bore axis respectively). The azimuth lead and elevation lead are given by

$$\Lambda_A = A_0 - A, \quad \Lambda_E = E_0 - E.$$

For the second (sight) system, let π_{SE} be the plane containing $\mathbf{r}$ and perpendicular to the azimuth plane, and let π_{ST} be the plane containing $\mathbf{v}_0$ and perpendicular to π_{SE}. If $\mathbf{c}$ is a unit vector along the intersection of these two planes, then the angle between $\mathbf{r}_1$ and $\mathbf{c}$ is called the sight elevation lead, Λ_{SE}, and the angle between $\mathbf{c}$ and $\mathbf{v}_{01}$ is called the sight traverse lead, Λ_{ST}. One has the formulas

$$\sin\Lambda_{ST} = \sin\Lambda_A\cos E_0,$$
$$\sin E_0 = \cos\Lambda_{ST}\sin(E + \Lambda_{SE}).$$

The third (gun) system is similar to the second in that π_{GE} is a plane containing $\mathbf{v}_0$ and perpendicular to the azimuth plane, and π_{GT} contains $\mathbf{r}$ and is perpendicular to π_{GE}. Then

$$\sin\Lambda_{GT} = \sin\Lambda_A\cos E,$$
$$\sin E = \cos\Lambda_{GT}\sin(E_0 - \Lambda_{GE}).$$

The fourth (plane of action) system uses a plane π containing $\mathbf{v}_G$ and $\mathbf{r}$. If π_0 is a plane containing $\mathbf{v}_0$ and perpendicular to π, and if $\mathbf{c}$ is a unit vector determined by the intersection of these two planes, then the angle $\Lambda_{\parallel}$ between $\mathbf{r}_1$ and $\mathbf{c}$ is called the lead *in* the plane of action and the angle $\Lambda_{\perp}$ from $\mathbf{c}$ to $\mathbf{v}_{01}$ is called the lead *out* of the plane of action. This system is particularly useful in theoretical studies since many of the tactically significant situations occur in a sensibly fixed plane of action and so $\Lambda_{\parallel}$ accounts for most of the total deflection. The other systems are appropriate for mechanizations of the fire control problem.

The discussion in this section will be restricted to the gun elevation and traverse system, which is typical of the last three systems in that deflection is obtained by a pair of rotations in orthogonal planes. Introduce unit normals $\mathbf{c}_T$ and $\mathbf{c}_E$ to π_{GT} and π_{GE} respectively, so directed that

$$\mathbf{c}\times\mathbf{r}_1 = \mathbf{c}_T\sin\Lambda_{GT}, \quad \mathbf{c}\times\mathbf{v}_{01} = \mathbf{c}_E\sin\Lambda_{GE}.$$

Then the leads Λ_{GT} and Λ_{GE} are computed by the formulas

$$\sin\Lambda_{GT}\cos\Lambda_{GE} = \mathbf{c}_T\cdot(\mathbf{r}_1\times\mathbf{v}_{01}) = \mathbf{c}_T\cdot\boldsymbol{\lambda}_0$$
$$\sin\Lambda_{GE} = \mathbf{c}_E\cdot(\mathbf{r}_1\times\mathbf{v}_{01}) = \mathbf{c}_E\cdot\boldsymbol{\lambda}_0. \qquad (20)$$

When equation (20) is combined with formulas such as (19) suitable expressions for Λ_{GT} and Λ_{GE} arise after rather massive manipulation.[13b]

2.4.4 Gun-Roll Error

Sights such as the K-3 use gun tracking rates and base their mirror system on the gun. Hence the gun elevation and traverse system is appropriate in describing such equipment. But this system has a peculiarity, accentuated with elevation, which leads to an appreciable error in deflection.[14] The angular

velocity ω of $\mathbf{r}$ has components ω_T and ω_E corresponding to rotations of the gun-target line in the traverse and elevation planes. Now consider the gun coordinate system. It will have angular velocities ω_0, ω_{0T}, and ω_{0E}. But in systems measuring gun rates, only ω_{0T} and ω_{0E} are obtained. The roll of the bore axis, ω_0, is not measured. Hence the angular speeds assigned to $\mathbf{r}$ by the gun rates are

$$\omega_T = \omega_{0T} - \dot{A}_{dT}, \quad \omega_E = \omega_{0E} - \dot{A}_{dE},$$

since the rate of change of deflection gives the *relative* speed of gun-target line and bore axis. But ω_0 should also contribute to ω_T and ω_E. The resulting errors are called the *gun-roll errors* and are given approximately by [18f]

$$\epsilon_{dT} \doteq -A_{dT} A_{dE} \tan E_0, \quad \epsilon_{dE} = A^2_{dT} \tan E_0.$$

If A_{dT} is 0.1 radian, then for elevations greater than 45°, the elevation gun-roll error is greater than 10 milliradians, which is significant.

2.4.5 Timeback Method

This chapter devoted itself to the computation of deflections based as much as possible on present data, since these quantities are all that are available to a fire control system at the instant of fire. In experimental and theoretical studies it usually happens that complete knowledge of the paths of gun mount and target is available. In such cases one may choose to circumvent the above formulas and return to first principles. That is, one can start by choosing a point of impact. Then either future range or air range is known exactly and therefore time of flight and kinematic deflection are known, as is the required position of the bore axis to generate the assigned point of impact. Hence the present position corresponding to the chosen point of impact is determined since the path is expressed as a function of time. A table can be built up of deflection versus the time parameter of the path in which one may interpolate at pleasure. The method is usually called the *timeback* method. It will be discussed again in Section 7.3.3. Whether formulas or timeback are used in the calculation of deflection, systematic computing aids in the form of tables and graphs are available.[18g]

2.5 SUMMARY

The introduction, Section 2.1, gives the following reasons why a systematic theory of deflection is required (1) to supply norms for approximations, (2) to furnish rapid computational procedures using legitimately available data, and (3) to determine the nature of possible mechanizations.

Section 2.2 considers the simple case of a target, moving at constant speed, on a straight line which lies in a common plane with the gun-mount velocity. The formulas deduced will be used repeatedly in the remainder of the text to elucidate various points.

Section 2.3 permits the target to move in a curved path. The form of the acceleration correction factor is computed. This section is a foundation for Chapter 4.

Section 2.4 uses vector methods to treat the general problem in which mount and target may move in arbitrary space paths. Particular coordinate systems used in fire control are discussed. A brief explanation of the phenomenon of gun roll is given.

Chapter 3

PURSUIT CURVES

3.1 INTRODUCTION

3.1.1 Pursuit Curves in Modern Warfare

There are many situations in which an object moving along a path of its own choosing is pursued by another object moving in a path constrained to point instantaneously either directly at the pursued (pure pursuit), or at some point in the vicinity of the pursued in accordance with some definite law (deviated pursuit). In one homely and classical example, the pursuer is a dog in a field and the pursued is the dog's master who walks along the edge of the field. If the dog were blind, he might run toward the sound of a whistle blown continuously by his master. The former case is pure pursuit, and the latter is an example of deviated pursuit.

In modern warfare pursuit curves arise in three types of situations:

1. Homing missiles may continuously change heading under radio, optical, or acoustic guidance unwillingly supplied by the target.

2. Aircraft, directing rockets or large-caliber projectiles at fixed ground targets, may find themselves in an air mass moving with respect to the ground. If the motion of the air mass is reversed and then given to the target a pursuit curve arises.

3. The standard fighter aircraft of World War II employed a heavy battery of guns fixed in the aircraft to fire sensibly in the direction of flight.[a] To change the direction of fire the aircraft itself must be flown in the new direction. Consequently, unless the fighter is directly behind or ahead of its airborne target, it must, ideally, fly on such a correctly banked turn that the correct and changing aiming allowance is continuously made.[b] By doing this the bullet pattern is held on the target until destruction is effected.[10]

Case 1 has been studied with the purpose, among others, of determining turning rates, since certain missiles have control limitations in this respect. Case 2 has been considered in assessing the effect of the path on the aiming problem (rockets), and in determining terminal conditions for bomb release (75-mm cannon-firing path for the B-25H). These two cases do not come within the scope of this account of aerial gunnery. An appreciable number of the techniques exposed below are, however, applicable to such studies.

3.1.2 Reasons for an Elaborate Investigation

The rationale of the elaborate pursuit curve investigations outlined in this chapter is discussed below.

1. The primary function of the defensive gunnery of a bomber is to prevent its parent bomber from being shot down by attacking fighters. Consequently it is reasonable to require the greatest accuracy of the defensive fire control under the circumstances of the ideal attack described as Case 3 above. The fire control must predict the future position of the target on such courses quite closely. To determine future positions requires the computation of aerodynamic lead pursuit curves. [It will be made clear in the sequel how such computations affect (1) the generation of position firing rules of thumb (Chapter 4), (2) the choice of percentage for an own-speed sight

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[a] Hence, fire from fighters is called *fixed gunnery*. If guns can be positioned freely with respect to the direction of flight, *flexible* (or free) *gunnery* arises. This is usually true for bombers.

[b] The alternative for the attacker is to hold a fixed direction of flight so arranged that the target will fly through a stream of bullets. Because of the rapidity with which the target will pass through the bullet stream and the spacing between successive bullets, very few hits may be scored. This strafing attack has been dismissed by all air forces as tactically inefficient in view of the limited vulnerability of the target to present calibers and rates of fire.

in a particular tactical circumstance (Chapter 4), and (3) the calibration of the *time-of-flight setting* for rate type sights (Chapter 5).]

2. From the point of view of offensive or fixed-gun fire control a study of pursuit curves leads to an appreciation of the effect of aerodynamics on the aiming problem, and to appropriate calibration of the lead computing sights used by fighters.

3. Pursuit curve studies have culminated in the detailed analysis of Section 3.4. Frequently one wishes to use the simpler methods of Sections 3.2 and 3.3. Since the complete picture is at hand, approximations may be tested as desired.

4. Finally, the design in the large of defensive fire control systems requires a knowledge of those limitations and possibilities of fighter attacks such as are given in Section 3.5. Design in the large means, here, the choice and disposition of armament and the optimization of performance of the fire control system over the appropriate range-azimuth-elevation cells.

3.2 THE ELEMENTS OF PURE AND DEVIATED PURSUIT THEORY

3.2.1 Assumptions and Coordinates

It will be assumed throughout this section that the pursued chooses to follow a straight line path at a constant speed v_B. Further, neglecting aerodynamic effects or deliberate throttle variation, the speed of the pursuer is also to be a constant v_F. (The velocity subscripts indicate that a bomber and a fighter are to be the objects of the primary physical realization.) If the laws of deviation are restricted to those in which the pursuer always homes on some variable point on the pursued's track, it follows that the curves considered must be plane curves. The plane containing the two tracks is called the *plane of action*. Under the assumptions made above, the geometrical results are independent of the angle between the plane of action and a horizontal plane through the pursued's track.

In studying these curves, the most natural coordinate system to use is an itinerant one whose origin is held on the moving object under pursuit. In such a relative coordinate system, the velocity vector of the pursued is transported to the pursuer and there reversed in sense. The pursuer then moves as dictated by the vector resultant of its own velocity and this reversed velocity.

Appropriate tactical variables are the range ρ and the angle θ measured positively from the rearward track of the pursued up to the line joining the participants. Experience has shown that Cartesian coordinates are not efficient.

3.2.2 Equations for Pursuit Curves

The above circumstances are put down in Figure 1. The deviation angle δ is specified separately as some

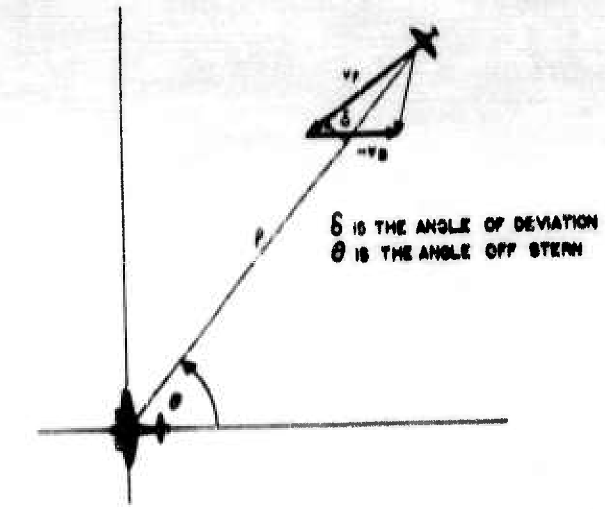

FIGURE 1. A typical instant of deviated pursuit.

function of θ. (More generally, it would be a function of both ρ and θ.) Three instances of such functions concern us in this section. They are (1) $\delta = 0$, which is pure pursuit,[a] e.g., homing by a connection through light or radio; (2) δ = constant, which is fixed lead pursuit, e.g., a fighter using a fixed average lead in attacking a bomber[6]; and (3) $\delta = \nu \sin \theta$, which is a variable lead pursuit,[88] e.g., when ν is a positive constant, a fighter attacks with a variable lead, computed on the assumption that his ammunition has a mean fixed velocity over the ranges in question; and, when ν is a negative constant, a missile homes acoustically on a target,[100] so that ν is the constant ratio of the speed of the pursued to the speed of sound. Later sections successively elaborate the deviation function to make it more realistic.

For these three cases the equations of the relative path may now be deduced. From Figure 1 the rate of range closure is

$$\frac{d\rho}{dt} = -v_F \cos \delta + v_B \cos \theta, \tag{1}$$

[a] Certain Japanese documents[100] indicate that this was doctrinal procedure for the Japanese Air Force during World War II.

and the rate of rotation of the range line about the pursued as pivot is

$$\frac{d\theta}{dt} = \frac{1}{\rho}\,(v_F \sin\delta - v_B \sin\theta). \tag{2}$$

The effect of the pursued's speed, as shown by the last equation, is to cause the pursuer to crab toward a point astern of the pursued. The pursuer is said, picturesquely, to be "sucked flat." When equation (1) is divided by equation (2), time vanishes and the speed ratio $\mu = v_F/v_B$ appears as a natural parameter. The equation

$$\frac{d\rho}{\rho} = \frac{-\mu\cos\delta + \cos\theta}{\mu\sin\delta - \sin\theta}\,d\theta \tag{3}$$

is integrable for the deviation functions in question. The results are:

Pure pursuit [89]

$$\frac{\rho}{\rho_0} = \frac{\tan^{\mu}\theta/2}{\sin\theta}. \tag{4}$$

Fixed lead pursuit [89a]

$$\frac{\rho}{\rho_0} = \left[\frac{1-\mu\sin\delta_0}{\sin\theta-\mu\sin\delta_0}\right]\cdot \tag{5}$$

$$\left[\frac{1+\sqrt{\dfrac{1+\mu\sin\delta_0}{1-\mu\sin\delta_0}}\cdot\tan\left(\dfrac{\theta}{2}-\dfrac{\pi}{4}\right)}{1-\sqrt{\dfrac{1+\mu\sin\delta_0}{1-\mu\sin\delta_0}}\cdot\tan\left(\dfrac{\theta}{2}-\dfrac{\pi}{4}\right)}\right]^{\frac{\mu\cos\delta_0}{\sqrt{1-\mu^2\sin^2\delta_0}}}.$$

Variable lead pursuit [89b]

$$\frac{\rho}{\rho_0} = \left[\left(\frac{\cos\delta+\nu\cos\theta}{\cos\delta-\nu\cos\theta}\right)^{\mu\nu/2}\cdot\right.$$

$$\left.\left(\frac{\cos\delta-\cos\theta}{\cos\delta+\cos\theta}\right)^{\mu/2}\frac{1}{\sin\theta}\right]^{1/(1-\mu\nu)}. \tag{6}$$

For the variable lead pursuit a close and useful approximation is obtained by putting $\cos\delta = 1$ in equation (3). The simpler formula is

$$\frac{\rho}{\rho_0} = \left[\frac{\tan^{\mu}\theta/2}{\sin\theta}\right]^{1/(1-\mu\nu)}. \tag{7}$$

It is clear that equation (5) collapses into equation (4) when $\delta_0 = 0$ and that equations (6) and (7) also reduce to equation (4) when $\nu = 0$, i.e., when the bullets used have infinite velocity. When $\theta = 90°$, $\rho = \rho_0$ in each of these four equations. Consequently ρ_0 is another natural parameter — the *proximity* parameter. It is the range on the beam, and each pursuit curve can be extended backward or forward as required to give its characteristic ρ_0.

Figure 2 illustrates the relative positions of these three types of curves, and Figures 3 and 4 supply convenient nomograms for the computation of pure pursuit courses. A large number of pursuit curves have been computed and graphed.[8, 141, 180, 89]

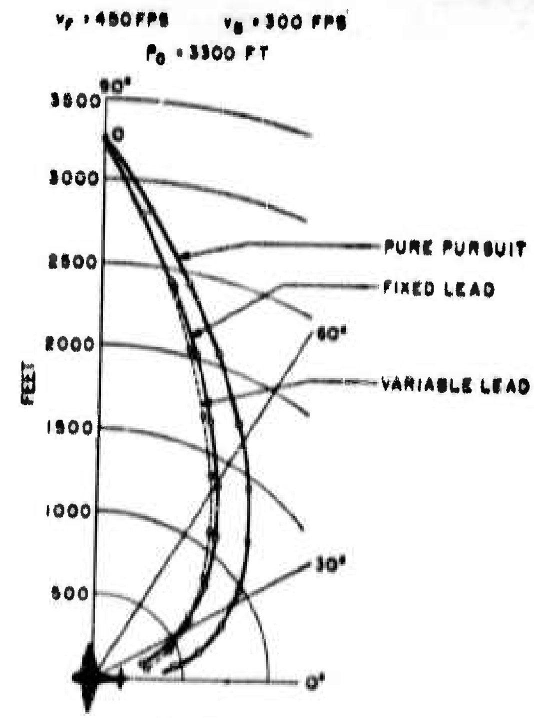

FIGURE 2. Relative pursuit curves.

2.2.3 Bifurcated Pursuit

In the preceding derivation the factor ν was the ratio of the pursued's speed to the speed of the bullet fired by the attacking fighter [the deviation function comes, in fact, from eq. (3), 2.2.2]. Normally, therefore, ν will be much less than 1. Certain implications of $\nu > 1$ are worth exploring with conceivable situations of the future in mind. Attack must be from a forward direction if rupture is to result, and for this region two separate pursuit curves are quite possible.[70] This is demonstrated most easily by the construction of Figure 5. In this figure $\nu = v_B/v$ and is greater than 1. After drawing the boundary line, a semicircle of any convenient radius R is constructed as shown. The line connecting the pursuer F to the pursued B cuts the circle at P_1 and P_2. The lines FP_2

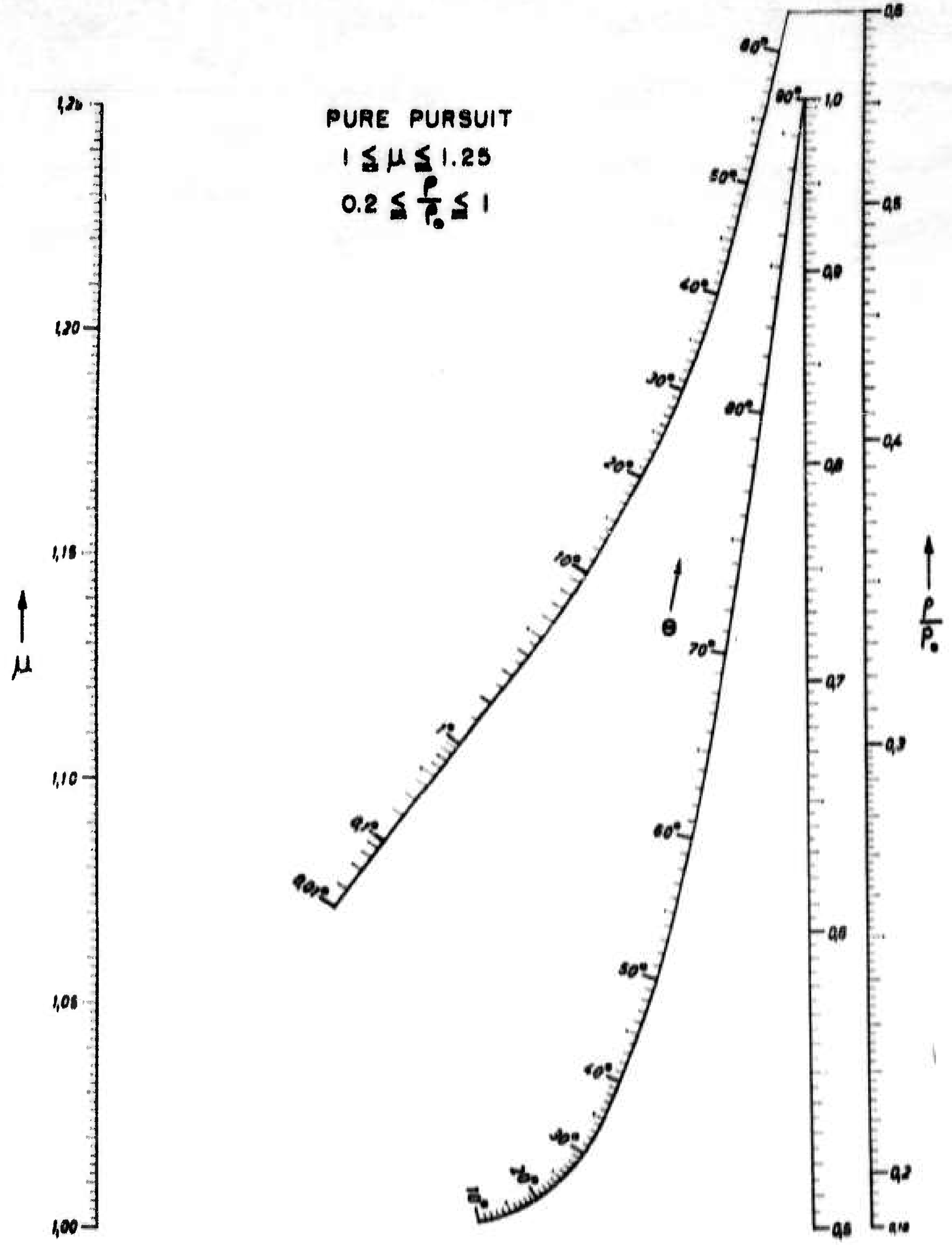

Figure 8. Nomogram for pure pursuit.

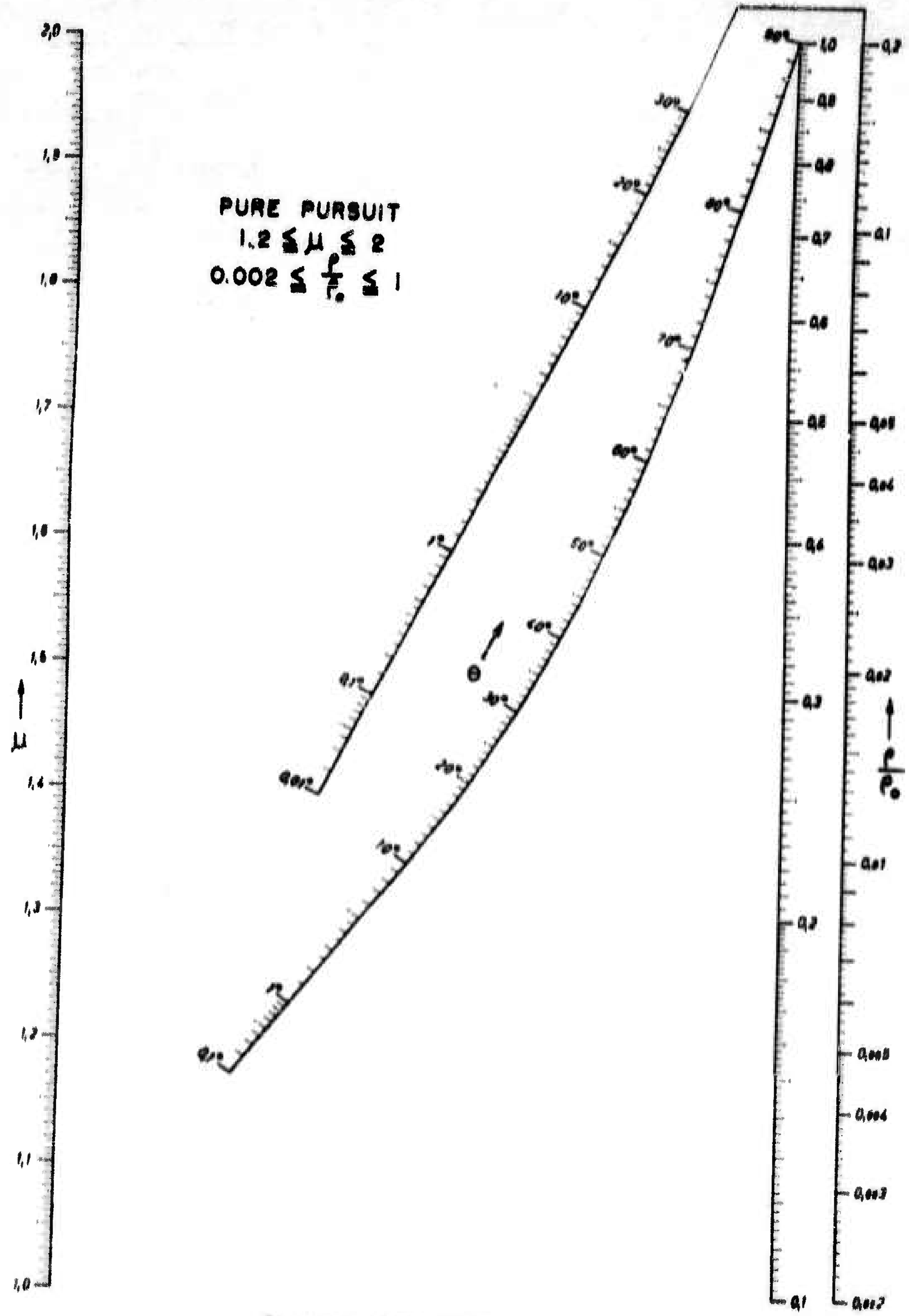

FIGURE 4. Nomogram for pure pursuit.

and FP_4 are parallel, respectively, to P_1M and P_2M. The pursuer can move in either of these two directions. Consider FP_2. Then we should have

$$\frac{v}{v_B} = \frac{\sin\theta}{\sin\delta} = \frac{FP_2}{BP_2}.$$

But the construction achieves this since

$$\frac{FP_2}{BP_2} = \frac{P_1M}{BM} = \frac{R}{BM} = \sin\phi = \frac{v}{v_B}.$$

And the alternative has exactly the same treatment.

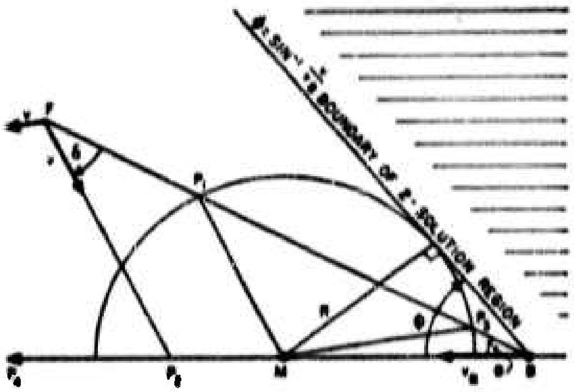

FIGURE 5. Bifurcation of pursuit.

3.2.4 Methods of Introducing Time as a Parameter

A basic disadvantage of the solutions (4) through (7) of the pursuit curve problem is that range and angle off are not given explicitly as functions of time. It is frequently necessary to have such dependence at hand. For example, in determining exactly the deflection to take against a fighter on such a curve one must, by cut and try, match up time along the curve from a chosen present position to the required future position of impact, with the time of flight over the range to this future position. There are three ways of getting points on the curve labeled with the appropriate time:

THE IMPLICIT METHOD

If ρ in terms of θ is substituted in equation (2), upon integration t as a function of θ will result. In fact, for the pure pursuit, one has

$$t = \frac{\rho_0}{2v_B}\left(\frac{\tan^{\mu-1}\theta/2}{\mu-1} + \frac{\tan^{\mu+1}\theta/2}{\mu+1}\right). \qquad (8)$$

By graphing or tabulating, θ is known implicitly for any t.

THE LOCAL METHOD[86]

Both ρ and θ may be expanded in power series valid in the neighborhood of any specified point on the curve. The derivatives required in these expansions are readily obtained by repeated differentiation of equations (1) and (2). Over intervals of time corresponding to normal times of flight of projectiles, the convergence is rapid.

THE MIDPOINT METHOD

This procedure renounces analysis and reverts to an approximation to the geometrical definition of the curve in question. It will suffice to sketch the method for a pure pursuit curve. Working in a fixed coordinate system, determine the position of the pursued at intervals of, say, one-quarter second. Over the first time interval let the pursuer move in a straight line from his initial position toward the midpoint of the first interval of pursued's motion. This yields a new position for the pursuer from which he can move in a second straight-line segment over the next time interval toward the midpoint of the second interval of pursued's motion. Continuing in this fashion a table of positional values given explicitly in terms of time is built by the most elementary of computing means. (This method has been used extensively in the production of synthetic motion pictures for use in flexible gunnery training devices.[90])

3.2.5 Centrifugal Force and Isogees

It is important to assess the centrifugal force to which the pursuer is subject in traversing pursuit courses. On one hand, the circle of curvature can be put to use as a replacement for a segment of the curve (over the time of flight of a bullet from a defending bomber) in deriving approximate deflection formulas. On the other hand, a knowledge of centrifugal load leads to an estimate of the angle of attack[d] of the fighter and its effect on the course flown. Finally this force or load gives the boundaries in regard to range and angle that a fighter may reach before physiological or structural limitations become operative.

For a pure pursuit curve, the radius of curvature R is given by $R = v\,dt/d\theta$, since the tangent to the circle of curvature is also the line to which the angle

[d] The angle of attack is the angle between the direction of motion of the aircraft and some reference line (such as a wing chord or a gun's bore axis or a longitudinal axis) that lies in the plane of symmetry of the aircraft.

off θ is measured. The centrifugal load n_c in units of gravity is found from

$$n_c = \frac{v_F v_B \sin\theta}{g\rho}. \qquad (9)$$

If n_c is given successive constant values, then the curves of equal load (*isogees*) that arise are, clearly, circles tangent to the pursued's track and of radius $v_F v_B / 2gn_c$. If such a family of isogees is drawn and if a family of pursuit curves is superposed, as in Figure 6, one may readily find the load for any curve at any

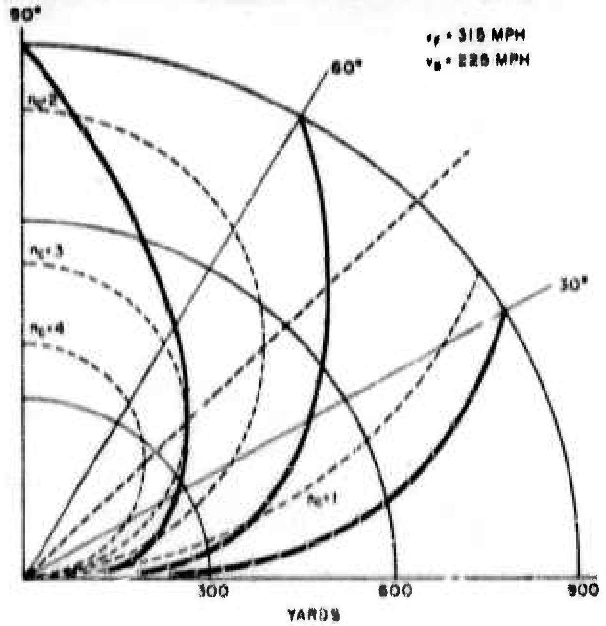

FIGURE 6. Isogees and pursuit curves.

point. As a sample conclusion it is immediately evident that with high-speed aircraft close approach at any material angle off the bomber's tail is prohibited by the high loading. This can be adduced as one argument for dispensing with all armament other than that of the defending tail of an ultra high-speed bomber.[131] With such a double family one could also trace out the growth and decay of load. The maximum load, when such exists, is found with ease analytically by maximizing n_c. The simple result is that $n_{c\,max}$ occurs at an angle off determined by

$$\cos\nu = \frac{\mu}{2}. \qquad (10)$$

This locus appears as a dotted radial line in Figure 6.

It is possible, in view of equation (9), for the fighter to reduce the load to which he is subject by deliberately using a slow speed during the firing run. As the fighter's speed decreases the maximum load diminishes steadily.[a] It may be presumed that the difficulty of the fighter's aiming problem is reduced, and that he may do his firing at greater angles off the bomber's stern, which is equivalent to offering a more difficult shot for the defense because of the higher angular rate. In addition, a longer period of sustained fire is available.[10, 139] The tactical disadvantage is that the fighter will close range up to a certain point and then will fall back.

Since a lead pursuit curve is usually a better approximation to the curve actually flown than is a pure pursuit curve, it is sometimes desirable to calculate the centrifugal load for the lead pursuit. This formula, which is a companion of equation (9), is

$$n_c = \frac{v_F v_B}{g\rho}(1-\mu\nu)\sin\theta\left(1-\nu\frac{\cos\theta}{\cos\delta}\right). \qquad (11)$$

3.2.5 Total Load Factor

For use in the next section, which calculates a more realistic deviation function, the total load on an aircraft is required. Evidently, this total load n is a suitable vector sum of centrifugal force and that component of the gravitational force lying in a plane perpendicular to the direction of motion. Forces are summed in this way because the lift, which must support the effectively heavier aircraft, lies in that plane. In the formulas given below for total load n and bank angle R (roll), the turn angle Y (yaw) is measured in a horizontal plane after projection of the flight path, and the dive angle P (pitch) is measured in a vertical plane from the horizontal projection down to the flight path. It is assumed that the aircraft is flying cleanly — with no slip or skid — at a speed v. Then[139]

$$\tan R = \frac{v\dfrac{dY}{dt}}{g + v\sec P\dfrac{dP}{dt}}, \qquad (12)$$

[a] It is a curious fact that if the *fighter's* speed is held constant and the *bomber's* speed is changed, there is a certain bomber speed which yields a least maximum load for the fighter.[131] Indeed, if $n_{c\,max}$ is expressed as a function of μ by combining equations (10), (9), and (4), then a true minimum is found to occur at the single root of

$$\frac{\mu}{2}\ln\frac{1+\frac{\mu}{2}}{1-\frac{\mu}{2}} = 1.$$

This root is 1.8.

and

$$n = \frac{\cos P + \dfrac{v}{g}\dfrac{dP}{dt}}{\cos R}, \tag{13}$$

in units of gravity.

For special flight paths, equations (12) and (13) reduce to the following forms, which are useful in various computations:

1. Circle of radius r in a horizontal plane.

$$\tan R = \frac{v^2}{gr} \qquad n = \sqrt{1 + \left(\frac{v^2}{gr}\right)^2}.$$

2. Circle of radius r in a vertical plane.

$$\tan R = 0 \qquad n = \cos P + \frac{v^2}{gr}.$$

3. Circle of radius r in a plane of action of inclination $\bar{\omega}$.

$$\tan R = \frac{\cos \bar{\omega}}{\dfrac{v^2}{gr} + \cos \theta \sin \bar{\omega}}$$

$$n = \sqrt{\cos^2 \bar{\omega} + \left(\frac{v^2}{gr} + \cos \theta \sin \bar{\omega}\right)^2},$$

where θ is the angle between the direction of flight and a horizontal line in the plane of action.

4. Helix with horizontal axis and sinusoidally varying speed. (This is the case of corkscrew avoiding action by a bomber. The details[10] are complicated but straightforward and will not be set down.)

3.3 THE EFFECT OF ANGLE OF ATTACK ON PURSUIT CURVES

3.3.1 Deviation Function and Trajectory Shift

In Section 3.1.2 it was pointed out that a knowledge of the exact curve flown by a real and perfect fighter permits the defending fire control to calculate future positions of the target. Since these future positions depend intimately on the deviation δ of the fighter's velocity from the gun-target line (Figure 1) it is necessary to analyze δ more carefully than was done in Section 3.2.2 [deviation (3)].

As footnoted in Section 3.2.5 (footnote d), an angle α exists between the bore axis of a fighter's gun and the direction of motion. This angle of attack consists of a fixed and a variable part. The fixed part is attributable to the installational setting which allows for gravity drop. The variable part is caused by a change in the load factor n (Section 3.2.6), which requires a change in the angle of attack of the wings to supply a balancing lift for this change in aircraft weight. It follows that the direction of departure of the fighter's bullet is along the diagonal of a parallelogram determined by the propellant muzzle velocity $\mathbf{v}_0$ and the fighter velocity $\mathbf{v}_F$. In

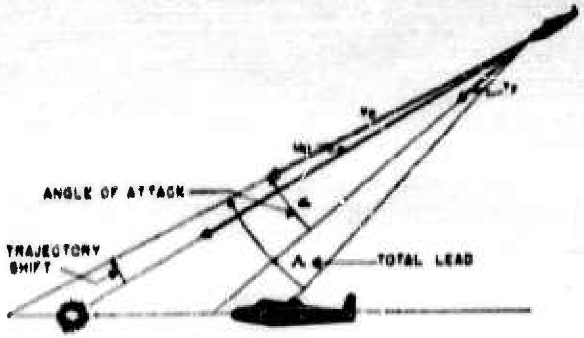

FIGURE 7. The effect of angle of attack of the bore axis.

Figure 7, it is assumed that α lies entirely in the plane of action. (This it will not do, in general, because the aircraft is banked.) The deflection problem is solved by equation (1) of Chapter 2, which gives

$$A = \frac{v_R}{\bar{u}} \sin \theta + \frac{v_F}{v_0 + v_F}\,\alpha. \tag{14}$$

In Section 3.2.2 [deviation (3)] only the first term on the right of equation (14), the normal lead, was used for the deviation. The correct deviation function is

$$\delta = A - \alpha.$$

The pursuit curve generated by this δ is called an *aerodynamic lead* pursuit. The next problem is to explore α.

Before leaving equation (14), however, the meaning of the equation from the point of view of the fighter pilot is given by Figure 8. In this representation, it is not assumed that α lies in the plane of action. Instead, the aiming allowance required by the second member on the right of equation (14) lies in the plane of symmetry of the fighter. It is called the *trajectory shift*.[10] Its size is also determined by an exploration of α.

3.3.2 Angle of Attack in Terms of Load and Indicated Airspeed

The resultant of all pressures on an aircraft wing — of the lower than atmospheric pressures on the upper

surface and of the equal to or slightly greater than atmospheric pressure on the lower surface — is resolved into a lift, perpendicular to the direction of motion, and a drag, parallel to the direction of motion. If the bullet in Figure 5 of Chapter 1 is replaced by a wing profile, that diagram illustrates this situation also. The yaw of the bullet is equivalent to the angle of attack of the wing. The lift [f] L, in pounds, is given by

$$L = C_L \frac{\rho}{2} S v^2, \tag{15}$$

where C_L = lift coefficient (dimensionless),
ρ = air density (slugs per cubic foot),[g]
S = wing area (square feet),
v = true airspeed (feet per second).[h]

[f] The drag is given by a similar expression:

$$D = C_D \frac{\rho}{2} S v^2 \quad \text{(See Section 1.2.2.)}$$

[g] The aerodynamic air density ρ is an NACA standard. It differs from the ballistic standard ρ_0, and must not be confused with the *relative* ballistic air density which is also denoted by ρ (Section 1.2.3). Both ρ (NACA) and ρ_0 (ballistic) vary at a given altitude as the temperature and humidity change.

Ballistic and NACA altitudes for given ballistic relative air density.

Relative air density	Ballistic altitude (feet)	NACA altitude (feet)
1.0	0	921
0.8	7,005	8,015
0.6	16,175	16,964
0.4	29,013	28,961
0.2	50,900	44,502

Standard atmosphere based on NACA Report No. 218.

Altitude (feet)	ρ_0/ρ	$\sqrt{\rho_0/\rho}$
0	1.000	1.000
2,000	1.061	1.030
5,000	1.161	1.077
10,000	1.354	1.164
15,000	1.590	1.260
20,000	1.877	1.370
30,000	2.675	1.636
40,000	4.080	2.021

[h] An airspeed meter measures $\frac{1}{2}\rho\,(TAS)^2$, where TAS is the true airspeed. It is calibrated to read TAS at sea level ($\rho = \rho_0$). Hence if IAS is the indicated airspeed,

$$IAS = \sqrt{\frac{\rho}{\rho_0}}\; TAS\,.$$

[i] The additional superelevation allowance for gravity drop is about one-fifth of L_2.

Experiment and theory show that the lift coefficient is an almost linear function of the angle of attack of the wing chord over a range almost up to stall. Since the guns are installed at a fixed angle with respect to the wing chord to which angle of attack

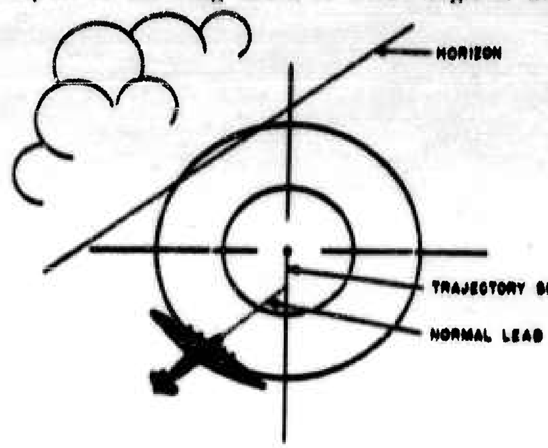

FIGURE 8. View through fighter sight of total deflection.

is usually measured, we may take the angle of attack α of the mean bore axis of the fighter's battery in the form

$$C_L = K_1\alpha + K_2,$$

where K_1 and K_2 are constants.

If an aircraft is subjected to a load factor n, we must have $L = Wn$, where W is the normal weight of the aircraft. Consequently,

$$\alpha = \frac{L_1 n}{(IAS)^2} - L_2, \tag{16}$$

where L_1 and L_2 are constants given by

$$L_1 = \frac{2W}{K_1\rho_0 S}, \quad L_2 = \frac{K_2}{K_1}.$$

By an aerodynamic argument [67, 117] which will not be reproduced, L_1 may be calculated by the formula

$$\frac{L_1}{W} = 69.1\left(\frac{1}{S} + \frac{1.8}{b^2}\right), \tag{17}$$

where S is the wing area in square feet and b is the wing span in feet. When equation (17) is used in equation (16), α will be in radians, IAS in miles per hour, and W in pounds. Next, L_2 depends on the installational angle of the guns. If we suppose [81] that the guns are adjusted so that at a particular level flight speed, $(IAS)_0$, they are horizontal,[i] then

$$\frac{L_1}{L_2} = (IAS)_0^2. \tag{18}$$

The value assigned to $(IAS)_0$ has frequently been 250 mph. Using this value and standard airplane dimensions, average values of L_1 and L_2 are given by Table 1 for certain fighters of World War II.

TABLE 1. Angle of attack constants for World War II fighters.

Class	Types	L_1	L_2
G_1	Me109, Me110G, FW190A, Ju88C	3,500	0.059
G_2	Me209, Me210, Me410	4,700	0.075
J_1	Zeke, Hamp, Oscar I and II	2,007	0.033
J_2	Tojo, Nick, Tony, Average American	3,100	0.050

3.3.3 Vicious Circle of This Approach

Formula (16) requires a knowledge of n, and to determine the load the curvature of the path must be known. A vicious circle is completed since the path cannot be determined until δ (and so α, and so n) is known. The correct resolution of this difficulty is given in Section 3.4.2. In the early literature,[19] the centrifugal load for a pure pursuit or a lead pursuit, i.e., equations (9) or (11) or some average of these, at the point in question, has been used. From the practical point of view, refinement at this point is somewhat absurd since the exact speed of the attacker cannot be known, and centrifugal load is always proportional to this speed. The only justification is found in a desire to avoid bias in calculations which are to be used to meet an average tactical situation rather than a particular one.

With such an approximation for n and under the assumption of constant speed, it is possible to integrate the differential equations (1) and (2), using a δ determined by equations (14) and (16), by numerical methods. This will not be done since defensive gunnery is interested only in a segment corresponding to the time of flight of the bomber's bullet. The required deflection can, in fact, be made out as a function of range and angle off of a fighter, rather than as a function of position along a particular curve. (See Section 4.2.2.)

Further analysis of aerodynamic lead pursuit curves by these methods has treated[170] the components of α in and at right angles to the plane of action. The perpendicular component causes the aircraft to move slightly below the instantaneous plane of action (or sag).

The patchwork theory of this section is correctly revised in Section 3.4. The detail supplied here has intrinsic value and is an exemplar of approximation methods in this field.

3.3.4 Qualitative Effect of Angle of Attack

The effect of angle of attack on a pursuit curve may be summarized qualitatively by giving the relations among pure, lead, and aerodynamic lead pursuit curves that originate at the same point and have the same v_B and v_F: (1) at long ranges, the load is low (close to 1), the angle of attack is small, and the aerodynamic curve almost coincides with the lead pursuit; (2) as the range closes, the load generally increases, the angle of attack increases, and, since the shifted trajectory must lead to impact, the fighter's

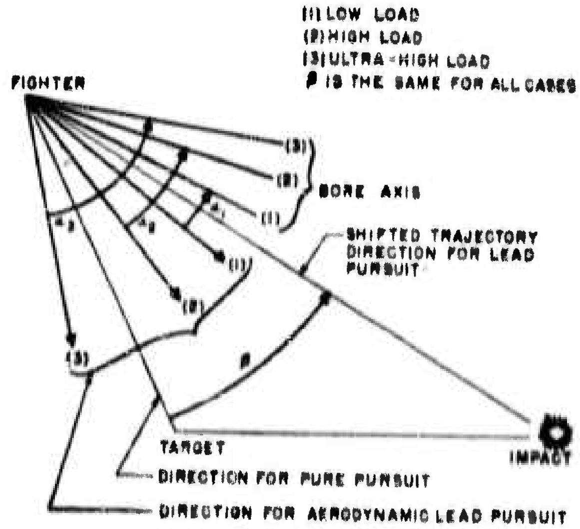

FIGURE 9. Effect of load on fighter's direction of motion.

velocity is directed more toward the bomber than it would be for a lead pursuit, so that the aerodynamic curve edges over toward the pure pursuit; and (3) for very high loads, the angle of attack may be so great that the fighter's velocity is directed behind the bomber, and yet the shifted trajectory leads the bomber by an amount sufficient to cause a hit. These three facts are partially illustrated by Figure 9.

3.4 TRUE AERODYNAMIC LEAD PURSUIT CURVE

3.4.1 New Variables — Fighter Speed, Course Curvature

In Section 3.3 the real analytic problem was sidestepped. Of the various approximations made in that section those regarding the load factor and constant speed on the part of the attacking fighter are crucial. Both do violence to the real dynamics of the situation. In the first instance, given an unknown curve, its rate of change of direction should enter naturally as an unknown. In the second instance the aircraft should be permitted to change speed naturally under the forces of thrust, drag, and gravity. Consequently, speed should also be a variable of the description.

The improvements to be made may be recognized most readily by considering the simple case of an attack, made in a vertical plane,[21] by a fighter on a bomber which moves on a straight and level track at constant speed. If a vertical plane is used, trigonometric details do not obscure the new approach. The new ideas are also kept clear by assuming that the bore axis of the fighter's gun is kept *on* the target, so that deflectional and ballistic notions are not present.

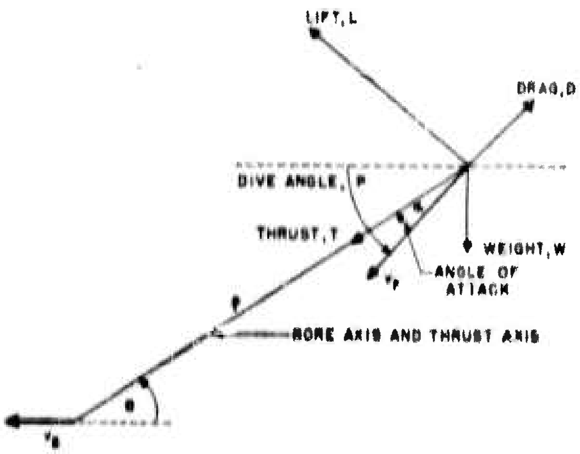

FIGURE 10. A typical instant of true aerodynamic pursuit.

Figure 10 shows the force system [1] on the airplane, and indicates that a further simplifying assumption — the bore axis and thrust axis coincide — has been made. The tangential and normal dynamical equations are respectively

$$\frac{W}{g}\frac{dv_F}{dt} = W \sin P + T \cos \alpha - D, \qquad (19)$$

and

$$\frac{W}{g}\frac{v_F^2}{R} = -\frac{W}{g} v_F \frac{dP}{dt} = L - W \cos P + T \sin \alpha, \qquad (20)$$

where R is the radius of curvature. The kinematical equations of pursuit are

$$\frac{d\rho}{dt} = -v_F \cos \alpha + v_B \cos (P - \alpha), \qquad (21)$$

$$\frac{dP}{dt} - \frac{d\alpha}{dt} = -\frac{1}{\rho}\left[v_F \sin \alpha + v_B \sin (P - \alpha)\right]. \qquad (22)$$

To utilize this set of nonlinear equations it is necessary: to determine the aerodynamic constants for a particular airplane at a particular throttle setting; to assign suitable initial values to the variables, v_F, P, α, ρ; and to proceed with the integration by a systematic process. Definite formulas are available,[24] enabling one to proceed directly from the performance values of propeller efficiency, maximum engine brake power, and the corresponding maximum level flight speed at a certain altitude, when combined with the weight and airplane geometry, to the constants T, D, L of equations (19) and (20). Turning next to the initial values, v_F, P, and ρ may be specified at pleasure at $t = 0$.

It is not at all evident, however, what initial value for the angle of attack α should be selected. This choice is connected with the nature of that part of the flight path to which some license is permitted, which just precedes initiation of the guns-bearing phase. Thus, depending on the choice of α in the usual range from 2° to 12°, it would appear that a *family* of curves is possible. Fortunately, however, numerical integration demonstrates the existence of a *boundary layer effect.* That is, the dive angle versus time curve will show a sharp hook over an interval of about one-half second. This means that different initial choices of α generate a funnel leading to a unique value of P and continuing as a single curve. By extrapolation of this single curve back to the initial time, what may be called a natural initial value for α is determined. As the final point in the discussion of the set of equations, it may be noted that although numerical integration is feasible a judicious blend [10] of graphical and nomographic procedures is enlightening.

[1] Aerodynamic forces acting on the tail surfaces of the fighter are neglected since the gun is kept on target by assumption and a description of how this is done by elevator moments is not required.

3.4.2 Complete Three-Dimensional Equations

The discussion of the true aerodynamic lead pursuit curve of Section 3.4.1 may now be extended to three dimensions. The treatment is complicated by the banking of the fighter and by the introduction of ballistic considerations in determining the lead taken by a perfect fighter pilot at each instant. (A final complication, that of allowing for sideslip by means of an additional term involving a cross force, has been considered in one account.[26] This leads to more variables than equations — because of the freedom permitted the pilot in the amount of slip — and does not furnish a unique curve. There is experimental evidence[89, 131] that fighter pilots can fly courses cleanly.) It will be assumed below that there is no sideslip. In addition to this stipulation, matters are further qualified by supposing that the bomber proceeds on a straight and level track at constant speed, that the fighter's throttle setting is left unchanged, and that the effect of gravity on the fighter's bullets may be neglected. The last assumption is tenable because the effect of gravity on time of flight is negligible and the drop itself, over the ranges in question, is sensibly removed by a slight elevation of the gun over the sight line. Bullet patterns are not considered. It is supposed that variable aim is taken to produce continuous bullet impact at one point of the bomber target.

The most convenient set of equations that has been derived[39a] is set up with respect to the rectilinear trajectory traversed in space by the bullet in going from the fighter to the point of impact. This air range is denoted by r and has an azimuth ϕ measured counterclockwise from the forward direction of bomber travel at the point of impact and an elevation θ with respect to the horizontal plane. The angle α denotes the angle of attack of this trajectory, i.e., the angle between the direction of motion of the fighter and the trajectory at the moment of departure. The angle ϵ_1 is the angle fixed at installation, from thrust axis to gun. The bank angle β_1 of the fighter is the angle from that perpendicular to the trajectory that lies in a vertical plane to the perpendicular to the trajectory that lies in the fighter's plane of symmetry.

The ballistic elements present are: v_0, the muzzle velocity of the fighter's bullet; u_0, the velocity of departure of that bullet; $(v_0 + v_F)$; and $b = 0.00180\rho/c_0$ where ρ is the relative ballistic air density and c_0 is the appropriate ballistic coefficient. In addition to these primary terms, two secondary combinations may be introduced. The angle $\alpha_1 = (v_0 + v_F/v_0)\alpha - \epsilon_1$ represents physically the angle from thrust axis to the aircraft's direction of flight. Again, in units of feet and feet per second, the time of flight t_f is given by equation (3) of Chapter 1 and is

$$t_f = \frac{\dfrac{r}{\sqrt{u_0}}}{\sqrt{u_0} - br}\,.$$

The derivative of this with respect to time is simply

$$\dot{t}_f = \frac{\dot{r}}{(\sqrt{u_0} - br)^2}\,.$$

With these letters the tangential equation is

$$\frac{W}{g}\frac{dv_F}{dt} = W(\cos\alpha\sin\theta + \sin\alpha\cos\theta\cos\beta_1) + T\cos\alpha_1 - D. \qquad (23)$$

It is evident that this summing of forces in the flight direction gives an equation of the same form as equation (19). Next, because two angles are required to specify r, two equations are required in the normal directions. These equations, taken together, form an analogue for the two-dimensional case [equation (20)]. The first equation is

$$-\frac{W}{g}v_F\left(\sin\beta_1\cos\theta\frac{d\phi}{dt} + \cos\beta_1\frac{d\theta}{dt} + \frac{d\alpha}{dt}\right) = L - W(\cos\alpha\cos\theta\cos\beta_1 - \sin\alpha\sin\theta) + T\sin\alpha_1. \qquad (24)$$

The structure is clear and the projection factors (in parentheses) are exposed. The third equation contains only accelerational and gravitational forces and will therefore be homogeneous in W. It is

$$-\frac{W}{g}v_F\left[(\sin\theta - \cot\alpha\cos\theta\cos\beta_1)\frac{d\phi}{dt} + \cot\alpha\sin\beta_1\frac{d\theta}{dt} - \sin\alpha\frac{d\beta_1}{dt}\right] = W\sin\beta_1\cos\theta. \qquad (25)$$

We must expect to find three kinematic equations also. The required deviation function is found indirectly by noting that the distance from the position of the bomber at the moment of fire to the future impact point along the bomber's track is $v_B t_f$ where t_f is the time of flight over r. It follows that the speed of the point of impact is $v_B + v_B\dot{t}_f$. It is such a ghost point which is being pursued since all work is relative to the trajectory. Consequently by projection of the

fighter speed and of the ghost speed on the trajectory the range rate equation

$$\frac{dr}{dt} = -v_F \cos\alpha - v_B(1 + i_f)\cos\phi\cos\theta \quad (26)$$

arises. Finally, two equations for the rates of change of azimuth and elevation of the trajectory are written down via projections normal to r. These equations are

$$\frac{d\phi}{dt} = -\frac{1}{r\cos\theta}\left[v_F \sin\alpha\sin\beta_1 - v_B(1 + i_f)\sin\phi\right] \quad (27)$$

and

$$\frac{d\theta}{dt} = -\frac{1}{r}\left[v_F \sin\alpha\cos\beta_1 - v_B(1 + i_f)\cos\phi\sin\theta\right]. \quad (28)$$

It will be remembered that $\dot{r}$ will occur in the left-hand members of the last three equations through i_f.

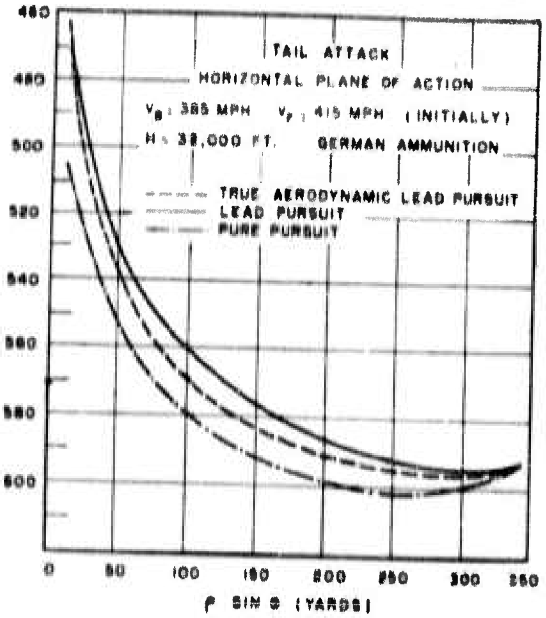

FIGURE 11. Comparison of pursuit curves.

Instead of azimuth and elevation of the trajectory it would be possible to use as angular coordinates the angle off bomber's track of the trajectory and the elevation angle of the (instantaneous) plane of action. This would be an heuristic rather than a technical improvement. It would bring the situation in symmetry with the usual pursuit curve description in terms of range and angle off, and would expose sag (Section 3.3.3) by showing the slow rotation of the plane of action.

It is also evident that by proper choice of the function giving distance between bomber and the ghost point, arbitrary behavior on the part of the attacking pilot, rather than perfect lead behavior, can be introduced painlessly.

Systematic and precise methods of numerical integration of such systems are available,[39b] and a considerable number of courses covering modern tactical ranges including high-speed bombers and fighters has been computed.[40] (An example of a true aerodynamic lead pursuit is given in Figure 11.) Computed courses have been carefully compared with those actually flown.[39c] During a sight assessment program at the Patuxent River Naval Air Station, an F6F-3 fighter equipped with a Mark 23 gyro gun sight was flown in pursuit of a bomber whose speed was 130 knots at an altitude of 6,000 ft. From camera records the range, azimuth, and elevation of the fighter with respect to the bomber, and the fighter's lead, at quarter second intervals, were known. To test the theory, it is reasonable to select those attacks in which the fighter's lead was well-nigh perfect. When this is done the coincidence between computed and observed values is quite remarkable, showing an average absolute range difference of about 9 yd, and average absolute differences in angle of about 10 mils.

In concluding this section, it is to be emphasized that the rounded character of this theory should make it immediately applicable to future problems in aerial warfare whether they deal with aircraft or missiles.

3.5 CERTAIN TACTICAL CONSIDERATIONS

3.5.1 Combat Maneuvers by Bomber

Throughout this chapter, the bomber under pursuit has moved on a straight line course at constant speed. This rectilinear assumption is tenable in the light of the requirements of massive formations, the nature of bombing runs, and consideration of range of operation. However, whenever a bomber must act alone or in a very small formation, as may well happen at night, on patrol, or on pathfinding tours, it may suffer saturation and coordination attack by enemy fighters, against which support fire power cannot be brought to bear. Violent and frantic evasive action suggests itself as a defensive measure. But further thought shows that although such random course changes may make the aiming problem of an attacking fighter more difficult and may destroy the coordination of two or more attackers, it

will at the same time deceive the gunners of the bomber itself. Instead of evasive action the concept of *combat maneuvers* arises. This is a deliberate, planned, and properly timed maneuver designed to reduce the possibility of damage to the bomber [102] by (1) offering the fighter a changing deflection in amount and in line, (2) shortening the attack, (3) increasing the loading on the fighter, and (4) making use of a violent and turbulent slipstream. Since the maneuver is planned and practiced, one may provide defending gunners with a specific set of simple rules for return fire,[78, 102] which may therefore be reasonably effective. The most common combat maneuvers are steep diving turns and corkscrews. Only the corkscrew will be discussed. This maneuver has the added advantages of maintaining mean track and (sensibly) height, and of being the best counter against a coordinated attack.

It is not profitable to attempt an exact mathematical solution of the problem of a fighter following a bomber in the corkscrew. The fighter cannot lay off deflection nearly as well as he may for the rectilinear case. This means that he will point in a random fashion in the neighborhood of the lead point, as air experiments show.[108] When necessary, one may rapidly appraise the situation by an obvious three-dimensional analogue of the midpoint method of Section 3.2.4. A standard corkscrew to port may be described as follows:[102, 109] (1) diving turn to port, changing course about 30°, losing perhaps 1,000 ft, and building indicated airspeed up to 220 mph, (2) climbing turn to port gaining perhaps 700 ft, and losing speed to 180 mph, (3) rolling and continuing with a climbing turn to starboard gaining perhaps 300 ft, and losing speed to 150 mph, (4) diving turn to starboard until speed is about 190 mph, (5) rolling and continuing with a diving turn to port until speed is about 220 mph. And the cycle may be continued. Note that after item (3) the course should be back on the opposite side of the original heading by 30°. About 500 ft will be lost during the cycle, and 50° will be about the maximum bank. The time per cycle is about 47 sec, which allows 4 sec for each change and for each roll, and allows 8, 9, 5, 6 sec respectively for the four phases of a cycle.

Effect of High Mach Numbers on an Attack

The trend in military aviation is toward higher and higher speeds and altitudes for both bombardment, and, consequently, pursuit operations. When a high-speed fighter attempts to attack a high-speed bomber on a pursuit curve, it is apparent from equation (9) that high loading will result at material angles off stern, and at those ranges within which ordinary ammunition is effective. Although it is not known exactly how load and growth of load affect the pilot's aiming, there still exist upper load bounds which may not be exceeded for physiological and aerodynamical reasons. Blackout (but not the weight of the limbs) may be inhibited by the use of suitable antipressure suits. The aerodynamical restriction, however, being a question of wing design, is not so readily dismissed. This restriction may be considered in some detail.[110]

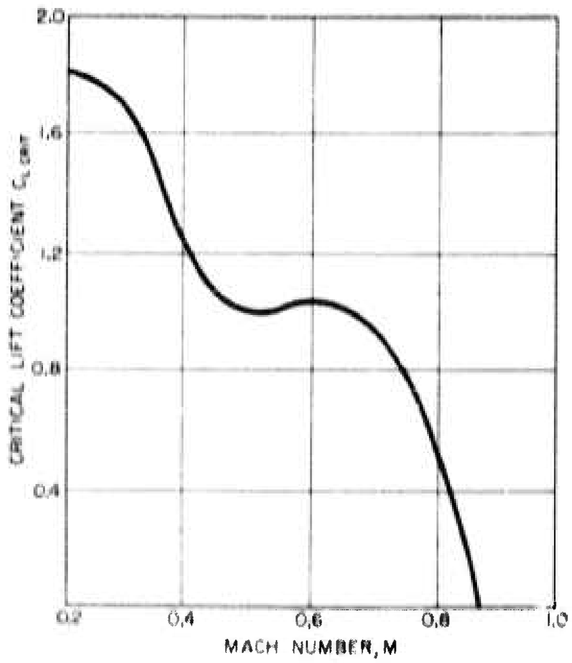

FIGURE 12. Typical critical lift coefficient curve.

The Mach number M is the ratio of the speed of a body to the speed of sound. Since the speed of sound decreases with altitude from about 761 mph at sea level to about 660 mph at 50,000 ft (true airspeed), those modern aircraft designed to perform best at altitude must accept Mach numbers approaching unity. For each wing there exists a curve giving a critical value for the lift coefficient C_L in terms of the Mach number. At any given Mach number, an attempt to exceed this critical value will lead to excessive shudder and vibration which may result in

structural damage.[8] Consequently, if the effective weight Wn of the airplane requires a lift coefficient in excess of $C_{L\,crit}$, shuddering flight must be accepted. Figure 12 shows a typical curve. It resembles the critical curve for jet fighters.

With the aid of the $C_{L\,crit}$ curve, regions of comfortable flight in coordinates of load versus indicated airspeed may be obtained. The *IAS* of stall at $M = 0.2$ is an efficient parameter in the discussion. A readily deduced approximate formula for the stalling speed v_{00} at $M = 0.2$ is

$$v_{00} = 16\sqrt{\text{wing loading}},$$

where v_{00} is in miles per hour, and the loading is in pounds per square foot. With the aid of equation (15), the curve of critical load, n, versus *IAS*, v, for a given altitude, is given by

$$n = \frac{C_{L\,crit}\frac{v}{a}}{C_{L\,crit}(0.2)}\frac{v^2}{v_{00}^2} = F(v),$$

where a is the *IAS* of sound at the chosen altitude. An $F(v)$ family, with the altitude as parameter, is a nested set of arches with legs set on the line $n = 1$. As the altitude increases, the region of comfortable flight so bounded shrinks materially. Such curves, when used in conjunction with the load factor formulas of Section 3.2.5 and the third form of Section 3.2.6, define a region, bounded by a closed range versus angle off curve, within which a fighter cannot penetrate on a pursuit curve attack.

[8] For very low Mach numbers, an attempt to exceed $C_{L\,crit}$ leads to the usual type of stall.

3.6 SUMMARY

Section 3.1 discusses the tactical setting of pursuit curves in aerial warfare, and considers the reasons why a detailed study of such curves is a prerequisite for gunnery investigations.

Section 3.2 deals with the geometry of pure and deviated pursuit curves. The centrifugal forces experienced in such curves is computed, and general formulas for the total force — gravity plus centrifugal — are given.

Section 3.3 reviews and refines the theory of fixed gunnery in order to determine more exactly the angle by which the attacking fighter deviates from pure pursuit. The angle of attack of the fighter's guns is explored in detail, since this determines the deviation as much as does the deflection taken by the fighter.

In Section 3.4, the anatomy of the true aerodynamic lead pursuit curve is considered. The important points are that the fighter speed and the curvature of the course are now variables of the problem.

Section 3.5 discusses avoiding action on the part of the bomber and the aerodynamic limitations on the fighter in making a pursuit curve attack.

Chapter 4

OWN-SPEED SIGHTS

4.1 INTRODUCTION

4.1.1 Positional and Rate Deflection Formulas

The deflection formulas deduced in Chapter 2 are essentially of two types. One type uses velocities and angles to express the lead and the other employs angular rates measured at the gun. Since the first type is sensibly positional, simple mechanization appears plausible. The more complicated problems introduced by the necessity of measuring rates will be dealt with in Chapter 5. This chapter considers only developments of the non-angular-rate formulas.

4.1.2 Prediction of Approach Angle

It was pointed out in Section 2.2.3 that positional formulas, of which equation (1) is typical, require a value for the approach angle of the target, and that this value is not readily measured. The alternative is to predict the approach angle. This can be done by assuming that a quite special tactical situation is at hand. The argument follows.

The primary function of the defensive gunnery of a bomber is to prevent its own aircraft from being shot down. Supporting other members of a formation by cross fire is a secondary function, and decimating an enemy fighter force is tertiary. A fighter attacking on a pursuit curve is most likely to shoot down the bomber, so that the defensive gunnery — in exercising its primary function — must deal adequately with such attacks. Fortunately, approach angle and course curvature can be predicted for pursuit curves, and this prediction may be utilized in the design of a mechanism. By taking advantage of such a special and dangerous situation, simple fire control may be developed which may be operated with a minimum of manipulation error on the part of the gunner. This is the basic logic of the chapter.

It is obvious that fire control should commit itself to such prediction of the attacker's behavior only if warranted by the current tactical situation and, even then, only if more flexible control is less effective because of difficulties in design, production, or manipulation. Since, in addition, such commitment implies that the defense lags the offense (it permits the opponent to set the tactics before it can be designed), it follows that this type of dependence on enemy cooperation is a stopgap procedure. This chapter is the history of such a stopgap — the own-speed sight.

A specialized weapon is designed to meet an expected *average* of a given class of tactical situations, e.g., average fighter speeds and average attack ranges. A proponent of a more flexible weapon, individualized to account for variations in the opponent's behavior, may justly seize upon this point.

4.1.3 Definition of Own-Speed Sight

An own-speed sight is a mechanism that displaces the bore axis of a gun from the line of sight by an angle $v_G \sin \tau / v_0$, where the gun-mount velocity v_G and the muzzle velocity v_0 are preset. The variable size of this angle depends, then, only on the positional angle τ, or the angle off of the target with respect to the aircraft's direction of motion. The displacement angle is laid off toward the rear of the firing aircraft in the plane of action. Since mount velocity and propellant velocity combine according to such an angle in yielding the bullet's velocity of departure, a bullet directed by this mechanism will depart along the line of sight. To hit a fixed ground target on a calm day, the gunner need only hold the sight on target and fire. There is no ranging with an own-speed sight. To hit a fighter attacking on a pursuit curve, some percentage of v_G (generally less than 100) is set in to decrease the full own-speed deflection. The reason is that the fighter deviates forward of the line of sight (of the instant of fire) during the bullet's time of flight. Much of this chapter is concerned with this percentage factor.

4.2 DEFLECTION AGAINST PURSUIT CURVES

4.2.1 Percentage Factor for Pure Pursuit

For a pure pursuit curve the approach angle α is always zero and the target speed is constant. Since this curve lies in the plane of action, formulas (8a) and (8b) of Chapter 2 apply and yield

$$\sin A = \frac{v_G}{v_0}\sin\tau - \tfrac{1}{2}qt_f\frac{v_T}{v_0}\omega_T .$$

For pure pursuit $\omega_T = \dot\tau$ and, by equation (2) of Chapter 3 with notational changes, $\omega_T = (v_G \sin\tau)/r$. Hence

$$\sin A = k_0\frac{v_G}{v_0}\sin\tau$$

$$k_0 = 1 - \tfrac{1}{2}qt_f\frac{v_T}{r} . \qquad (1)$$

This introduces the percentage factor k_0 referred to at the end of Section 4.1.3. For a pure pursuit curve computations show[19a] that

$$\left(\frac{r}{qt_f} \doteq v_0\right) \quad k_0 \doteq 1 - \tfrac{1}{2}\frac{v_T}{v_0}$$

is an excellent approximation and agrees with an earlier estimate.[14b] The estimate[21c]

$$k_0 = \frac{1}{1 + \dfrac{v_T}{v_0}}$$

is not so happy.

4.2.2 Percentage Factor for Deviated Pursuit

If the fighter, while still flying in the plane of action, has a deviation angle δ (see Figure 1 of Chapter 3), formulas (8a) and (8b) of Chapter 2 give

$$\sin A = \frac{v_G}{v_0}\sin\tau - q\frac{v_T}{v_0}\sin\delta - \tfrac{1}{2}qt_f\frac{v_T}{v_0}\omega_T\cos\delta . \quad (2)$$

In this case $\omega_T = -(\dot\tau + \dot\delta) = [v_G \sin(\tau+\delta)]/r$. But for a lead pursuit curve we may take $\sin\delta = (v_G \sin\tau)/\bar u_T$, where[a] $1/\bar u_T$ is rarely greater than 1/2,500. Thus it is numerically reasonable to let $\omega_T = (v_G \sin\tau)/r$, and $\cos\delta = 1$ to obtain

$$\sin A = k_1\frac{v_G}{v_0}\sin\tau$$

$$k_1 = 1 - q\frac{v_T}{\bar u_T} - \tfrac{1}{2}qt_f\frac{v_T}{r} . \qquad (3)$$

4.2.3 Percentage Factor for Aerodynamic Lead Pursuit

Consider now an aerodynamic lead pursuit curve in the sense of Section 3.3. Interpret the angle of attack α of the mean bore axis of the fighter's guns as that component of the true α which lies in the plane of action. (An exact analysis has been made.[29b]) The deviation angle δ is

$$\delta = A_T - \alpha = \frac{v_G}{\bar u_T}\sin\tau - \frac{v_{0T}}{v_{0T} + v_T}\alpha . \qquad (4)$$

It is now evident that appropriate combination of equation (4), equation (11) of Chapter 3 (extended to give n by combination of n_c with gravity), and equation (2) will yield the deflection to take against a fighter attacking on an aerodynamic lead pursuit curve, as a function of altitude, range, angle off, speeds, ballistics of the two types of ammunition, and fighter type. Consideration of the required algebra shows that it will not be possible to factor out $(v_G \sin\tau)/v_0$ as was done in determining k_0 and k_1. But one can still write

$$\sin A = k_2\frac{v_G}{v_0}\sin\tau ,$$

and the dependence of k_2 on the speed of the gun mount *will be very slight* indeed. In fact, if the ammunition used by the duelists is given, the percentage factor k_2 will depend on fighter type, altitude (affecting the behavior of the fighter and ballistics), range (affecting lead and ballistics), fighter speed, and angle off.

4.2.4 Variables on Which Percentage Factor Depends

If only one of the last four variables is varied, then the following qualitative predictions about the variation in k_2 can be made, on the basis of the physics of the situation. (1) If the altitude is increased, the lead required of the fighter pilot decreases since his bullets are faster, that part of the total lead required of the bomber to allow for target motion decreases because

[a] The subscript T means that a distinction between the fighter's ammunition and the bomber's ammunition must be made.

of less motion and faster bomber bullets, and both effects cause the required percentage to *increase*. (2) If the range *increases*, there is less lead on the fighter, less angle of attack, his path is closer to a lead pursuit, more allowance for target motion is required, the k_3 *decreases*. (3) If the fighter speed *increases*, the greater target motion forward more than compensates for the increased load which tries to make the flight path come closer to pure pursuit, and k_3 *decreases*. (4) In the forward hemisphere the future ranges are less than in the rear hemisphere and mean bullet velocities are higher, so that the percentage is somewhat *greater* in the forward hemisphere. (5) As angle off tends to 0° or to 180°, starting at a point somewhat forward of the beam because of item (4), load decreases, the path is closer to a lead pursuit, more allowance for forward target motion is required, and the percentage *decreases*. In sum, the defending gun moves forward from its full own-speed position to allow for target motion forward of the line of sight at the instant of fire. It will still point behind the line of sight at a percentage of the full value that varies as discussed above.

These predictions are borne out by careful computation[29a] and may be made very evident graphically.[31a] For a fighter of Class J_3 (Table 3 of Chapter 3) at a *TAS* of 315 mph, and attacking at 22,000 ft, the way in which k_3 varies with range and angle off is

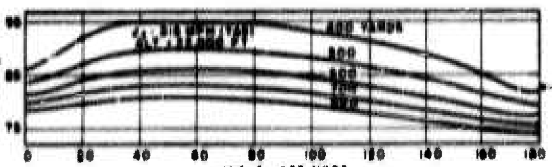

FIGURE 1. Variation of k_3 with range and angle off.

shown in Figure 1. The fighter's ammunition is characterized by v_0 = 2,357 fps and c_b = 0.24, and the bomber's ammunition by v_0 = 2,700 fps and c_b = 0.46.

4.2.5 Choice of Optimum Percentages

One of the most important ways in which an own-speed sight is kept simple is by avoiding range input. The second way in which simplicity is obtained is by making the deflection it supplies vary angularly only with the sine of the angle off. Hence if a fixed percentage of own speed is to be used, a value of k_3 must be selected that corresponds to some average range and angle off.

The choice of optimum percentage against a fighter of a given class attacking at a given speed at an assigned altitude may be made as follows.[31b] Since $A = (k_3 v_G \sin \tau)/v_0$, $dk_3 = (v_0 \csc \tau dA)/v_G$. If an error of ±5 milliradians can be tolerated in deflection, then, for example, at v_G = 225 mph and v_0 = 2,700 fps (caliber 0.50 AP M2 ammunition), $dk_3 = \pm 0.04 \csc \tau$. If the curves $\pm 0.04 \csc \tau$ (Figure 2) are drawn

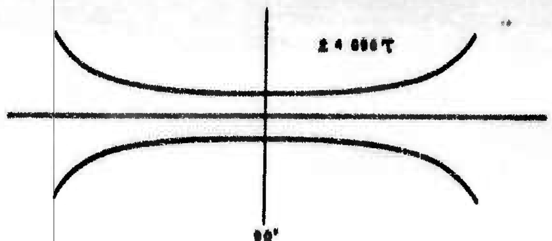

FIGURE 2. Overlay for Figure 1.

on tracing paper, then the band defined by these curves may be used as an overlay on Figure 1, and one may see what parts of the percentage curves fall to lie between the two cosecant curves. The fanning out at the ends of the band is proper since a large error in percentage at a small angle off means a large percentage error in a small deflection which can give the same error in aim as a small percentage error in a large deflection. The overlay can now be moved up and down until the most important zone is covered.

For the example used, a percentage of 83 is selected as optimum. It will be observed that the 400-yd curve from 30° to 140° is not covered by this choice, nor is the 500-yd curve from 50° to 100°. This is not serious since at these points a fighter is under high loads. (Combat plots of attacks as a function of range and angle off show very few attacks on the beam at close range.) In choosing the percentage, long ranges[b] in the forward hemisphere are emphasized. This makes the percentage exact at expected typical positions of the attacker.

A summary of optimum percentages[31c] chosen by the above principles is given by Tables 1 and 2.

The technique of this subsection shows that it is not necessary to use a special percentage for a particular gun position, such as nose or tail, with a

[b] Frontal attacks are initiated by longer ranges because of their short duration, and fire against them at long present range means the future range is short because of the rapid closure.

restricted cone of fire. It also shows that no real gain in accuracy would result by elaborating the mechanism to vary the percentage as some function of angle off. However, it is clear that variation with altitude and fighter speed is significant and should be taken into account.

TABLE 1. Average percentages of own-speed deflection.

Fighter class	G_1		G_2		J_1		J_2	
Fighter speed (*TAS*)	300	400	300	400	300	400	300	400
Altitude								
0	70	61	71	62	68	61	69	61
10,000	77	68	80	70	74	67	76	68
20,000	85	75	89	78	80	73	84	75
30,000	92	82	97	85	86	78	91	81
40,000	100	89	106	93	92	84	98	88

TABLE 2. Rules for modification of average percentages.

	Fighter class			
	G_1	G_2	J_1	J_2
1. The percentage at 20,000 ft for 350-mph fighters is	80	83	76	79
2. For every 10,000-ft *increase* in altitude *add*	7	8	6	7
3. For every 100-mph *increase* in fighter speed *subtract*	9.5	11	8	9
4. For every 100-yd *increase* in beam range of 700 yd *subtract*	3	5	2.5	3

A final reduction of the above tables has been suggested cooperatively by the Research Division of the Army Air Forces Central School for Flexible Gunnery and the Applied Mathematics Group at Columbia.[30] First decide on the *type* of fighter with which one must contend, e.g., class J_2. Secondly, assume that this fighter will operate at essentially the *same IAS* at all altitudes, e.g., 275 mph. This will correspond to an increasing *TAS* at increasing altitude in accordance with the footnote to Section 3.3.2. Thirdly, construct a chart such as that illustrated in Figure 3, in which the slope of each line is the proper k_0^{-1}. In this chart the optimum percentage has been absorbed to produce a fictitious own speed $v_0' = k_0 v_0$, and full own-speed shooting can be done with v_0'. Finally, a navigator has available at all times his *TAS* and altitude. He is in a position to relay to gunners, via such a chart (and an intercom), the appropriate v_0'.

4.3 VERIFICATION OF THEORY

4.3.1 Pro and Con Combat Evidence

The own-speed sight, with some optimum percentage of the true airspeed of the gun mount set in, is useless against target paths which differ markedly from aerodynamic lead pursuit curves. In justification, then, of this specialised weapon, it must be established that pursuit curve attacks were the usual attacks during the period when this method of fire control was to be used. It was German opinion[c] that 95 per cent of all attacks of World War II, made by German fighters during daylight on Allied bombers, were pursuit curve attacks with an attempt at properly varying deflection. More objective evidence is supplied in Figure 4. A rather random selection[d] was made from a collection of 285 attacks (1944) by German fighters. The analysis was carried out by the assessment of combat gunnery film. Records of these attacks are also available which show the aim wander and plot angle off versus range. When hits are distributed over such great range intervals, it is obvious that some facsimile of a pursuit curve is being flown.

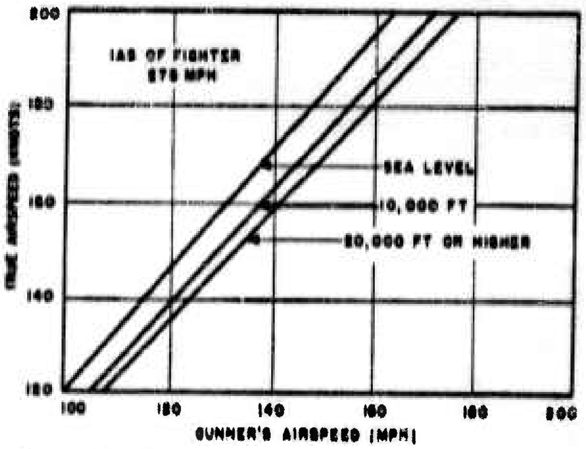

FIGURE 3. Calculation in the air of own-speed setting.

There are three parts to the counterevidence. First, the Japanese seemed to prefer the use of fixed

[c] Interrogation of Th. W. Schmidt, Director of Film Analysis Section, by E. W. Paxson at Coburg, Germany, July 5 to 8, 1945.

[d] Any bias in this selection is in the direction of choosing attacks with many hits as opposed to those in which bursts were fired throughout the attack with indifferent success. But even in the latter case the pilot was trying to fly lead pursuit. This represents a deviation from the average for which the fire control is designed.

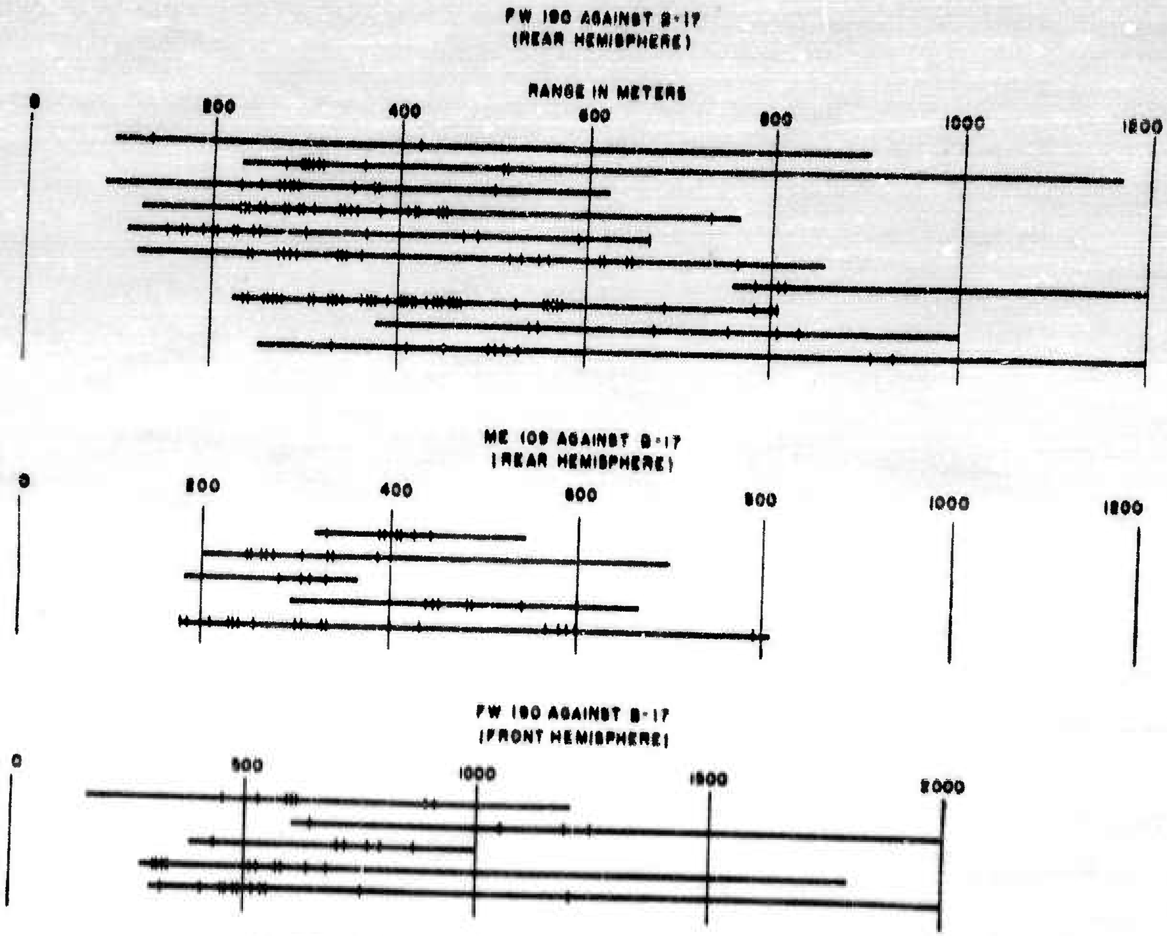

FIGURE 4. Bursts and hits within the burst as a function of range.

medial deflection pursuit curve attacks.[18] It does not follow that own-speed sights will perform badly against such target paths. Second, against high-speed bombers (B-29 at a *TAS* of 300 mph at 24,000 ft), there is experimental evidence that the leads taken by a fighter executing a frontal attack will differ[88] from those predicted by pursuit curve theory. A bias in performance of an own-speed sight would occur. Third, the closing months of World War II saw increased emphasis in both Europe and the Pacific on offset gun attacks. (See Chapter 8.) Again, saturation attacks from the rear by massed fighter formations (see Section 4.6) may require much support fire.

One tends to conclude that the emphasis on defensive deflection of the own-speed type for positions such as waist, tail, and nose of moderate-speed bombers was not misplaced throughout most of World War II.

4.5.2 Check of Optimum Percentages by Airborne Experiment

At the request of the Army Air Forces Board, a check on the validity of the theory of optimum percentages described in Sections 4.2.3 and 4.2.4 has been carried out.[89] Experimental data giving the bearing, range, speed, and deflection taken, for an attacking fighter, are available from careful camera analyses arising from assessment studies of lead computing sights. (See Chapter 7.) Since the path actually flown by the fighter was accurately known it was possible to compute the correct defensive deflection at each instant. An experimental percentage, k_{exp}, was chosen which minimized the sum of the squares of the errors in the plane of action. On the other hand, to calculate an optimum percentage, k_{opt}, the correct fighter type, average speed, and

altitude only, were used. (Since the fighter used caliber 0.50 AP M2 or API M8 in taking deflection, Tables 1 and 2 do not apply, since they were based on typical German and Japanese 20-mm ammunition.) Points at an interval of one-half second were taken. A point is an O point if the fighter's lead gave a mean point of impact *on* its target (a sphere of radius 7 yd). A point is a B point if it comes *before* an O point by 2 sec or less. All other points are N points. Ranges were up to 900 yd only. The results are given in Table 3. The agreement is satisfactory.

TABLE 3. Theoretical and experimental own-speed percentages.

Source	Data for attacks	Type of point	Number of points	100$k_{s\,opt}$	100$k_{s\,exp}$	Average error in plane of action using $k_{s\,opt}$ (milliradians)
AAFPGC Eglin Field (ST 2-44-120)	7 rear quarter, 6 beam, 7 nose, 3 strafing v_B = 178 mph (B-24) v_F = 348 mph (P-63) Alt = 8,000 ft (averages)	O	25	79.6	76.3	6.6
		O, B	41	79.6	76.5	6.2
		O, B, N	138	79.6	87.3	32.8
AAFPGC Eglin Field (ST 2-44-22)	1 rear quarter, 1 beam, 5 nose v_B = 207 mph (B-17) v_F = 348 mph (P-63) Alt = 8,000 ft (averages)	O	9	81.8	70.9	8.1
		O, B	17	81.8	68.2	12.4
		O, B, N	48	81.8	63.9	18.7
Patuxent River Naval Air Sta. (Mk 23–Mk 8 assessment)	16 beam and rear quarter v_B = 150 mph (B-24) v_F = 300 mph (F6F) Alt = 6,000 ft (averages)	O	61	84.5	83.9	6.0
		O, B	83	84.5	83.5	5.7
		O, B, N	157	84.5	80.6	7.3

For the second source, the sparseness of correct lead points means that the theory is not expected to hold well. In the opposite direction the good agreement in the third class can be attributed to the use of a lead computing sight by the attacking fighter which led to excellent shooting on his part.

4.3.3 Analytical Checks

It is also possible to validate the approximate theory analytically. A large number of perfect aerodynamic lead pursuit curves has been computed [40] as sketched in Section 3.4.2. For each of these the correct lead to be taken in the plane of action is given. The only course that can be checked immediately without calculation is Course L_6. This is a flat tail attack, made at an average speed of 415 mph at an altitude of 31,000 ft by a class J_s fighter, on a bomber flying at 385 mph. The ammunition used by both fighter and bomber agrees with that used in calculating Table 1. By interpolation in that table, k_s = 0.80. The corresponding deflection A = 108 sin τ agrees with the exact values with an average error of about one milliradian. A systematic check of this nature is at hand,[80a] but is academic compared with that of Section 4.3.2. The errors in the plane of action, when particular values of $k_{s\,opt}$ are used, are of the order of 2 milliradians.

4.3.4 Earlier Work

Earlier efforts at validating the theory[80, 80a] are mentioned bibliographically to complete the picture of experimental evidence. This work compared 85 per cent of own-speed deflection with correct deflection and suffered from many crudities.

4.4 POSITION FIRING

4.4.1 Necessity of Eye-Shooting Methods

It is unfortunately true that the development of weapons and of adequate methods to control those weapons rarely keep pace. Control is usually a poor second. For example, hand-held guns were installed in waist, nose, and tail of heavy bombers, but no me-

chanical provision was made for the difficult problem of determining the deflection to take with such guns. Under such circumstances an almost intolerable burden is placed on a Service Training Command which must produce gunners who can estimate approximately the required deflection by eye. The earliest methods of *eye shooting* were based on a perception of tracking rates. Such methods are properly considered in Chapter 5. This section deals with a translation into eye-shooting terms of the own-speed concept exploited in this chapter. The logic of Section 4.1.2 is applicable here.

4.4.2 Derivation of Rule of Thumb

Supposing that a bomber is under pursuit curve attack, the required defensive deflection is given by

$$\sin A = k_g \frac{v_G}{v_0} \sin r.$$

If it can be agreed that the bomber will operate approximately at a fixed altitude and a fixed speed v_G, and if a standard type fighter is expected to attack at an approximately known speed, then by means of Table 2 a value for k_g may be selected and $\sin A$ will depend only on the angle off r. During World War II, over Europe conditions standardized very well at: v_0 = 2,700 fps, v_G = 225 mph, altitude = 22,000 ft, v_T = 325 mph, fighter type G_1, G_2. Hence from Table 2,

$$100k_g = 80 + \left(\frac{25}{100}\right)9.5 + \left(\frac{2{,}000}{10{,}000}\right)7 = 83.8 \quad (G_1),$$

and

$$100k_g = 83 + \left(\frac{25}{100}\right)11 + \left(\frac{2{,}000}{10{,}000}\right)8 = 87.4 \quad (G_2).$$

A value of 0.85 has commonly been adopted for k_g under these conditions. Then

$$\sin A = 0.104 \sin r.$$

If a unit of 35 milliradians (called one RAD) is used we have values shown in Table 4. If the gunner is

TABLE 4. Deflection over Europe.

r (degrees)	90	45	22.5	11.25	0
		135	157.5	168.75	180
A (milliradians)	104	73.5	39.8	20.3	0
A (RADS)	2.87	2.10	1.14	0.58	0
A (rounded RADS)	3	2	1	½	0

provided with a ringsight which is aligned with the bore axis and has two (or three) rings subtending one and two (and three) RADS at his eye, then under the above conditions, if he can estimate the angle between the fore and aft axis of his own aircraft and the line out to the head-on fighter, he knows from Table 4 how much deflection to take. Since the angle off is in the plane of action, the amount of deflection is independent of the elevation of the plane of action. Doctrinally,[179, 180] he looks at the fighter's position relative to the bomber and decides whether the target is on the 3, 2, 1, or ½ RAD cone (Figure 5). If

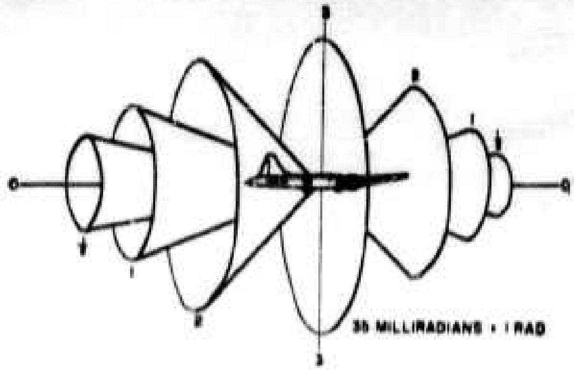

FIGURE 5. Key cones for position firing (Army Air Forces).

the angle off is not a key angle, linear interpolation is assumed,[61] and, in fact, as the attack develops and the target slides from one cone to another the gunner is expected to change his deflection continuously. In the Royal Air Force (and early Eighth and Ninth Air Forces) version of position firing, the gunner held a constant deflection of 3 RADS over the zones 60° to 90° and 90° to 120°, 2 RADS over the zones 30° to 60° and 120° to 150°, 1 RAD over the zones 10° to 30° and 150° to 170°, and 0 RADS over the zones 0° to 10° and 170° to 180°. This is called the *zone system*[166, 169] and antedated position firing as described above.

The amount of deflection is now established. It remains to consider the line along which this deflection is laid off. Since the gunner's eye is in the plane of action, which is assumed to remain sensibly fixed during the guns bearing phase of the attack, and since the vector $0.85\mathbf{v}_G$ is assumed to lie in this plane, parallel to $\mathbf{v}_G$ and reversed, it follows as illustrated by Figure 6 that *except when the target is at an angle of A/2 ahead of the beam*, the point of aim is always on a line connecting the target to the point on the gunner's horizon dead astern. When the attack is in the forward hemisphere the target is positioned linearly between the pipper of the sight and a point

on the horizon dead ahead. This applies to straight and level flight and will be supported by the apparent direction of motion of the target as seen by the gunner. In certain cases of avoiding action (Section 3.2.6, form 4) no apparent motion may be evident over a time interval of perhaps a second. In this instance of a *lead plateau*, i.e., constant lead, the pipper is held between the target and the extended fore-and-aft axis of the bomber.

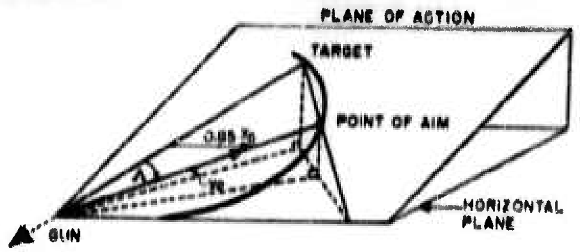

FIGURE 6. Line of deflection in position firing.

4.4.3 Variations in Standard Rules

The above discussion was predicated upon very special operating conditions. It is abundantly evident from Section 4.2.4 that changes in bomber speed, in operational altitude, and in fighter speed and type, require concomitant modification in the eye-shooting rules which the gunner is asked to memorize and apply. These changes have been made for various aircraft and conditions.[82, 90, 84, 152]

4.5 OWN-SPEED SIGHTS

4.5.1 Types

The mechanical application of the principle of compensating for the speed of the gun mount by introducing an angle $(k_g v_G \sin \tau)/v_0$ between bore axis and sight line is as old as military aircraft. The first version — a *wind vane* on the front end of a gun — is apparently a British design of 1915 which was immediately exploited by the Germans.[219] Such a sight uses the *air stream* to maintain a vector $k_g \mathbf{v}_G$ parallel to the flight velocity $\mathbf{v}_G$. Modern own-speed sights obtain the angle off τ by takeoffs from gears fixed in the aircraft. The required lead angle is then obtained either by a mechanical construction of the vector triangle[181] $\mathbf{u}_0 = \mathbf{v}_0 + k_g \mathbf{v}_G$ or by a gear and cam calculation of deflection formulas for lead in azimuth and elevation.[204] These types will be called the *vector sight* and the *algebraic sight* respectively. A fourth type, a *linear correction sight*, has been designed for the tail stinger of a B-17.[107] Catalogues of the numerous versions of each type are available.[81, 106, 217, 218] This section will discuss the Sperry K-13 and K-11 sights which are, respectively, examples of the vector and algebraic types.[84]

4.5.2 The K-13 Vector Sight

The inputs to the K-13 vector sight are $k_g v_G = v_G^*$ (which is set in on a dial by the gunner), and the azimuth and elevation of the bore axis of the gun with respect to the aircraft (which are supplied automatically to the mechanism by flexible cables from the turret gears). The output system consists of a collimated reticle image (infinity focus) appearing on a combining glass which rotates about a horizontal axis perpendicular to the bore axis to produce the vertical component of deflection. Preceding the combining glass is a mirror which rotates about an axis parallel to the bore axis to produce the lateral component of deflection as a displacement of the reticle image on the combining glass. This optical output system is seen in Figure 7. The sight is designed for caliber 0.50 AP M2 ammunition with a muzzle velocity of 2,700 fps. The actual mechanism of the K-13 is also shown in Figure 7. The points B and O are fixed and the length BO may be taken to be proportional to v_0. The length OC varies according to the input kv_G. The inputs $A_G(GA)$ and $E_G(GE)$ rotate the point C so that OC remains parallel to the aircraft's fore-and-aft axis. The mechanically constructed angles $\Delta_L(TLD)$ and $\Delta_V(TVD)$ are translated into the correct rotations of the mirror and combining glass of the optical output system. However, when a mirror rotates through an angle, a reflected ray changes its direction by twice that angle. This must be accepted and corrected by reducing OB and inserting suitable linkages. A slight error arises which has nothing to do with theory or manipulation.

The sight was designed to use $k_g = 0.855$. This indicates that theory lagged behind design. No harm is done since it is possible to obtain the appropriate v_G^* from Figure 3 and use this on the *TAS* dial set for strafing (100 per cent own speed). Similarly, if it is necessary to use ammunition with a muzzle velocity different from 2,700 fps, one has only to use $(2{,}700\ v_G^*)/v_0$ as input. The *ratio* of OC to OB then yields v_G^*/v_0 as it should.

The K-13 has no range input. Hence any super-

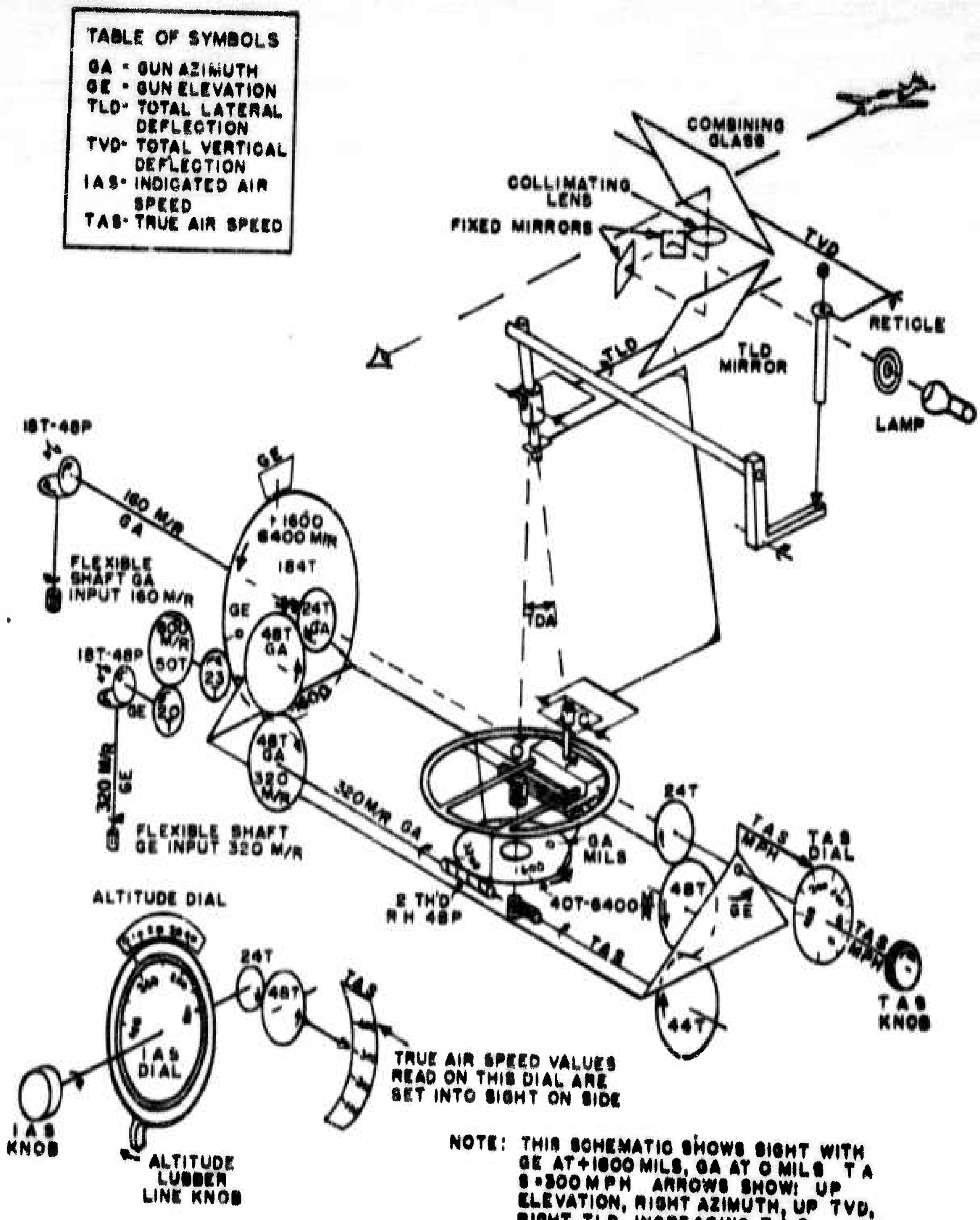

FIGURE 7. K-13 vector sight schematic. (Courtesy of Sperry Gyroscope Company.)

elevation allowance for gravity drop can depend only on direction of fire. The design superelevation used is 6 cos E_G milliradians, with a slight dependence on azimuth. (The dependence is a minimum on the nose, since future ranges will be a minimum, and a maximum on the tail, with a spread of 2 milliradians for $E_G = 0$.) Taking, roughly, gravity drop to be $5t^2$ yd, and a mean velocity $\bar{v} = 2{,}500$ fps, it follows that the allowance of 6 milliradians corresponds to a range of approximately 800 yd. If it is decided that this is too great, it may be reduced by setting the elevation dial appropriately during alignment of gun and sight.

The nature of the K-13 is such that there is no restriction on the azimuths it will accept and there is but minor restriction on elevations (+85° to −85°). It is suitable for use at any gun position.

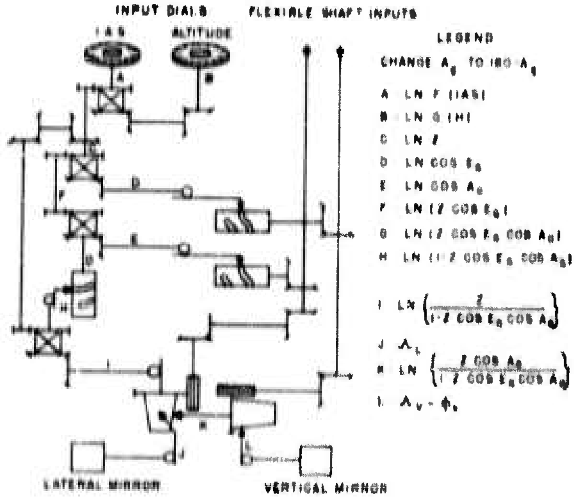

FIGURE 8. K-11 algebraic sight schematic. (Courtesy of Sperry Gyroscope Company.)

4.5.3 The K-11 Algebraic Sight [61]

The inputs to the K-11 algebraic sight differ from those for the K-13 in that IAS and altitude are to be inserted instead of $k_0 v_0$. The optical output system is the same.

It is an easy exercise in spherical trigonometry to show that, with good approximation,

$$A_L = \frac{Z \sin A_G}{1 + Z \cos A_G \cos E_G},$$

$$A_V = \frac{Z \cos A_G \sin E_G}{1 - Z \cos A_G \cos E_G},$$

where

$$Z = \frac{k_0 v_0}{v_0},$$

Instead of physically constructing angles A_L and A_V as does the K-13, the K-11 mechanically works through these formulas as indicated in Figure 8.

In such circuits (1) differentials add algebraically two input rotations, (2) one-dimensional cams give a function of a single variable, (3) two-dimensional cams give a function of two variables, and (4) a rack and pinion translates a rotation into a displacement. It follows that multiplication must be effected by logarithms. Hence, in particular, a log cosine cam is restricted in size since log cos 90° is negatively infinite. Consequently, algebraic sights can only be used in nose and tail positions or, more generally, in some restricted region, the K-11 being restricted to a cone of radius 60° about the nose.

The original design of the K-11 called for inputs of altitude and IAS. It computes TAS. The design k_0 is given in the following Table 5.

TABLE 5. Design k_0 of K-11 algebraic sight.

Altitude (feet)	k_0 (AP M2)	k_0 (API M8)
0	0.805	0.850
6,000	0.835	0.888
12,000	0.850	0.913
18,000	0.873	0.928
24,000	0.870	0.934
30,000	0.877	0.932
36,000	0.894	0.918

It is again evident that design preceded theory. If it is desired to use the optimum v_0^* from Figure 3 one proceeds as follows. Let σ be relative air density (NACA). Then the $v_0(TAS)$ used by the K-11 is $k_0 \cdot [IAS] \cdot \sigma^{-\frac{1}{2}}$, where k_0 is that of Table 4. But one wants to set on the IAS dial v_0^*. Hence $k_0[IAS] \cdot \sigma^{-\frac{1}{2}} = [IAS]$ and it follows that the altitude dial may be set and left at the altitude corresponding to the solution σ of the equation $\sigma^{\frac{1}{2}} = k_0(\sigma)$. For AP M2 ammunition, this altitude is 10,450 ft and for API M8 ammunition it is 7,100 ft. (In the actual sight IAS is set opposite altitude. Hence v_0^* is set opposite these values of altitude.) A second method is to put the sight at "strafe," which is 100 per cent own speed, and set v_0^* opposite zero altitude. With the sight in hand it is easily seen that the two schemes are identical.

4.5.4 The Class B Errors of Own-Speed Sight

The errors in gun pointing committed by a gunsight are due to (1) errors in the inputs, (2) mechanical errors, such as back lash, caused by the construction, (3) errors attributed to engineering compromises in the design, (4) errors in the gunner's manipulation, and (5) errors caused by a design based on inaccurate formulas.

Input errors arise by using an incorrect v_0^* or v_0 (over the life of a barrel, v_0 may drop 200 fps). Any percentage error in v_0^* or v_0 appears as the same percentage error in lead as differentiation of $A = (v_0^* \sin \tau)/v_0$ shows. Input errors in A_G and E_G may also occur because the gun was incorrectly boresighted, because the aircraft has an angle of attack,[13a] or because the aircraft is moving forward crabwise. Since A_G and E_G are picked off relative to the aircraft and not relative to the true velocity vector, the guns may not lie in the correct plane of action.

Errors of types (2) and (3) are analyzed by laboratory bench tests.[e] The results are not described here. Type (4) errors are dealt with in airborne experimental programs. (See Chapter 7.) This section considers only errors of the theoretical design of type (5).

These theoretical faults are called *Class B errors*. Suppose that a sight is perfectly constructed according to the blueprints, that it is operated perfectly, and that there is no dispersion. The bias in gun pointing that still exists is caused by the systematic failure of the sight in computation. When the own-speed sight is used against a pursuit curve, the choice of $k_g = 0.85$ or even $k_{g\,opt}$ must lead to Class B errors in the plane of action, since the factor chosen has only an average value. There is also an error normal to the plane of action because the path of the fighter sags below the plane of action of the instant of fire and because the gravity drop allowance is not a function of range.

It is desirable to express these errors as a function of range and angle off [21a] rather than as a function of time along particular courses. Closed deflection formulas such as equation (19) in Chapter 2 make this possible. As an example of this point-function technique, consider the family of all aerodynamic lead pursuit curves, with specified v_B, v_F, and altitude, which lie in a horizontal plane of action. The correct lead for any τ and r may be computed by equation (19) in Chapter 2 with the approximate theory of Section 4.2.3. Then, for an own-speed sight set at $k_g = 0.85$ and at $k_{g\,opt} = 0.77$, the Class B error in the plane of action [14a] can be presented cellularly as in Figure 9.

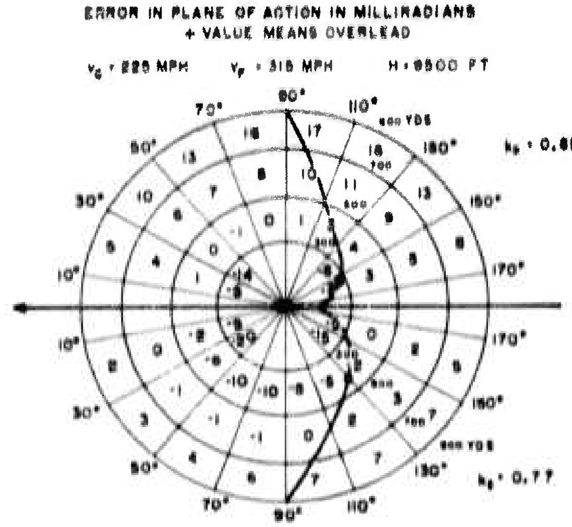

FIGURE 9. Typical Class B errors of an own-speed sight.

By superposition of any relative pursuit curve on Figure 9, the way in which the error varies along that curve may be inferred. Characteristically, the sight overleads initially, leads properly at midranges, and then underleads. When a different percentage is used, the range at which the lead is correct changes. But the bullet pattern still walks slowly through the target. This is obviously preferable to a fixed bias in pointing since the dispersion pattern rapidly becomes dilute as the target moves from the MPI. Figure 9 shows that the range at which the pattern is centered is tactically better for $k_{g\,opt} = 0.77$ than for $k_g = 0.85$. (For this example the lead required by sag, normal to and downward from the plane of action, varies from 1 to 5 milliradians. It is possible to combine these values with the required allowances due to gravity and windage jump and assess Class B performance normal to the plane of action. This will not be done here.)

4.6 SUPPORT FIRE

4.6.1 Support Fire Situations

A presumable implication of a tightly massed bomber formation is that cross or support fire is quite possible. The guns of one bomber can be

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[e] Such tests were made at Northwestern University under Contract OEMsr-1270 as Project No. 14.

brought to bear on a fighter attacking another bomber of the group. Massed firepower[f] has, in fact, been viewed with dejection by opposing air forces.[174]

If the specialized own-speed sight is to be used in support fire the pipper can no longer be held on the target and the sight must be used in combination with some eye-shooting rule. During the closing months of World War II, over Europe there was a need for such application. The German Air Force massed fighters astern of a bomber formation and rode up the tails of all bombers almost simultaneously. This was the Company Front attack.[88, 178] The nature of the tactic masked out upper and lower turrets, leaving tail and waist positions to counter this saturation attack. Other support fire situations arise when a neighboring bomber is under pursuit attack or under direct frontal attack. Assuming that waist and tail positions are equipped with own-speed sights (and not even a majority were) that sight must be used for a job that completely violates its design. There was no alternative since fire control capable of meeting general attack paths was not available for nonturreted emplacements.

4.6.2 Use of Own-Speed Sights

Suppose for the moment that the speed input to the own-speed sight is full *TAS* of the gun mount, i.e., $k_g = 1$. Then the bullet will go along the gunner's line of sight in the air mass with an initial velocity of u_0. Hence the gunner must displace his point of aim to allow for the crossing speed $v_T \sin\alpha$ of the target. The deflection he must take, by eye, is $(v_T \sin\alpha)/u$ radians. Even this does not take into account the correction required by the curvature of the target's path.[190] However, it has been suggested that for many important support fire situations the target will be on a near level with the supporting bomber and will fly sensibly in a straight line.[6, 88] For this case, if the target speed and the average bullet speed may be fixed approximately, the gunner's problem is reduced to the estimation of the approach angle α. The amount of his deflection is then given by some such memorized sequence as[187]

α (degrees):	0	10	20	30	45
Lead (RADS):	0	1	2	3	4

This deflection is laid off along an extension of the target's fuselage (*not* in the direction of the target's *apparent* motion). Ingenious methods of estimating α have been given, which note the relative position of an empennage as seen superposed on wings.[88] The ability of a gunner to recognize support fire situations[188] and to estimate approach angles[188] have been studied by controlled experiments with groups of gunners.

Much of the literature on support fire[80, 189, 188] is concerned with variations in rules such as those quoted above and with pro and con arguments about shifting the own-speed sight, when used in support, from $k_{g\,opt}$ to $k_g = 1$. The subject is probably a dead end as far as future developments are concerned. It will not be pursued further here.

[f] But it is quite essential that a distinction be made between a plot of the *number* of guns that can be brought to bear on each point of a sphere surrounding the bomber unit, and a plot of the *accuracy* in deflection with which supporting guns will point at positions on that sphere.

4.7 SUMMARY

Section 4.1 introduces own-speed sights as devices designed to counter the special case of a pursuit curve attack on a defending bomber. It discusses the arguments for and against such equipment.

Section 4.2 shows that in shooting against a pure pursuit curve a large fraction of full own-speed allowance is made since the target path curves forward but slightly during the bullet's time of flight. This section continues with a discussion of the factors affecting deflection to be taken against an aerodynamic lead pursuit curve, gives optimum rules summarizing detailed calculations, and reduces the problem to simple charts which could be used in the air.

Section 4.3 discusses the validation of the theory of the previous section in three directions (1) the demonstration that curves approximating pursuit curves were flown extensively in combat, (2) the agreement between the predictions of theory and the deflections demanded by paths actually flown by fighters during gunsight assessment programs, and (3) the agreement between the predictions of the crude theory and the exact analytic theory of Section 3.4.2.

Section 4.4 considers position firing, with the aid of which the gunner is to estimate his deflection by eye, i.e., he is his own compensating sight.

Section 4.5 discusses briefly the mechanical features of the vector and algebraic types of own-speed sight, and partially reviews Class B errors.

Section 4.6 is concerned with support fire with an own-speed sight in which eye-shooting rules are combined with the automatic compensation for own speed.

Chapter 5

LEAD COMPUTING SIGHTS

5.1 INTRODUCTION

5.1.1 First-Order Nature of Sights

Fire control should be flexible enough to supply accurately, quickly, and continuously the aiming allowance required by a target traversing an arbitrary path relative to the gun mount. Unlike the own-speed sights of the preceding chapter, the mechanisms considered below do not assume that the target's approach angle is behaving in the highly special way required by an aerodynamic lead pursuit curve. In this sense, lead computing sights can handle arbitrary target paths. They are, however, inflexible in another and important sense. Basically, they assume that the target's track relative to the gun mount is a straight line (sensibly straight over the time of flight of the bullet). Operation is founded on the theory of Sections 2.2.4 and 2.2.5. Despite the calibration concept (Section 5.3.5), which presumably permits the sight to deal with a class of curved paths, these *first-order* mechanisms do not solve the general fire control problem adequately, since accuracy against one class of target paths is gained at the expense of lowered performance against some other class. Besides describing those lead computing sights used in inhabited turrets during World War II, it is a responsibility of this chapter to enlarge upon and justify the statements made above. Somewhat more generally, the chapter is to relate lead computing mechanisms to the points of the initial sentence of this introduction.

5.1.2 Rate Deflection Formulas Mechanized

The most important deflection formulas of Chapter 2 were those based on the angular rate of the gun-target line. The reason is that the introduction of angular rate deletes the target's approach angle which is not measured. In simplest form the lead was the difference between a kinematic deflection and a ballistic deflection. The latter deflection depended mostly on gun position and range so that one would expect it to be mechanized, conceptually at least, in a fashion independent of the kinematic deflection. Consequently, we must consider as a basic computing device a machine (1) which will convert range (and altitude and perhaps other variables) obtained stadiametrically or by radar into a time-of-flight multiplier t_m, (2) which will obtain the present angular rate ω of the gun-target line by a gyroscope, variable speed drive, or a tachometer, and (3) which will combine these to yield the kinematic deflection $t_m\omega$.

The curvature correction factor h of equation (12) of Chapter 2 depends not only on future range, as does t_m, but also on the direction and amount of curvature. The machines considered in this chapter can take curvature into account only by assuming some particular average behavior on the part of the target. In particular, information on rate of change of range is neither given to nor accepted by these devices. The attitude is adopted that h and t_m, and even trail, are at our disposal and are to be chosen to give optimum performance over a special class of target paths. However, this procedure is not nearly as restrictive here as was its equivalent in the case of the own-speed sight. (Suppose for example that the curvature correction appropriate to a pure pursuit curve is adopted. Then, even if the target is not flying a perfect guns bearing attack, we would still expect to have a good approximation to its actual curvature. The same remarks apply to support fire. The topic is investigated in more detail below.)

In obtaining the angular rate ω it is important to distinguish between the actual angular rate of the gun-target line in the air mass and the tracking rate as determined at the gun position. For example, if gun mount and target are chasing each other in a circle of radius R at equal speeds, $\omega = v_0/R$. The tracking rate, on the other hand, is zero. (For this reason, kinematic deflection is a somewhat inaccurate

term.) In general, any mechanism which measures angular rates with respect to the gun mount will err whenever the axes of that mount are in curved flight.

5.1.3 The Disturbed Reticle Principle

The sights considered in this chapter are of the *disturbed reticle* [a] type. The gunner controls the position of the bore axis of his gun either manually or by motor control. The computing unit lays out the deflection between the bore axis and the line of sight. The gunner has, then, only indirect control over the line of sight which he is required to keep on target. In general, a given motion of the gun results in a different motion of the line of sight. One disturbs the line of sight instead of controlling it.

5.2 APPARENT MOTION EYE SHOOTING

5.2.1 Historical Reason

In the preceding chapter it proved possible to supply rules to a gunner provided only with a fixed sight by which he became his own compensating sight. Similarly, it is possible to give rules by which the gunner becomes his own rate-time sight. This procedure was in effect in our air forces in the early days of World War II. It preceded position firing and all gunsights with the exception of the Sperry K-3 installed in the Sperry upper turret of the B-17. (Actually, the K-3 itself was only a pilot model and not really intended for extended use.[87]) Although the digression of this section is primarily of historical interest, it demonstrates what must be done when design of a control mechanism lags the weapon installation and is not coordinated with that installation. In some form or other it is not unlikely that such situations will always arise during a war. Eye-shooting methods are of dubious efficiency in the air. An inordinate amount of training is needed to achieve any results at all. A close analogy is flying by the seat of the pants in weather of zero visibility.

[a] Modern gunsights whether computing, or own speed, or fixed, use an optical system in which a reticle pattern scratched on a plate located in the focal plane of a collimating lens system appears as an image on a combining glass mounted at an acute angle to the line of sight. To the observer, the reticle appears to be at infinity, i.e., on the target, and the user has no problem in close-far distance adaptation nor need to hold a fixed eye to sight distance to preserve the angular dimensions of the reticle. The eye may be moved about over the field.

5.2.2 The "Elephant" Method

An early method of the type referred to above was the USN *Elephant* method.[184, 185] The gunner (1) tracks with pipper on target until the range is 650 yd (stadiametric estimation), (2) stops pipper and observes how far the target moves across the sight framework in two-thirds of a second (while the word *elephant* is said aloud), and (3) positions target an equal and opposite distance on the other side of the pipper and starts firing. The two-thirds of a second deflection is decreased in proportion to the ratio of actual range to 600 yd. This procedure may be considered analytically. If the target is on a pure pursuit curve the required deflection is

$$A \doteq \left(1 - \frac{1}{2}\frac{v_T}{v_0}\right)\frac{v_G}{v_0}\sin\tau$$

by Section 4.2.1. The angular rate of the gun-target line is

$$\omega = \frac{v_G \sin\tau}{r},$$

where r is the range. If ω does not change radically the gun is held still for

$$t_m = \frac{A}{\omega} = \left(1 - \frac{1}{2}\frac{v_T}{v_0}\right)\frac{r}{v_0} = k_0\frac{r}{v_0}$$

seconds, where t_m is a time-of-flight multiplier.[41] If $r = 1{,}800$ ft and $v_0 = 2{,}700$ fps, then t_m is *approximately* two-thirds of a second. The factor k_0 varies[68] from 0.85 to 0.95 with 0.90 as an acceptable mean. Suppose now that the gunner makes an error dt in estimating time. Then, assuming that he can determine and lay off the resulting motion perfectly, his error is

$$\left(\frac{2}{3}\frac{r}{600} + dt\right)\omega - t_m\omega = v_G \sin\tau\left(0.112 + 1000\frac{dt}{r}\right)$$

milliradians, where r is in yards and v_G is in yards per second. If for *all* gunners the average dt is zero, the average shooting is biased by about $12\sin\tau$ milliradians, for a bomber at 225 mph, because of the neglect of the curvature correction in the rules. If the rules were adjusted to allow for this, and if, then, the average *absolute* error in estimating time were $|dt| = 0.15$ sec, at 600 yd, the average absolute error in lead would be $27.5\sin\tau$ milliradians.

It was appreciated[184] that a correction must be made for approach angles that are not sensibly zero. The gunner is to use some point on the extended fuselage of the target from which to lay off his de-

flection as arrived at above. This is obviously an allowance for ballistic deflection. It is doubtful that a combat gunner could do this. Further discussion is possible,[4] but in view of the real practical difficulties in carrying out the process in the air (jumping sight, etc.) the subject will not be elaborated.

5.2.3 The "ABC" Method

The USAAF used a method quite similar to the Elephant method. This was the *ABC* method, in which *ABC*, when said aloud, was presumed to have a duration of *three-quarters* of a second. The discrepancy in duration of time in the two methods is explained by noting that general firing conditions, as opposed to pursuit curve attacks, were in mind. The bullet was to have an average speed of 2,400 fps over 1,800 ft. It is evident that this errs in exactly the wrong direction in countering pursuit curves. Against a target on a parallel course at the same speed it gives no lead (like the Elephant method) and so does not even allow for trail. The method, in fact, will give the correct lead when the target is crossing at right angles to the bomber's track at a range for which $\bar{v} = 2{,}400$ fps.[4] For this case

$$A = q\frac{v_T}{v_0}\sin\alpha,\ \omega = \frac{v_T}{r}\sin\alpha,\ t_m = q\frac{r}{v_0} = \frac{r}{\bar{v}} = \frac{3}{4}.$$

For a target on a rectilinear course the error of the method when used perfectly is $\omega\left(\frac{3}{4}\frac{r}{1{,}800} - t_m\right)$

where[4]

$$\omega = \frac{v_G\sin\tau - v_T\sin\alpha}{r}$$

$$t_m = \frac{r}{v_0}\left[1 - \frac{t}{\dfrac{v_G\sin\tau}{v_T\sin\alpha} - 1}\right]$$

(See Sections 2.2.1 and 2.2.3.) After computation, it is seen that the errors are really serious.

5.2.4 The "Apparent Speed" Method

The two preceding methods fixed a time interval and determined how far the target moved in the frame of the sight. It is possible to fix the distance to be moved, e.g., the ringsight radius, and determine how long this motion takes. This is the *Apparent Speed* method (USN). A ringsight whose angular radius is 35 milliradians gives the deflection, *without trail*, to take against a target with a speed advantage of 50 knots, abeam, and on a parallel overtaking course at a range for which the average speed of the bullet is 2,413 fps. (Formula (1) of Chapter 2 with $v_0/q = 2{,}413$ gives $A = 35 + 0.045v_G$ milliradians, where v_G is in feet per second.) Now, for example, if a target at 2,000 ft takes 0.5 sec to cross the radius, its crossing speed is $(35)(2)/(0.5)(1.69) = 83$ knots (a memorized figure of 80 is used) and the deflection laid off is $80/50 = 1\frac{3}{5}$ RADS (1 RAD = 35 milliradians). This method is quite equivalent to the *ABC* method.[4]

5.2.5 Critique

The rules of thumb discussed in this section are ingenious but unsatisfactory on both theoretical and practical grounds. Theoretically they neglect trail and curvature effects and will not function when the gun mount is on a curved course. Moreover, it is assumed that the angular rate observed will persist while deflection is laid off. Ballistics are not permitted to vary with range and altitude. On the practical side they are difficult to apply at any except key ranges, even assuming the target would drift in a smooth path across an airborne sight.

5.3 BASIC THEORY OF LEAD COMPUTING SIGHTS

5.3.1 Range and Tracking Rates and Smoothing[3]

Since ballistic effects usually constitute a minor part of the total deflection, and since these effects may be presented quite accurately by subsidiary mechanisms, let us consider generically the nature of a lead computing sight designed to produce a kinematic deflection which is the product of a time-of-flight multiplier, t_m, and the present angular rate of rotation, ω, of the line connecting gun to target.

In order for a sight to obtain t_m it must be supplied with some sort of continuous estimate of the present range to the target. The range is usually obtained stadiametrically by the operator. This means that an optically presented reticle image of some pattern (parallel bars, dots in a circle, a ring) can be changed in size by the gunner. The remaining two elements of the required simple proportion are the actual size of the target and the size of the target in the scale of the instrument. The operator must set in the latter small instrumental dimension before the engagement,

The changes of the reticle required to keep the target framed, when compared with the fixed input size, permit the sight to obtain the range. The range ρ supplied in this fashion by the operator will not, in general, be the correct present range r. The difference $\rho - r$ is the ranging error and is a function of time. Two ideas immediately occur. Why not *smooth* the range error $\rho - r$ before using it? (This is in analogy with communications engineering practice in which irregular and oscillating terms are called *noise* and are subjected to smoothing circuits.) Smoothing of the range is not done in the devices considered in this chapter. And why not use the rate of change of range $\dot{\rho}$ to estimate future range? This estimate would be made by $\rho + t_m\dot{\rho}$, just as future *position* is estimated by $t_m\omega$. In practice, ρ itself is poor and jumpy and $\dot{\rho}$ is not used. With radar input of range both of these ideas become more attractive.

In order for the sight to obtain ω it must be supplied continuously with the angular position τ of the target. Given the position continuously, the sight can obtain the required rate $\dot{\tau} = \omega$. The gunner must keep the center of the reticle image (the pipper) on target. He will generally err in this and give to his sight σ, which is the angular position of the pipper, and is irregular and oscillatory. The difference $\tau - \sigma$ is called the tracking error or noise. For angular position, both of the points of the previous paragraph are carried out. This datum *is* smoothed *and* the angular *rate is* computed, as, of course, it must be.

The simultaneous requirements of ranging and tracking (and triggering) are mutually inhibitory. Performance on either one improves markedly if the other is missing. In general, tracking is better than ranging in the sense that the error is smaller and more rapidly random. Thus smoothing on the tracking in angle is feasible.

In this chapter we shall assume that the gun mount is in uniform motion. This is equivalent to saying that the tracking rate $\dot{\tau}$ is equal to the true angular rate of the gun-target line ω. Then those sights which measure rates relative to the gun mount and those that measure it relative to the air mass may be subsumed under the same theory.

5.3.2 The Exponential Smoothing Circuit

The most common smoothing system is the *exponential smoothing circuit* which can be realized in fire control electrically, mechanically, and gyroscopically. In fact, this is the only type of circuit exploited in the devices of this chapter. In Figure 1 the basic angles of the discussion are illustrated. (To make the argument transparent, it is assumed that the motions of gun mount and target are coplanar.

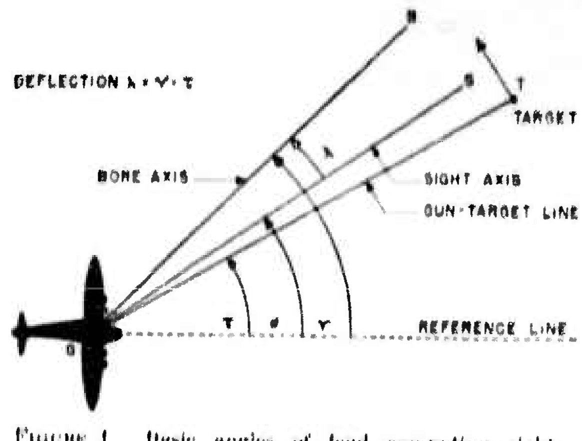


FIGURE 1. Basic angles of lead computing sight theory.

The notation λ instead of A_k will be used throughout this chapter for kinematic deflection.) If the tracking were perfect, $\sigma = \tau$ and $\lambda = \gamma - \tau$, so that $\dot{\lambda} = \dot{\gamma} - \dot{\tau}$. The lead produced should be $\lambda = t_m\dot{\tau}$. Hence, the sight equation would be

$$t_m\dot{\lambda} + \lambda = t_m\dot{\gamma}.$$

As will be seen below, the equation actually mechanized by lead computing sights is not the preceding one but is

$$(1 + a)t_m\dot{\lambda} + \lambda = t_m\dot{\gamma}.$$

The constant a leads to smoothing and is a consequence of the mechanization adopted. From this equation one obtains

$$at_m\dot{\lambda} + \lambda = t_m\dot{\sigma}, \qquad (1)$$

where λ is now the actual lead produced by the sight since one takes $\sigma = \gamma - \lambda$ to obtain equation (1). This is the basic equation of the theory.

5.3.3 Physical Realization of the Sight Parameter

The constant a is called the *sight parameter*. It can be given a physical interpretation. From equation (1), it follows that

$$\lambda = t_m(\dot{\sigma} - a\dot{\lambda}).$$

If a fourth line is added to Figure 1, making an angle $\sigma - a\lambda$ with the reference line, it follows that the

sight is computing lead as if the target were on this fourth line. Describing this line by GY, it is evident that as the system rotates about G, the *ratio* of the angle YGS to the angle SGB remains constant at a. As the deflection changes, the proportionate position of GB, GS, and GY does not change. In gyroscopic sights, the line GY has a definite existence. It is the spin axis of the gyro.

5.3.4 Discussion of the Circuit

A discussion of equation (1) is necessary and interesting.

WEIGHTED AVERAGES

We can first see in what sense smoothing or averaging is effected. The time-of-flight multiplier is a function of time. But the change to a new variable z by $dt = t_m dz$ will eliminate t_m, and an equation with constant coefficients arises. It is

$$a\frac{d\lambda}{dz} + \lambda = \frac{d\sigma}{dz}.$$

The integrating factor is $e^{z/a}$, and multiplication by this and integration by parts give

$$\lambda = \lambda_0 e^{-z/a} + \frac{1 - e^{-z/a}}{a(e^{z/a} - 1)}\int_0^z e^{z/a}\frac{d\sigma}{dz}\,dz, \qquad (2)$$

where $z = \int_0^t \frac{dt}{t_m}$, $z(0) = 0$, and $\lambda(0) = \lambda_0$, with time originating at $t = 0$. The integral in equation (2) suggests the exponentially weighted average of $d\sigma/dz$. The total weight is

$$w(z) = \int_0^z e^{z/a}\,dz = a(e^{z/a} - 1).$$

If we use $(d\sigma/dz)_{av}$ to mean the average of $d\sigma/dz$ weighted in this sense, then equation (2) can be written[2]

$$\lambda = \left(\frac{d\sigma}{dz}\right)_{av} + e^{-z/a}\left[\lambda_0 - \left(\frac{d\sigma}{dz}\right)_{av}\right]. \qquad (3)$$

The entire second term of equation (3) may be called the *transient*, since it becomes small as z increases. The weight function $e^{z/a}$ is such that decreasing significance is attached to information further and further back in the past. If a is large, $e^{z/a}$ falls off slowly, and the smoothed present value depends more heavily on the past values.

DAMPING OF OSCILLATORY TRACKING

From a somewhat different point of view the smoothing effect of the circuit can be clarified by a simple example. Suppose that the target is moving in a circle around the gun at uniform angular speed ω. Then t_m is constant. Suppose that the tracking rate oscillates around the correct value according to $\dot{\sigma} = \omega + K \sin nt$. Then the *steady state* solution of equation (1) may be written

$$\lambda = t_m\omega + \frac{Kt_m}{\sqrt{1 + a^2n^2t_m^2}}\sin\left(nt - \tan^{-1} ant_m\right).$$

If a were zero the lead produced would be

$$\lambda = t_m\omega + Kt_m \sin nt.$$

The ratio of the lead error amplitude with a nonzero a to the amplitude with zero a is

$$\frac{1}{\sqrt{1 + a^2n^2t_m^2}}.$$

This effect is called *damping*.

DECAY OF FALSE LEADS AND SLEWING ROUTINES

The initial lead of the sight λ_0 may well be false, since under slewing to get on target the sight feels that a fast target is at hand. It is important to know how rapidly such false leads decay if the rapidity of lead computation is to be assessed. Suppose the sight axis is displaced from the bore axis by an amount λ_0. If the gun is not moving, $\gamma = 0$. Hence by the equation connecting lead to gun position, we have

$$(1 + a)t_m\dot{\lambda} + \lambda = 0$$

and

$$\lambda = \lambda_0 e^{-\frac{t}{(1+a)t_m}}.$$

The response of the circuit to a particular form of gun

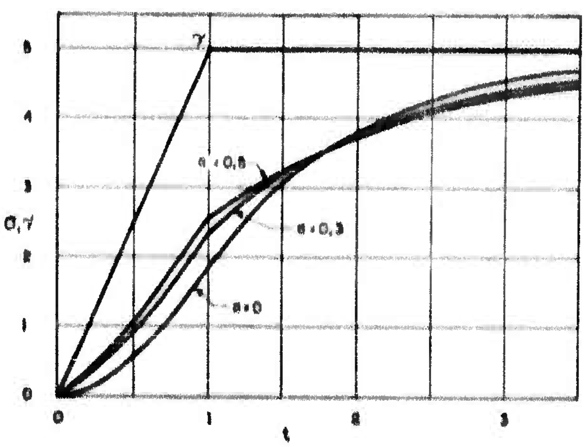


FIGURE 2. Drift of sight line back to bore axis.

motion is shown in Figure 2. It follows from the above equation that λ decays to $1/e$ of its original value in $(1 + a)t_m$ sec. This time is called the *time*

constant of the sight mechanism. (If $t_m = 1$ sec, the resulting time constant $1 + a$ is sometimes called the *sight response factor*.[206]) Since t_m decreases with decreasing range, in slewing the gunner is frequently instructed to hold the range setting at a minimum until he is on target with the sight pipper.

In an attempt to minimize the time required for the transient (or false lead) to decay, slewing routines other than the simple one mentioned above have been proposed. The problem is approached analytically.[120] Suppose that a target has a constant angular velocity $\dot{r}_0$. The true lead is then taken to be $t_m\dot{r}_0$. Let the gun position be, originally, r_0 *behind* the target. Supposing that the maximum slewing rate of the turret is A radians per second, that gun motion is required which minimizes[80] the time interval in which the gun reaches and holds its correct position ($\gamma = r_0 + t\dot{r}_0 + t_m\dot{r}_0$) and in which the sight line also attains its correct position ($\sigma = r_0 + t\dot{r}_0$). The solution consists in slewing at the maximum rate until the gun is beyond its true position at a time t_1, and then in slewing back, again at the maximum rate $-A$, until it reaches the true position at a time t_2. The latter time t_2 is readily picked out by the gunner as the time after t_1 when the sight is on target. The real difficulty lies in telling the gunner how to determine t_1, i.e., how far to go past the target. This question has not been answered and this slewing routine has not become standard operational procedure.

Amplification of Tracking Errors

If attention is turned to the relation between tracking errors and gun errors, it will be seen that errors in the tracking are *amplified* when translated into errors in gun pointing. Consider the equation

$$(1 + a)t_m\dot{\sigma} + \sigma = at_m\dot{\gamma} + \gamma, \tag{4}$$

which arises from equation (1) if we put $\lambda = \gamma - \sigma$. Suppose that the sight line is oscillating according to $\sigma = A \sin nt$. Then, for constant t_m, the *steady-state* solution of the gunsight equation is

$$\gamma = MA \sin (nt - \phi),$$

where

$$M = \sqrt{\frac{1 + (1 + a)^2t_m^2n^2}{1 + a^2t_m^2n^2}}, \tag{5}$$

and ϕ is a phase angle whose small value need not be considered. M may be interpreted as the ratio of the amplitude of gun oscillation to the amplitude of the sight-line oscillation. It is called the *amplification factor* and is plotted for typical values in Figure 3. Amplification deceives the gunner, since he may consider his tracking good (amplitude of error 5 milliradians) and yet his guns may be sweeping over the target with an amplitude threefold increased. It is evident that as a approaches zero the amplification *increases*, and as the frequency $n/2\pi$ increases the amplification increases but tapers off to the value $(1 + a)/a$. This is a good reason why sights use circuits with nonzero parameters.

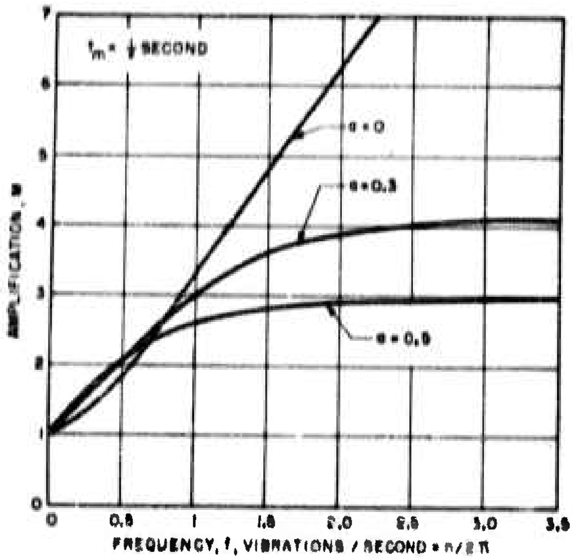


FIGURE 3. The amplification of oscillatory tracking errors.

In actual tracking, errors are committed not only along the path of the target but also at right angles to that path. It is natural to ask if the two components of noise can be treated independently insofar as amplification is concerned. The answer is yes.[10] (There is a *very* slight cross effect on the vertical amplification factor which reduces it by about 0.4 per cent.) For this two-dimensional case, however, it may be pointed out that for certain combinations of frequency and relative phase of the componential tracking noises, the resulting amplified gun motion may be such that the gun never gets on target, even though each component of the tracking crosses the target position. (Obviously, if $\sigma_L = A \sin nt$ and $\sigma_V = A \cos nt$ are the lateral and vertical tracking noises, the gun describes a circle of radius MA.) Tracking errors are in general not so regularized that this is matter for concern.

Operational Stability

The question of *operational stability* may be clarified through equation (4). Suppose initially that

$\sigma = \gamma = 0$. If the gun is given a sudden jerk, the response of the sight line is determined by

$$\dot{\sigma}_0 = \frac{a}{1 + a} \dot{\gamma}_0 ,$$

where $\dot{\gamma}_0$ is the initial jerk. Then (1) if a is positive, the initial sight response agrees in sense with the gun motion and the sight is called operationally stable, (2) if a is zero, the sight does not respond immediately, there is no impulsive effect and the sight seems sluggish, and (3) if a is negative, the sight is a fast starter, but in the opposite direction to the gun, and is called operationally unstable. In the third case the gunner is deceived since a small corrective jerk in what should be the right direction causes the sight to move initially in the opposite direction. Such sights can be built, however, and will function in the sense that if the guns are started and continue tracking, the reticle will ultimately start back in the right direction, overtake the target, and get into lead position. Nevertheless it does not seem advisable to use negative values of the sight parameter because of the disturbing effect on the gunner, and also because the weighting (see "Weighted Averages") would attribute more importance to old values than to the new ones.

Delay in Lead Computation

The considerations of the previous paragraphs were based on the interpretation of equation (1) as a smoothing or damping circuit. Such systems may also be thought of as *exponential delay* circuits in the sense discussed below. If equation (1) is differentiated and if the second derivative of λ is neglected, one obtains the expression

$$\lambda \doteq t_m\dot{\sigma} - at_m \frac{d}{dt}(t_m\dot{\sigma}) , \qquad (6)$$

as an approximation to λ. Admitting that tracking is correct, equation (6) shows that the circuit errs in producing the correct deflection $t_m\dot{\sigma}$ (unless t_m and $\dot{\sigma}$ are constant in time). Denoting the correct lead by λ_0, equation (6) is rewritten in the form

$$\lambda = \lambda_0 - at_m \frac{d\lambda_0}{dt} .$$

λ has *approximately* the value that λ_0, the correct lead, had at a time at_m in the past. (Put $dt = at_m$.) In this sense the sight's answer is always stale and becomes staler as a is increased. The delay must be accepted if smoothing is desired.

From one point of view, the delay is desirable. Suppose that instead of t_m we use the time of flight over present range t_p. Then, referring to Section 2.2.5, one recalls that on the incoming leg of a target course $t_p\omega$ is too large a kinematic deflection. If a delay circuit is used, the correction it makes to $\lambda_0 = t_p\omega$ is in the right direction, *if a is positive*, since on the incoming leg lead is increasing. A similar compensatory effect is exercised on the outgoing leg. In spite of this effect, it is now generally conceded that a sight should be calibrated, i.e., that t_m should be chosen to give best results over a given set of tactical circumstances. Hence this behavior cannot be construed as favoring only sights for which $a > 0$. *For any value of a, calibration can allow for delay.* (See Section 5.3.5.)

The exact solution of the type equation (6) is needed if delay is to be determined precisely. For equation (1), this exact solution is[a]

$$\lambda = t_m\dot{\sigma} - at_m \frac{d}{dt}(t_m\dot{\sigma})$$
$$+ e^{-z/a}\left[\lambda_0 - t_{m0}\dot{\sigma}_0 + at_{m0}\frac{d}{dt}(t_m\dot{\sigma})_0\right] + E(z), \qquad (7)$$

where $z = \int_0^t dt\ t_m$, and $E(z)$ is the error term.

Choice of Sight Parameter

From the discussion of this section, it is evident that there is no rational way in which an optimum value for a can be selected. There are factors which want a small and others which would prefer it to be large. If we first agree to use a positive value for a in order to have operational stability and proper weighting of the newest data, then the factors involved can be summarized as follows. For large a, smoothing and damping are better, but, on the other hand, perhaps too much weight is assigned to past information. As a increases, transients (false leads) take longer to decay, and the delay in putting out correct deflection is greater. These are not desirable features. On the other hand, as a increases, the amplification of gun motion is decreased which is probably desirable. An engineering compromise must be made. The sights of this chapter use parameters with values somewhere in the range 0.2 to 0.5.

Since t_m has an effect paralleling that of a (a and t_m occur as a product in connection with the above factors), it is reasonable to suggest that one should have a vary with range to effect compensations. This situation will be dealt with briefly in Chapter 8. But it is worth emphasizing now that operational

errors of simple manual ranging and tracking usually overpower such refinements. It is another case in point of improving inputs before refining the blind computing mechanism.

Aided Tracking

Before leaving this particular discussion of the basic sight equation, it may be appropriate to show how it is connected to tracking mechanisms.[9] There are three principal methods of relating handlebar motion to the motion of a gun. These are (1) *direct tracking* in which the angle γ through which the gun is displaced is directly proportional to the angular displacement μ of the handlebars ($\gamma = c_1\mu$), (2) *velocity tracking* in which the velocity with which the guns will move is proportional to the angular displacement of the handlebars ($\dot{\gamma} = c_2\mu$), and (3) *aided tracking* in which a displacement of the handlebars simultaneously displaces the guns and gives them a velocity ($\dot{\gamma} = c_1\dot{\mu} + c_2\mu$). Type 2 is realized by a motor whose speed depends on displacement of the control. Type 3 implies an exponential smoothing mechanism, since with c_2^{-1} and c_1 the respective analogues of a fixed t_m and a sight constant a, the theory of the aided tracking equation follows that of equation (1).

Investigations of aided tracking [10] have generally shown that aided tracking gives better results than do the other types. However, no optimum value for $c_1 c_2$, i.e., the analogue of at_m, has been arrived at. There is little to distinguish results for values within the range 0.2 to 0.8.

5.3.5 The Calibration Concept

It has been emphasized that t_m, the time-of-flight multiplier, is to be chosen to optimize the performance of the sight over some class of tactical circumstances. To illustrate this philosophy, consider the set of target paths which sensibly resemble pure pursuit curves. The deflection to take against a pure pursuit is by equation (1) of Chapter 4

$$A = k_0 \frac{v_T}{v_0} \sin \tau$$

$$k_0 = 1 - \frac{1}{2}\frac{v_T}{v_0}.$$

Consequently, if t_m were given the value

$$t_m = \frac{A}{\dot{\sigma}}$$

correct shooting against pursuit curves would occur.

However this simple procedure would not eliminate the time lag in lead computation nor would it allow for the fact that the actual sight will subtract from the kinematic lead that it produces, a lateral ballistic allowance β. To take these two facts into account start with the complete basic equation

$$at_m\dot{\lambda} + \lambda = t_m\dot{\sigma} - \beta, \tag{8}$$

where the lateral ballistic deflection β is given approximately, following equation (11) of Chapter 1, by

$$\beta = crpv_T \sin \gamma,$$

where c is a constant and r is present range. Now solve equation (8) for t_m, using the correct *total* lead A for a pure pursuit curve instead of λ. Then

$$t_m = \frac{A + \beta}{\dot{\sigma} - a\dot{A}}. \tag{9}$$

This "forcing" of the differential equation means that the sight's *total steady state* solution must give the correct deflection against a pure pursuit curve. The procedure [11] is equivalent to selecting t_m such that the sight's lagged kinematic deflection when combined with the sight's ballistic deflection, whatever it may be, will yield the correct lead A.

To reduce equation (9) to a form suitable for calculation, the expressions

$$r\dot{\sigma} = v_T \sin \sigma,$$

$$\dot{A} = \frac{k_0 v_T \cos \sigma \cdot \dot{\sigma}}{v_0},$$

$$\sin \gamma = \sin \sigma + A \cos \sigma$$

may be used. Then t_m is given by

$$\frac{t_m}{r} = \frac{k_0 + crp(v_0 + k_0 v_T \cos \sigma)}{v_0 - ak_0 v_T \cos \sigma} \quad (\sigma = \tau). \tag{10}$$

The actual sight mechanisms have been kept simple by using a circuit that computes t_m as a function of present range r, and altitude (relative air density ρ) only. Hence the variation of equation (10) with target and mount speeds, and with angle off σ are not taken into account. In particular, equation (10) is definitely asymmetric with respect to the beam ($\sigma = 90°$). There are three sources of the asymmetry (1) k_0, when given its exact value from equation (1) of Chapter 4, is seen to be larger in the forward hemisphere because t_f and q are both smaller (because of the shorter future range), (2) $\sin \gamma > \sin \sigma$ in the forward hemisphere and so the trail allowance is greater for a given angle σ in the forward hemisphere than it is for that angle at the rear (in the rear hemisphere $\sin \gamma < \sin \sigma$), and (3) since A

increases in the forward hemisphere (A decreases in the rear hemisphere), the time lag does not act to counterbalance the first two factors. In sum, the total lead put out by the sight is

$$\lambda \doteq t_m\dot{\sigma} - at_m \frac{d}{dt}(t_m\dot{\sigma}) - \beta.$$

If the t_m used is that appropriate to the beam ($\sigma = 90°$),

$$\frac{t_m}{r} = \frac{k_0}{v_0} + cpr,$$

then in the forward hemisphere $t_m\dot{\sigma}$ is too small, and both the delay term and the ballistic term subtract still more. The sight underleads. In the rear hemisphere $t_m\dot{\sigma}$ is too big, the delay term adds somewhat more and the ballistic term does not subtract as much as it should. The sight overleads.

In spite of the arguments for making t_m vary with σ, v_T, and v_G — as well as with r and p — enthusiasm for sight refinement must keep in mind that such elaboration may again be pointless in view of the errors in supplying inputs, and may give excellent results for the special class of curves assumed in the calibration but deteriorate on other types of target paths. A suggested calibration [a] (using averages over v_G, v_T, and σ) is, for $a = 0.43$, API M8 ammunition ($v_0 = 2870$, $k_0 = 0.440$), and ballistic deflection constant [b] $c = 0.0025$;

pr	0	3	6	9	(hundreds of yards)
t_m/r	0.0968	0.1030	0.1101	0.1164	(seconds per hundreds of yards)

Figure 4 gives more details of the calibration.

5.3.6 Class B Errors in General

The Class B errors of a fire control system were defined in Section 4.5.4 as those arising when a sight constructed exactly in accordance with design is operated perfectly with no dispersion. To assess these errors analytically for a given type of course, the calibration in the sense of Section 5.3.5 must be known. Early estimates [a] of Class B errors were made under the assumption that t_m was the actual time of flight over present range as obtained from ballistic tables. From the current point of view it is more significant to proceed in two directions. (1) Under perfect calibration in the sense of equation (10), how does the sight perform against target paths other than those assumed in the calibration, e.g., rectilinear paths? (2) Under an actual averaged calibration, how does the sight perform against the courses of the calibration?

Against a straight-line course

$$\sin A = \frac{v_G}{v_0}\sin\sigma - q\frac{v_T}{v_0}\sin\alpha$$

by equation (1) of Chapter 2, and

$$r\dot{\sigma} = v_G \sin\sigma - v_T \sin\alpha$$

by equation (9) of Chapter 2. One could get the optimum calibration against *straight-line* courses by inserting these expressions in equation (9). Alter-

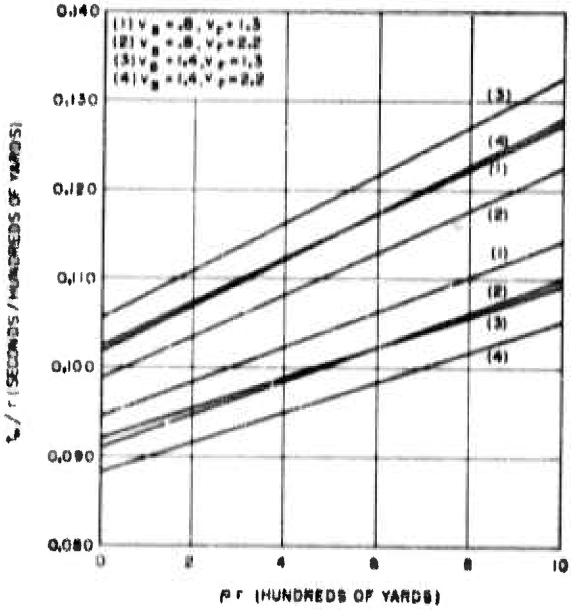


FIGURE 4. Calibration of a lead computing sight when used against pursuit curves. (Upper family: $\sigma = 0°$; lower family: $\sigma = 180°$.)

natively, as representative of direct calculation, consider a special case of support fire against a rectilinear company front attack (Section 4.6.4). Using one hundred yards and seconds as units, choose $v_G = 0.8$, $v_T = 2.2$, $\sigma = 150°$, $\alpha = 30°$, $r = 6$, $p = 0.5$, API M8 ammunition. A good value for q is 1.090, so that the correct *total* lead $A \doteq 81$ milliradians. For these conditions a sight calibrated perfectly against pure pursuit ($k_0 = 0.80$) would use $t_m = 0.580$ sec. Since the tracking rate is $\dot{\sigma} = 116.7$ milliradians per second, the kinematic lead produced is about 68

[b] $c = 0.008$ is the actual value if the future range were r (caliber 0.50 API M8 ammunition). The use of $c = 0.0025$ takes into account the fact that future range should be used in assessing ballistic deflection, and this range is about 85 per cent of r for pursuit curves.[4]

milliradians. In this case the ballistic deflection will be *added* by the sight. It is $\beta = 3$ milliradians. However, since the lead is increasing, the effect of lag is to decrease the deflection. This contribution [equation (6)] is about $(0.43)(0.586)(33.2) = 8$ milliradians. The total sight error is, therefore, 18 milliradians. Although this is a serious bias it must be kept in mind that dispersion, input errors, and Class A errors may still lead to hits. At least the deflection is on the right side of the target. (An own-speed sight, which can also be regarded as calibrated for pure pursuit curves, would give, if used with pipper on target, a deflection of 37 milliradians to the

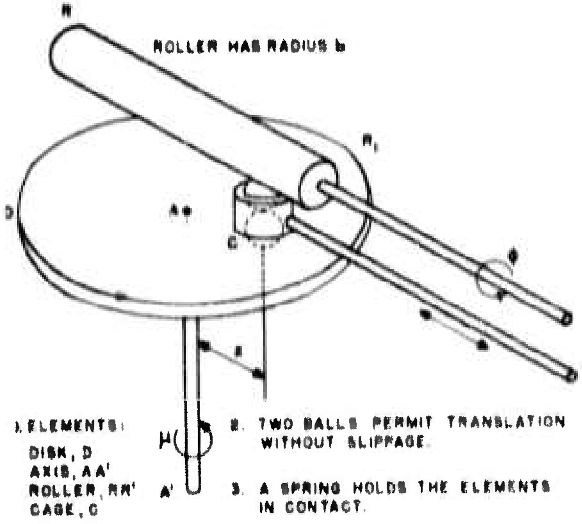


FIGURE 5. Ball-cage integrator.

rear of the target and so would commit a Class B error of 118 milliradians.)

It is appropriate to continue the discussion of Class B errors in those sections concerning particular sights.

5.4 MECHANICAL LEAD COMPUTING SIGHTS

5.4.1 Types

The realizations of the theory of Section 5.3 that will be considered here are called *mechanical* rate sights. The computing mechanism consists entirely of mechanical elements as opposed to electrical or gyroscopic units. These sights form the Sperry series: K-3, K-4, K-9, K-12, K-16. The pairings are natural since the K-4 and K-16 are lower ball turret versions of the K-3 and K-12 respectively. The K-9 is a layout redesign of the K-3 to make a Martin turret installation possible. All the other models go in Sperry turrets. Although the K-12 is also an improved version of the pilot model K-3, most of the emphasis of this section will be placed on the K-3, since this sight has received more study. All models are two component sights with separate mechanisms to compute the total lead laterally and vertically.

5.4.2 The K-3 Sight

A general knowledge of the mechanism is useful.

THE BALL-CAGE INTEGRATOR

The essential element is the *ball-cage integrator* illustrated in Figure 5. The speed of a point on the disk at the cage is $z\dot{\mu}$. This speed is transmitted to the roller which must have a speed $b\dot{\phi}$. Hence

$$\dot{\phi} = \frac{\dot{\mu} z}{b} \quad \left(\phi = \int \frac{z}{b}\, d\mu\right),$$

and the output angular velocity $\dot{\phi}$ is proportional to the input displacement z.

MECHANICAL EXPONENTIAL SMOOTHING

The ball-cage integrator can readily be made into an exponential smoothing circuit.[13] In the schematic

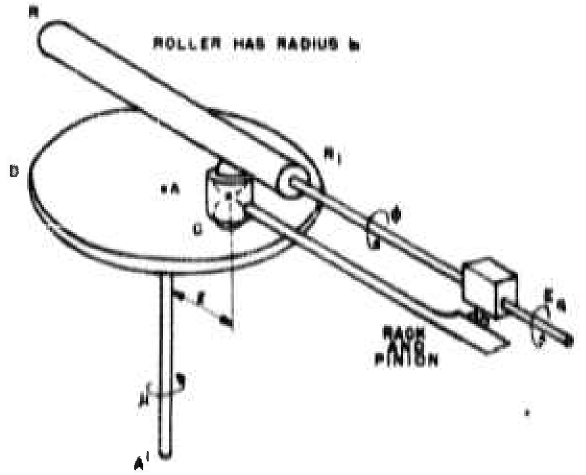


FIGURE 6. Mechanical exponential smoothing circuit.

of Figure 6 a differential (see "Ball-Cage Integrator" above) has been added. The rotational inputs are the elevation E_H and roller angle ϕ. Through a proper choice of gear ratios, the differential adds the two inputs algebraically in a desired proportion.

Thus

$$E_G = \phi + cz\,.$$

Then

$$\dot{E}_G = \dot{\phi} + c\dot{z}\,,$$

$$\dot{E}_G = \frac{\mu}{b} z + c\dot{z}\,.$$

If one takes[c] $c = 1 + a$ and varies the disk speed so that $\mu/b = 1/t_m$, then, calling the rack output z, the vertical kinematic lead λ_V, one has

$$(1 + a)\,\dot{\lambda}_V + \frac{1}{t_m}\lambda_V = \dot{E}_G\,.$$

This is the required equation, in the vertical component, connecting gun rate to deflection.

Another ball-cage integrator is used to make the disk speed vary as the reciprocal of the time-of-flight

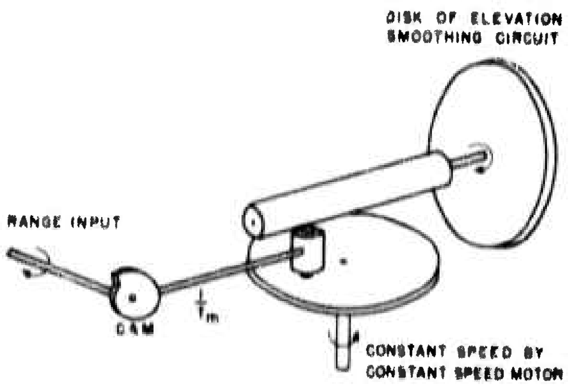


FIGURE 7. Method of making disk speed depend on $1/t_m$.

multiplier. This is shown in Figure 7. In this application, the ball-cage unit is used only as a device to transform a displacement into an angular velocity. It is evident that as the range approaches zero, $1/t_m$ approaches infinity. Hence the mechanism must abandon its computation at a point for which the speed would be excessive.[d]

LATERAL CIRCUIT

Near the zenith, a small change in target position leads to a large change in the azimuth of the target. Hence lateral deflection rather than azimuth deflection is computed. This is the deflection in a plane whose normal is perpendicular to the gun and which lies in a vertical plane containing the gun. The appropriate gun rate is, therefore, $\dot{A}_G \cos E_G$, since this is the magnitude of the rotational vector directed along the normal just described. The lateral circuit will be complicated by the required multiplication by $\cos E_G$. According to Figure 8, this multiplication

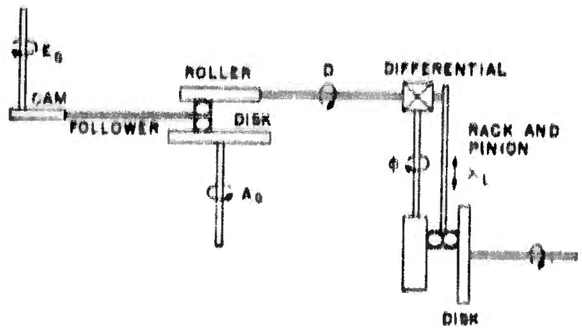


FIGURE 8. Lateral smoothing circuit.

is achieved by yet another ball-cage integrator and a cam. The output of the first roller is

$$\dot{D} = \dot{A}_G \cos E_G,$$

and at the differential

$$D = (1 + a)\lambda_L + \phi$$

$$\dot{D} = (1 + a)\dot{\lambda}_L + \frac{1}{t_m}\lambda_L\,,$$

so that

$$(1 + a)\dot{\lambda}_L + \frac{1}{t_m}\lambda_L = \dot{A}_G \cos E_G\,.$$

COMPLETE CIRCUIT

Lateral and vertical ballistics, β_L and β_V, are to be combined with λ_L and λ_V just before the lateral and vertical mirrors of the optical system are rotated. Two-dimensional cams are used to put in ballistics. (Such cams are irregular cylinders. By rotating such a cylinder, one input is made, and by displacing a follower along it a second input is possible.) Ballistic deflections actually depend on range, azimuth, elevation, altitude, and mount velocity. With two-dimensional cams average values for certain variables must be selected.[e] The lateral deflection β_L is taken

[c] The gear ratios are chosen so that $a = 0.22$ for the K-8 sight.

[d] In the K-8, the limiting range for accurate computation is 200 yd. Below that range, a speed corresponding to a constant value $t_m = 0.2$ sec is operative.

[e] For the K-8, the average values used for β_L are altitude = 18,000 ft and IAS = 200 mph; for β_V, an average range of 800 yd is used.

to depend only on range and A_G, and β_E to depend on E_G and A_G. The complete sight schematic is formed in Figure 9.

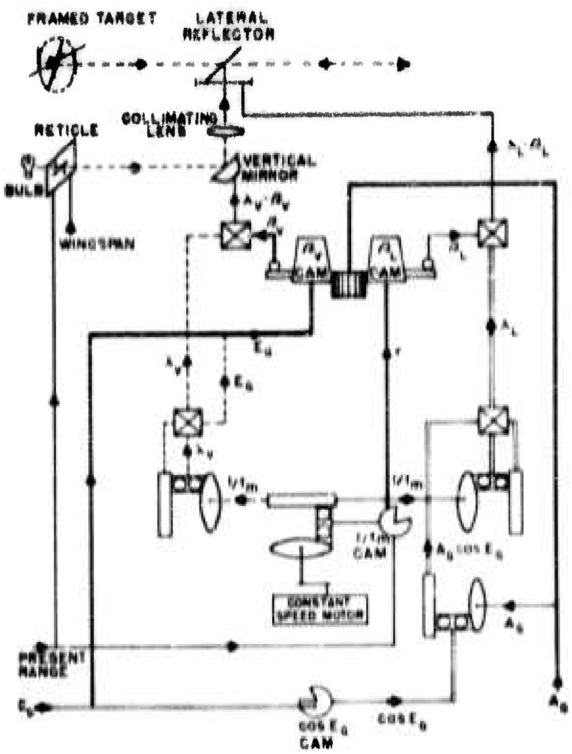


FIGURE 9. K-3 sight schematic. (Courtesy of Sperry Gyroscope Company.)

5.4.3 The K-12 Sight

Mechanically, the K-12 sight differs rather radically from the K-3. Only two ball-cage integrators are used so that the kinematic deflection outputs are λ_L/t_m and λ_E/t_m. Similarly the outputs of the ballistic cams are β_L/t_m and β_E/t_m. A linkage adder combines kinematic and ballistic deflections, and then t_m is multiplied out by a linkage multiplier (which is a level with a variable fulcrum positioned according to t_m). Linkages also permit the ballistic deflection to depend on all five variables rather than two. Caliber 0.50 API M8 ballistics are used, a is changed from 0.22 to 0.37, and it is possible to switch from a t_m appropriate to straight-line courses to $0.90t_m$ which fits pursuit curves. In the mirror system the lateral deflection is taken first, and the mirrors are perpendicular rather than parallel as in the K-3, improving the system performance by minimizing the error due to gun roll. (See Section 2.4.2.) Range is controlled by a button operated motor, giving velocity ranging instead of the K-3 direct ranging (see "Aided Tracking" in Section 5.3.4). The reticle consists of 12 dots (instead of a gate as in the K-3). Input intervals are: range 200 to 1,500 yd; target dimension 30 to 60 ft; IAS 100 to 450 mph; altitude 0 to 40,000 ft; gun rates up to 250 milliradians per sec; elevation up to 85°.[71]

5.4.4 Complete Sight Equations and Origin of Class B Errors

The Class B errors (Section 4.5.4) of an actual sight are attributed not only to approximations in the original theory but also to compromises made in the design and to the peculiarities of mechanization. Consequently, with a real sight, in discussing errors of this type it is customary to say that the sight behaves exactly according to blueprint specifications. Everything is to be included except the statistical features of dispersion, manufacturing and functioning variations, and input errors.

For the K-3 sight the kinematic deflections are computed by

$$\begin{aligned}(1 + a)t_m\dot{\lambda}_L + \lambda_L &= t_m\dot{A}_G \cos E_G\\ (1 + a)t_m\dot{\lambda}_E + \lambda_E &= t_m\dot{E}_G,\end{aligned} \tag{11}$$

and these are combined linearly with the ballistic deflections β_L and β_E to give the total deflections

$$\begin{aligned}\Delta_L &= \lambda_L - \beta_L, \quad \beta_L = Cr \sin A_G + D\\ \Delta_E &= \lambda_E - \beta_E, \quad \beta_E = F \sin E_G \cos A_G - G \cos E_G - H,\end{aligned} \tag{12}$$

where C, F, and G are constants. In working with these equations the blueprint t_m, β_L, and β_E are to be used. To complete the description of the sight the approximate *mirror equations*[71] are

$$\begin{aligned}A_G &= A + \Delta_L \sec E + M_L\\ E_G &= E + \Delta_E + M_E,\end{aligned}$$

where

$$\begin{aligned}M_L &= -\sec E \cdot \Delta_L\Delta_E\\ M_E &= \tfrac{1}{2}\Delta_L^2(1 - 2\Delta_E)\tan E.\end{aligned} \tag{13}$$

These describe how the optical system relates the azimuth and elevation of the sight line, A and E, to the angular coordinates of the gun, A_G and E_G.

It is readily seen how errors other than those of the underlying theory (Section 5.3.6) must arise. The complete equations for kinematic deflection[208] should contain an additional term on the right of equation (11) to account for *gun roll*. (See Section 2.4.2.)

Next, β_L and β_V from equation (12) do not use ρ, v_{0}, E_G, and ρ, v_0, r, respectively, as they should. Finally, in regard to equation (13), the error terms M_L and M_V show how the mirror system fails to do its job. And there are subtler phenomena than these. Near the zenith, lead is changing rapidly from one component to another, and the ballistics are also interchanging. The lag effect of the exponential circuit is enhanced, and spurious terms can be introduced into the lead computations. Again, there are *feedbacks* from the ballistics and from the mirror system. To understand this phenomenon, simplify the above mathematical description of the sight to read

$$(1 + a)t_m\dot{\Lambda} + \lambda = t_m\dot{\gamma} \quad \text{(smoothing)}$$
$$\Lambda = \lambda - \beta \quad \text{(ballistics)}$$
$$\gamma = \sigma + \Lambda + M \quad \text{(mirror)}.$$

These equations may be combined to yield

$$at_m\dot{\Lambda} + \Lambda = t_m\dot{\sigma} - \beta - (1 + a)t_m\dot{\beta} + t_m\dot{M}.$$

The last two terms are the ballistic and mirror feedbacks and are, of course, extraneous.

The analytical assessment of the Class B errors of a sight determines the total errors that the sight makes against some class of courses, and decomposes the total error into its several causes. This program has been carried out for the K-3.[21] One may take a class of, say, pure pursuit curves for which the exact deflection is known as a function of position. The tracking rates and ranges are also known as functions of position. The system (11), (12), and (13) is then to be solved for the steady state Λ_L and Λ_V. Since the solution is in symbolic terms, the identity of each part of the total error can be established. The process is lengthy and involves a sequence of approximations with the result that the final values have a probable accuracy of 3 milliradians.

In summary of this computation, the lateral error is given by

$$(A_G - A_G^*) \cos E = a \sin A + b \sin 2A + c \sin 2A \cos A, \quad (14)$$

where a, b, c are not constants. However a is independent of elevation, and b and c are independent of target speed. The vertical error is given by

$$E_G - E_G^* = d \cos A + e \sin^2 A + f \sin^2 A \cos A + g, \quad (15)$$

where e, f, and g are independent of target speed, and e and f are independent of range.

The lateral error may be interpreted as follows. (1) $a \sin A$ is attributed to the curvature of the target's path — it causes the sight to lead 10 per cent in excess when t_m is time of flight over present range; (2) $b \sin 2A$ is due to gun roll, or, if one does not wish to elaborate the basic equations (11) to include this, is due to the unsuitability of the mirror system; and (3) $c \sin 2A \cos A$ represents the combined effect of delay, feedback, and interchange. Similarly, the vertical error may be interpreted as follows. (1) $d \cos A$ is again in this component, the error arising from the neglect of the curvature effect in choosing t_m, although at short ranges the incorrect gravity drop (by equation (12) independent of range) is so large that it compensates for this error; (2) $e \sin^2 A$ is composed of errors due to gun roll, delay, feedback, and interchange,[c] (3) the third term is principally caused by the mirror system and its feedbacks; and (4) g represents a failure to make gravity drop vary with range.

Detailed tabulations of Class B errors for this sight have been made.[14,21] The cellular array of Figure 10 is illustrative of the results.

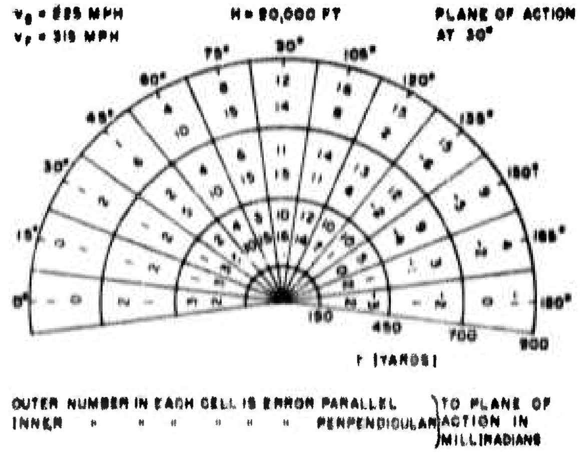


FIGURE 10. Class B errors of K-3 against pure pursuit curves (milliradians).

5.5 GYROSCOPIC LEAD COMPUTING SIGHTS

5.5.1 General Nature of a Single Gyro Sight

Mechanical lead computing sights are essentially computing machines or brains which work through certain formulas. Gyroscopic sights, which utilize

[c] Typically at $A = 90°$ and $E = 45°$, the error of 28 milliradians for this term breaks down to 15, 6, 4, 3 milliradians respectively.

the properties of a gyroscope, which employ eddy currents, and which construct physically (rather than calculate mathematically) the required deflection, are more like sentient beings. A much more elementary example of this point occurred in the comparison of vector and algebraic own-speed sights (Section 4.5.3).

A significant advantage of a gyro sight is that it will measure the true rate of rotation in the air mass of the line connecting gun to target. In certain instances, the tracking may actually be done by the motion of the firing aircraft. For example, in firing at a fixed ground object from an aircraft making a pylon turn on that target, the gunner has nothing to do. The gyro sight will establish the angular rate $\omega = v_a/r$, will combine this with a time of flight $t_m = qr/v_0$ to give a kinematic deflection, *behind* the target, of qv_a/v_0 and will subtract from this the lateral ballistic allowance which will decrease qv_a/v_0 to full own-speed allowance v_a/v_0 which is the required deflection. A mechanical sight, under these circumstances, could supply only a trail allowance ahead of the target. In bombers, this independence from aircraft motion in obtaining correct deflection is important, but in fighters it is imperative.

The sight considered in this section uses a single flexibly mounted gyro. (Certain sights used for antiaircraft purposes and for fighters — the Draper-Davis sight and the German EZ42 — employ two suitably constrained gyros and solve the problem componentially.) The single gyro sight gives the total kinematic deflection along the relative path of the target regardless of the direction of that path.

The gyro sight consists essentially of (1) an electromagnetically controlled gyroscope which measures kinematic deflection and also introduces ballistic corrections, and (2) an optical system which establishes the line of sight and *introduces the sight parameter*. We shall consider constructional details[90] but slightly. In operation, the gunner presets altitude, airspeed, and target span. During an attack, he tracks with pipper on target, keeps the target framed by the reticle pattern, and fires. Azimuth and elevation of the guns are automatically picked off the turret gear trains for use in determining ballistic corrections.

The single gyro sight for a turret is called (1) the Mark II C by the British, who originated the design, (2) the Mark 18 by the U. S. Navy, which modified the design for U. S. ammunition, and (3) the K-15 by U. S. Army Air Forces. The fighter version (Mark 21, Mark 23, K-14) is discussed in Section 5.6.

5.5.2 Method of Producing Kinematic Deflection

To arrive at the basic principles of the sight, it may first be explained how kinematic deflection is produced.[72, 78] Suppose that a gyroscope is mounted on a gun so that the spin axis is parallel to the bore axis of that gun, and so that the universal joint mounting (Figure 11) at O is the center of mass of the gyro

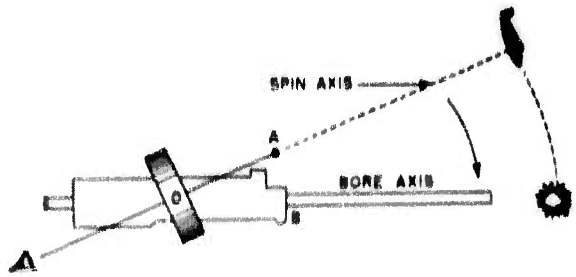


FIGURE 11. Principle of the gyro sight.

system and is also the point about which the gun rotates. Since the mounting is universal, as the gun is moved about O the gyro axis will remain pointing in its original direction in space, this being a basic property of gyroscopes (used for example in the gyro compass and flight-attitude gyro). But as shown in Figure 11, in tracking a target we require the gyro axis, if we consider that axis as the line of sight, to lag behind the guns an appropriate amount to generate deflection. This can be done only by making the gyro precess, since, in tracking, the line of sight must stay on target. To make the gyro precess in the plane of the tracking, a force F must be applied at a point such as A and directed at *right angles* to the plane of the tracking. (Witness the dip of a single engine aircraft in making a horizontal turn due to the action of the propeller and the horizontal forces on the tail surfaces.)

If I is the moment of inertia of the gyro, if T is the applied torque (equal to the force F times the distance $l = OA$), and if Ω is the angular velocity of spin, then, by a familiar formula of mechanics, the precessional rate $\dot{\sigma}$ is given by

$$\dot{\sigma} = \frac{1}{I}\frac{T}{\Omega}.$$

Since moment of inertia, spin, and the distance OA are normally fixed, a change in the rate of precession requires a change in the applied force F. Hence write

$$\dot{\sigma} = c_1 F,$$

where c_1 is the constant $l/I\Omega$. If σ and γ are measured from some reference line fixed in space,[a] e.g., pointed at a star, then, if the force F could be made proportional to the angular separation of gun and sight lines, we would have

$$\dot{\sigma} = c_1c_2(\gamma - \sigma).$$

The force F could be made proportional to the angle $\gamma - \sigma$ by attaching a spring from A to B and relying on Hooke's law. F would then be right in amount but wrong in direction. If c_2 could be made inversely proportional to a time-of-flight multiplier t_m (variable spring stiffness), we would have

$$\dot{\sigma} = \frac{c_1c_3}{t_m}(\gamma - \sigma).$$

Finally since $\gamma - \sigma$ is the lead λ, a design such that $c_1c_3 = 1$ would lead to

$$\lambda = t_m\dot{\sigma},$$

and kinematic deflection would be produced. The only things wrong with the scheme are (1) a spring mechanism would cause precession at right angles to the plane of tracking, and (2) no smoothing is produced, i.e., $a = 0$. The first point can be met by an ingenious application of electrical eddy currents. This is described in Section 5.5.3. Recalling Section 5.3.3, smoothing can be achieved not by using the spin axis as the line of sight, but by keeping the sight axis a fixed proportionate distance between spin axis and gun axis. It is one function of the optical system to do this. The method is given in Section 5.5.4.

5.5.3 The Physics of the Gyro System

Instead of a cylindrical disk, the actual gyro of the sight consists of a copper dome, a flat circular mirror, and a short connecting axle. This system revolves at approximately 3,000 revolutions per minute about a universal mounting to be considered at greater length below. The cap of this toadstool moves between the two poles of an electromagnet whose current can be varied through a variable resistance. This arrangement is schematized in Figure 12.

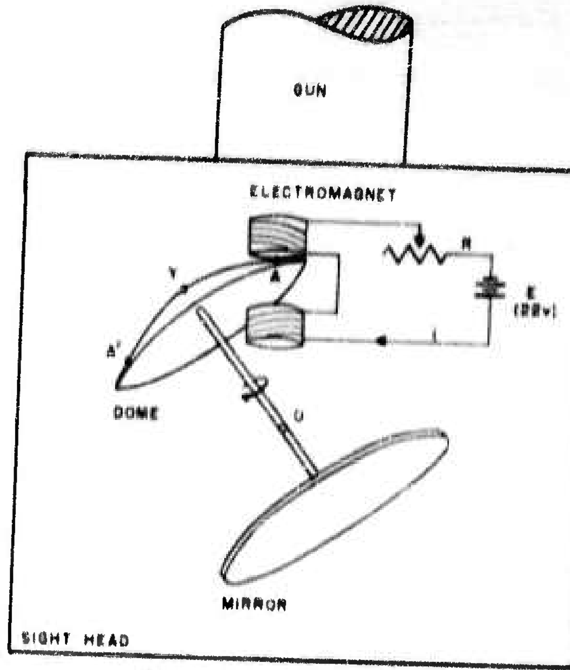


Figure 12. Gyro schematic.

The strength H of the magnetic field between the poles is

$$H = c_4 i$$

where c_4 is a constant, and i is the current in the coils. The lines of magnetic force pass through the dome, intersecting it in a circular area centered at A. The area of one instant is immediately replaced by another since the dome is spinning. Eddy currents are induced in the dome, since a circular annulus of the dome can be thought of as a coil of wire moving across a magnetic field. The current j induced in such a coil is given by

$$j = c_5Hv,$$

where c_5 is a constant and v is the velocity with which the coil moves.

The effect of these eddy currents can be made out *heuristically* by an examination of Figure 13. The induced eddy currents set up the equivalent of two magnets in the instantaneous dome segment, with S and N poles on the right and N and S poles on the left. The resulting four forces all act to oppose the motion with a force

$$F = c_6Hj,$$

where c_6 is a constant. The force F tries to slow the dome down. But the constant-speed motor supplies a couple with a force $F/2$ at A (Figure 12) opposing F and a force $F/2$ at A' in the same direction as F. Hence the speed of rotation is maintained and an

[a] See Figure 1.

unbalanced force $(F - F/2) + F/2 = F$ is left which is directed *down* into the paper in Figure 12. Hence precession will occur in the plane of the spin and bore axes as it should.

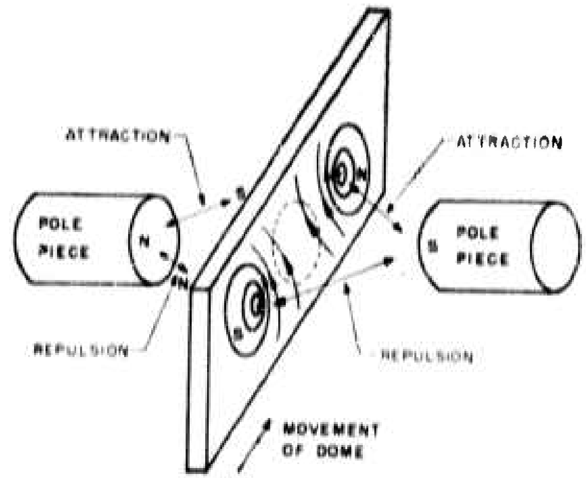


FIGURE 13. Eddy current drag.

Under tracking, then, as the bore axis is displaced, the spin axis tries to remain fixed, but the displacement causes a force F which leads to precession of the spin axis toward the bore axis. If the gun keeps rotating, the spin axis does not catch up and the two axes revolve with an angular displacement between them.

The applied torque, assuming that the moment arm $OY = OA = l$, is

$$T = l c_5 c_3 c_1^2 \frac{(22)^2}{R^2} v .$$

And if r is the (variable) distance YA we have

$$v = r\Omega = l\lambda\Omega ,$$

where $\lambda = \gamma - \sigma$. But the precessional rate

$$\dot{\sigma} = \frac{1}{I}\frac{T}{\Omega} = \frac{c}{R^2}\lambda ,$$

where $\hfill (16)$

$$c = \frac{c_5 c_3 (22 l c_1)^2}{I} .$$

Finally,[h] if R^2/c is chosen to be t_m, we have

$$\lambda = t_m \dot{\sigma} .$$

[h] In connection with c it should be observed that, since the 22-volt power supply is from the aircraft, a p per cent error in regulation means a $2p$ per cent error in the deflection output.

5.5.4 Optical System and Optical Dip

The optical system must introduce smoothing. The system is schematized in Figure 14. The reticle gate consists simply of one flat disk provided with a central hole and six radial slits placed against another disk provided with six curved slits. The reticle pattern consists, as a result, of a pipper and six diamond dots (due to the overlap of slits). By rotating a disk with foot pedals, the size of the pattern is changed, a target may be framed and present range supplied to the range coils.

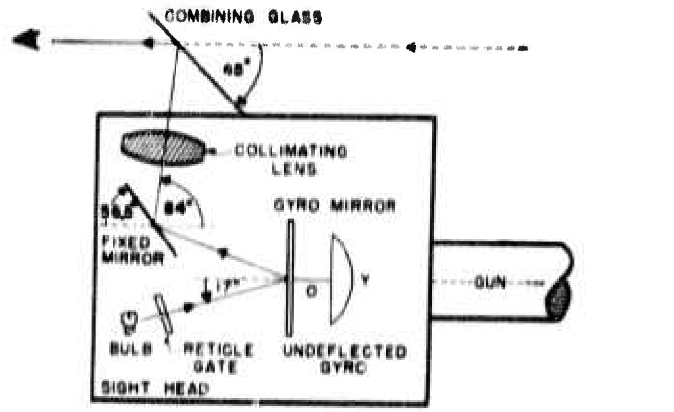


FIGURE 14. Optical system schematic.

The optical distance from reticle gate to lens is the focal length f (7.078 in.) since this is a collimating system. The optical distance from gyro mirror to lens is the image distance u (4.578 in.). The distance v to the (virtual) object is given by

$$\frac{1}{u} - \frac{1}{v} = \frac{1}{f} .$$

Consider a ray emanating from the image point (gyro mirror) and making a small angle α with the axis of the lens. Upon refraction by the lens the exit ray makes a (smaller) angle[i] β with the axis of the lens determined by

$$\frac{\tan\beta}{\tan\alpha} = \frac{u}{v} .$$

Combination of these relations yields

$$\tan\beta = \frac{f-u}{f}\tan\alpha = 0.353\tan\alpha .$$

This result is only approximate. If thick lens theory is used,[78] it is found, more exactly, that

$$\beta = 0.353\alpha - 0.0805\alpha^3 + 1.185\alpha^5 .$$

[i] Do not confuse this angle β with ballistic deflection, also frequently called β.

This point is labored since it will be seen immediately that the sight parameter a will not be constant but will vary in a peculiar way depending on the amount of deflection.

Suppose now that the gyro mirror is rotated in elevation through an angle λ_V. (The design is such that O is effectively in the mirror. See Figure 16.) The reflected ray from the gyro mirror is then displaced from its original position by twice this angle. Upon hitting the fixed mirror the reflected angle remains $2\lambda_V$ so that the entrant ray to the lens makes this angle with the lens axis, i.e., $\alpha = 2\lambda_V$. Hence the exit ray from the lens becomes a reflected ray from the combining glass and makes an angle

$$\beta = 0.700\lambda_V - 0.710\lambda_V^3 + 37.02\lambda_V^5$$

with the original position. Consequently the ratio of the angle from gyro axis to sight axis to the angle from sight axis to gun axis is

$$a = \frac{\lambda_V - \beta}{\beta}.$$

It is more important to consider a rotation of the gyro mirror in azimuth, since in this case not only is a a function of the deflection λ_A but also a spurious elevation angle is introduced leading to the phenomenon of *optical dip*, which is an error of the sight. The doubling principle used above depended on a rotation of the mirror about an axis perpendicular to the plane of incidence. For rotation around some other axis doubling is not quite accurate.[18] This may be explained crudely, for an azimuth rotation, as follows. In Figure 15, the deflected ray to the lens has further to go to get to the lens, and climbing at 17° it hits the lens a small distance above the central line of the lens. To the combining glass of the sight a *dip* has apparently been introduced. The glass thinks that the gyro has also changed in elevation. Furthermore, instead of full doubling, a factor 2 cos 17° appears. In fact, a simple trigonometric analysis of Figure 15 shows that for a displacement of the gyro axis, in azimuth only, of λ_A radians, the sight axis will move in azimuth by

$$(0.353)2 \cos 17^\circ\ \lambda_A \text{ radians}$$

and will always depress in elevation by

$$(0.353)2 \sin 17^\circ\ \lambda_A^2 \text{ radians.}$$

First approximation thin lens theory is used, i.e., $\beta = 0.353\alpha$.) In general, dip may be made out with sufficient accuracy by this approximate theory.

If the gyro axis is displaced from the gun axis by angles λ_A and λ_V in azimuth and elevation, then the sight line is displaced from the gun axis by

$$0.675\lambda_A + 0.200\lambda_A\lambda_V \text{ radians},$$
$$0.700\lambda_V - 0.1975\lambda_A^2 \text{ radians}$$

in azimuth and elevation, respectively. It is evident that dip is due to the design necessity of having a ray of light from the reticle strike the gyro mirror at 17°.[‡]

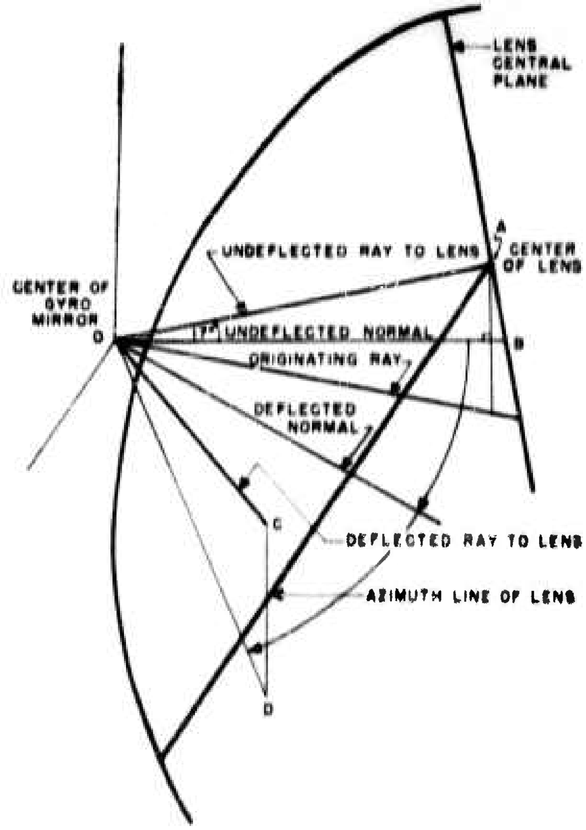


FIGURE 15. Explanation of optical dip.

To get at the way in which a changes, the exact theory may be used.[76, 198] The variation in a is given by the formula

$$\frac{1}{1-a} = 0.700d_m - 0.710d_m^3\lambda^2 + 37.02d_m^5\lambda^4,$$

where λ is the angle between gyro and gun axis and d_m is the distortion factor

$$d_m = \sqrt{1 - \sin^2 17^\circ \cos^2 \psi},$$

‡ In certain German single gyro sights, this is not true. The fixed mirror is half-silvered and dropped down to the OY axis (see Figure 14), and the reticle system is raised to this axis so that it is *behind* the fixed mirror. See Section 5.7.3.

ψ being the angle between the horizontal plane and the plane of the gun and gyro axes. The value usually assumed for a is 0.43, but it can readily vary from 0.41 to 0.48. This variation has a negligible effect on sight performance.

5.5.5 Mechanical Details

It is to the point to consider actual mechanical details since such knowledge leads to an understanding of the limitations inherent in mechanisms of this order and is of assistance in the general design of new equipment.

HOOKE'S JOINT

The actual gyro system employs a Hooke's joint [b] for its universal mounting. As indicated in Figure 16, the gimbal is a flat cross of metal whose input arm

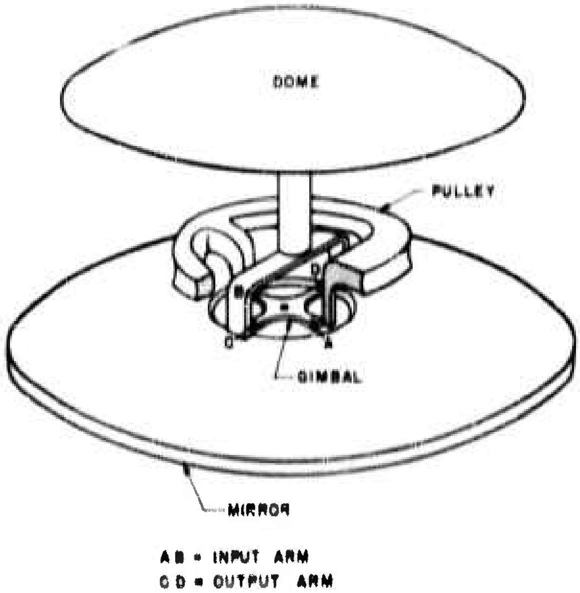


FIGURE 16. Actual gyro unit.

is pivoted to the driving pulley, while the rotor assembly is hung on the output arm. The pulley is placed in ball bearings in a panel connected to the sight head and in a vertical plane perpendicular to the bore axis of the gun. The axle can assume any angle with respect to the pulley by rocking about the four ends of the gimbal cross, and rotation is maintained at any such angle. This implies that the output arm must bob up and down 3,000 times per minute. The result of this *gimbal inertia* is a slight torque tending to push the axle still farther from the vertical. This tends to cause a *conical* precession of the axle. (The cross-wind force on a bullet which acts radially outward in the instantaneous plane of yaw is analogous.) Another torque which has a similar effect is caused by the rapid spinning of the mirror while in a cocked position. The asymmetry in the sight head leads to low air pressure on one side of the mirror and high pressure at the other. This is the *Venturi torque*. Additional torques are due to *air drag* and *frictional resistances*, and, of course, the *driving torque* as modified by the joint. These five torques are extraneous in the sense that the only torque we really want is the one due to eddy currents in the dome. Evidently, the analysis of the gyro system is complicated. The results [18] will not even be quoted. From the point of view of ultimate consideration of Class B errors, however, the conical precession referred to above implies that a displaced gyro axis does not return to the gun axis along a straight line. This leads to a dip additional to the optical dip of Section 5.5.4. Since its magnitude is approximately

$$0.025 t_m^2 \dot{\sigma} \text{ milliradian}$$

it follows that it can be positive or negative depending on the direction of tracking and so may partially balance, or augment, the optical dip.

ELECTROMAGNETIC SYSTEM

The electromagnetic system actually uses four sets of poles instead of the single pair of Figure 12. The range coils are not wound individually around each pole, but are wound around the entire unit of four. The fields between the several pairs are the same in magnitude and sense. Ballistic deflections are taken into account in a simple fashion. In addition to the enveloping range coil, each pair of opposite poles is wound with a separate ballistic coil (Figure 17). The fields produced are opposite by pairs because of the winding. Referring to Figure 3 of Chapter 1, it is seen that the two sets of ballistic coils are needed to produce the appropriate lateral and vertical components of the trail W. These components are, respectively,

$$W \sin A_G \quad \text{and} \quad W \cos A_G \sin E_G.$$

An additional coil, on P_1 and P_3 only, is needed for the gravity allowance. Under these four windings the four electromagnets become equivalent to *one* placed at some point between them, a magnetic center of gravity. The single point at which the total

[b] The proper oiling of this joint is of the utmost importance.

magnetic force seems to act is called the *magnetic center*, and the line from this point to the Hooke's joint is the *magnetic axis*. When ballistic currents flow, the ballistic fields, being opposite in sense, cause the magnetic center to shift. To the gyro the magnetic axis now plays the role of the gun axis and it is with respect to the magnetic axis that kinematic deflection is taken by the gyro. In this physical fashion the gyroscope constructs the difference between kinematic and ballistic deflections and properly positions the gun.

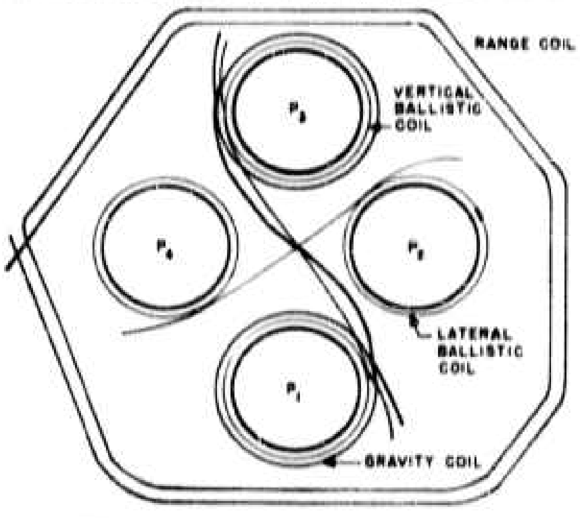
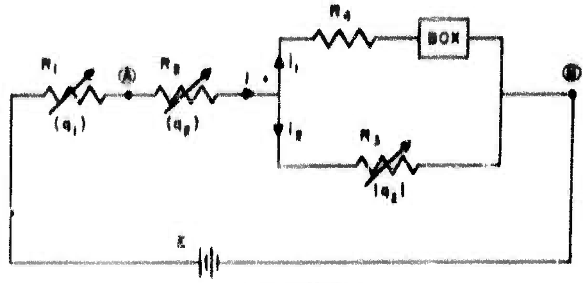


FIGURE 17. Range and ballistic coils.

Class B errors can occur because of the electromagnetic system. The lateral ballistic coils not only pull the magnetic center over as they should but they also deflect it slightly in elevation, introducing still a third dip. Again, as might be expected, the ballistic coils also act in a small way as range coils, changing t_m slightly. Finally, an important error could be introduced due to the variability of conductivity of the dome with temperature which changes radically from sea level to high altitude. (Lead will change by 0.4 per cent per degree C change in temperature.) Since the altitude input changes resistances, it follows that these resistances must be so chosen that the resulting currents will compensate for this effect.

ELECTRICAL CIRCUIT

The electrical circuit must take the inputs of range, position of target, etc., and supply suitable currents to the coils of the electromagnets. To translate properly the required formulas, nonlinear resistances and attenuating (multiplying) circuits are used. Suppose, in Figure 18, that the variable resistance R_1



$$e = R_2 + R_3kq_2$$
$$i_1 + i_2 = i = \frac{K}{R_1 + e} = \frac{K}{k_1}q_1$$
$$R_4i_1 = R_3i_2$$
$$\left(1 + \frac{R_4}{R_3}\right)i_1 = \frac{K}{k_1}q_1$$
$$i_1 = \frac{K}{k_1} = kq_2q_1$$
$$k_1 = k$$

FIGURE 18. Multiplying circuit.

is set by a dial number q_1, and the variable resistances R_2 and R_3 are set simultaneously by the dial number q_2. The problem is to design variable nonlinear resistances R_1, R_2, and R_3, and to choose a fixed resistance R_4 so that the current in the box is

$$i_1 = Kq_1q_2.$$

Now the resistance from A to B will always be constant for all values of q_2 if we choose R_2, R_3, R_4 such that

$$e = R_2 + \frac{R_3R_4}{R_3 + R_4},$$

where e is a constant. If R_3 is chosen so that

$$\frac{R_3}{R_3 + R_4} = kq_2$$

and if R_1 is chosen so that

$$\frac{E}{R_1 + e} = \frac{K}{k}q_1$$

we will have, as required,

$$i_1 = Kq_1q_2,$$

where k, e, and R_4 are at our disposal.

Using this principle, the complete circuit is laid out as Figure 19. It is evident that t_m is taken to depend only on range and altitude, and is made out by a simple summing of variable range and altitude resistances. So values of R_2 and R_3 must be determined experimentally to give optimum results and errors can naturally be expected.[109] The triple branching in the trail circuit corresponds to the

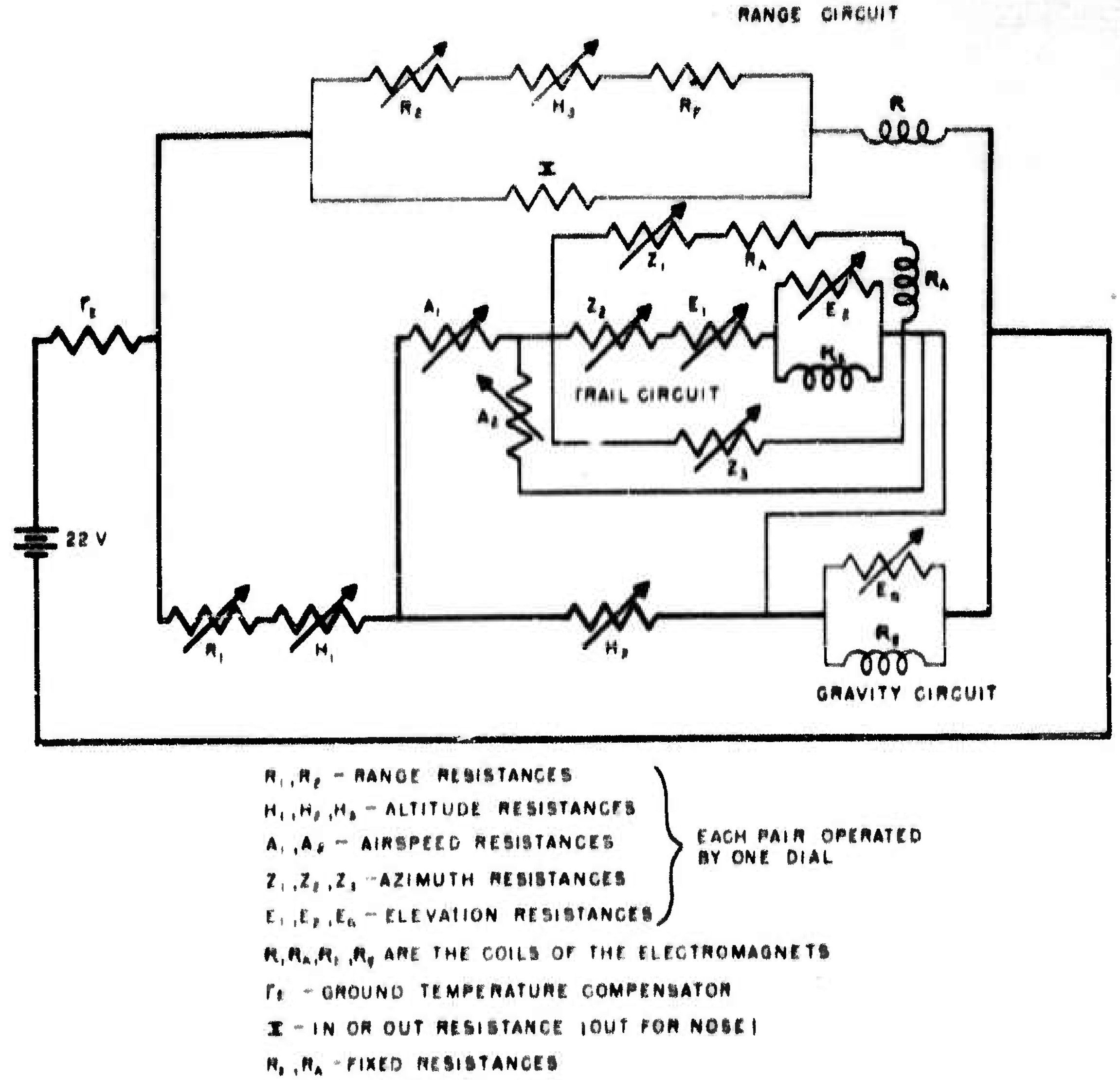


FIGURE 19. Complete electrical circuit.

multiplication of density, airspeed, and range required by the expression for trail (Formula (11) of Chapter 1). The final internal branchings introduce multiplication by cos A_G, sin E_G and sin A_G, respectively. Gravity drop depends on range and cos E_G but not on altitude or airspeed.

5.5.6 Specific Origin of Class B Errors

In getting at the Class B errors of the gyro sight a natural first step is to determine its behavior in symbolic language against a target on a general space path and with an arbitrary range r and tracking rate $\dot{\sigma}$.[16, 76–78] This requires a complete theoretical analysis of the instrument. Given such an analysis it is possible (1) to determine what design constants should be used to effect an optimum calibration, (2) to test the performance of the sight in a given class of tactical circumstances, and (3) to assign parts of the total error to their particular causes. In illustrating this program briefly, we shall restrict quotations to those pertinent to motion of target and gun mount in a horizontal plane.

On the surface of a unit sphere let U and V be the azimuth and elevation displacements of the gyro axis

from the gun axis which is assumed to lie in a horizontal plane. Then, if u and v are the azimuth and elevation of the sight axis with respect to the gun

$$U = (1 + a_1)u \quad V = (1 + a_2)v,$$

where a_1 and a_2 vary slightly, depending on u and v (Section 5.5.4). We shall take $a_1 = a_2 = a$ (constant). For gun rotation in a horizontal plane, the general equations become, after linearization,

$$a\dot{u} = -(1 + a)(p_R + Dp^2 + Ar_1^2 + Br_2^2)u - (1 + a)p_G v - \dot{\sigma} + Cpr_1$$

$$(1 + a)\dot{v} = (1 + a)p_G u - (1 + a)(p_R + Dp^2 + Br_1^2 + Ar_2^2)v + Cpr_2,$$

where p is the range coil current, r_1 and r_2 are ballistic coil currents (gravity neglected), A, B, C, D are constants determined by the geometric and material character of the electromagnetic system, p_G is a small positive constant (0.02) due to gimbal inertia and air currents, and p_R is a small positive constant (0.035) due to bearing friction and to air drag on the rotor. Neglecting only p_G^2, these equations have as *steady-state* solutions (assuming $\dot{\sigma}$ and range are constant)

$$u = t_{m1}\dot{\sigma} - p_G(1 + a)t_{m1}\beta_2 - \beta_1,$$

$$v = \beta_2 + p_G(1 + a)t_{m2}(t_{m1}\dot{\sigma} - \beta_1),$$

where

$$t_{m1} = \frac{1}{(1 + a)(p_R + Dp^2 + Ar_1^2 + Br_2^2)},$$

$$t_{m2} = \frac{1}{(1 + a)(p_R + Dp^2 + Br_1^2 + Ar_2^2)},$$

$$\beta_1 = Cpt_{m1}r_1,$$

$$\beta_2 = Cpt_{m2}r_2.$$

For a horizontal plane, $r_2 = 0$, since there is no vertical component of trail. (r_1 and r_2 were given above because the discussion is also applicable to a vertical plane.) Numerically [78] $A = 0.26$, $B = 0.18$, $C = 1.374$, and $D = 20$. To get at the character of the situation take $A = B = 0.23$. Put $t_m = 0.035/p^2$. The equations above do not include the effect of optical dip. If this effect is superposed, using the approximate thin lens results of Section 5.5.4, we obtain

$$u = 0.675(t_1\dot{\sigma} - \beta_1)$$

$$v = 0.1975(t_1\dot{\sigma} - \beta_1)^2 + 0.025t_1(t_1\dot{\sigma} - \beta_1),$$

where

$$t_1 = \frac{t_m}{0.035t_m + 0.7 + 3.5\beta^2}$$

$$\frac{\beta}{t_m} = \frac{\beta_1}{t_1}.$$

If there were no bearing friction or air drag the term $0.035t_m$ in t_1 would be missing. If the trail coil currents did not affect time of flight $3.5\beta^2$ would be missing. Then $t_1 = t_m/0.7$, and if the optical system were perfect, the number in u, 0.675 (which is $(0.353)2 \cos 17°$), would be 0.7 and we would have $u = t_m\dot{\sigma} - \beta$, i.e., a perfect sight in azimuth. Similarly, if the optical system were perfect, the first (optical dip term in v would vanish, i.e., sin 0° instead of sin 17° in $0.1975 = (0.353)2 \sin 17° \cos 17°$), and if Venturi torque and gimbal inertia were not present, the second electromagnetic dip term in v would vanish.

This discussion, however, is academic in the following sense. It assumes that the correct time-of-flight multiplier t_m is given by $t_m = 0.035/p^2$. As noted earlier, it is necessary to calibrate the simple range circuit to give optimum results over a set of tactical conditions, such as a family of pursuit curves. In doing this it is possible to ameliorate to a certain extent the above types of errors, but it will not be possible to make t_m exactly correct in any event. Without giving the numerical values of resistances adopted in the final circuit, the typical size of the total lateral and vertical Class B errors can be inferred from the summary in Figure 20.

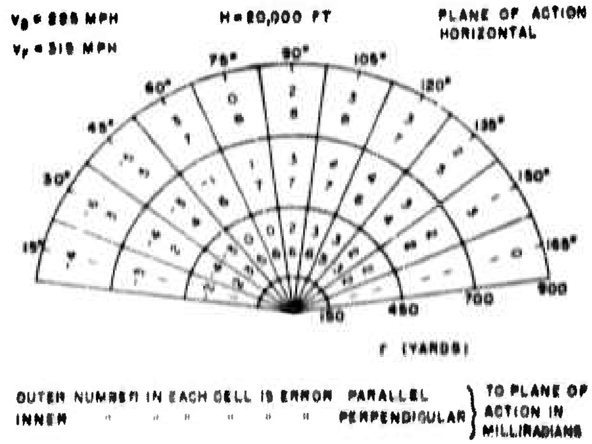


FIGURE 20. Class B errors of a gyro sight against pure pursuit courses.

It must be expected that an optimum calibration for pursuit curves will cause sight performance to deteriorate markedly when the sight is used against straight-line courses. This is the case.[79] The chart of Figure 21 shows by how much the optimum time of flight must be increased to give the optimum time

of flight against straight-line courses. (In each cell one number applies to an approach angle of 0° and the other to an approach angle of 90° as indicated in a typical cell.) These are also approximately the percentage errors in lead.

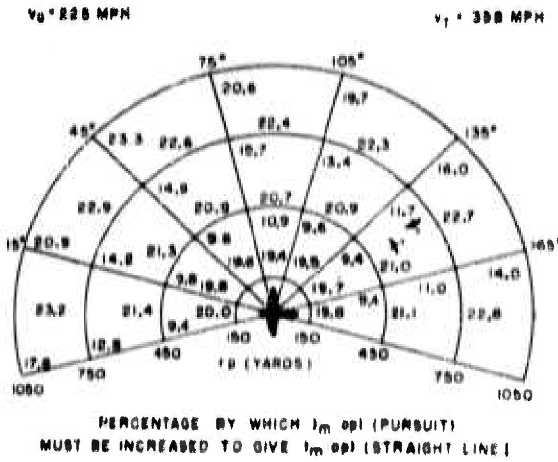


FIGURE 21. Relation between pursuit and rectilinear calibrations.

If a blanket percentage increase in t_m were decided upon, in passing from pursuit courses to rectilinear courses, no redesign would be necessary. The introduction of a suitably fictitious target dimension would accomplish the purpose.

5.5.7 Different Type Turrets

To conclude this section, the effect on the sight design of a change in the turret type may be considered. The classical turret, on which the design discussed above is based, positions its guns by rotations in azimuth and elevation.[1] As a consequence of this it was necessary to decompose ballistic deflection into lateral and vertical components. Tracking in an elevated plane of action is not easy, and the tracking pattern is frequently stepwise. If, on the other hand, a turret is employed in which one rotation is about a longitudinal axis of the aircraft (instead of the azimuth rotation about a vertical axis) then a plane of action is established in which the guns may make a second rotation. (If the elevation rotation is ϕ and the plane of action rotation is μ, then $\sin \phi = \cos E_0 \sin \theta$, $\sin E_0 = \sin \mu \cos \phi$, $\cos \theta = \cos A_0 \cos E_0$.) Tracking should be improved in such a watermelon turret. And the gyro sight is now simplified since all the trail is in the plane of action. But still a redesign of the electric circuit is required for good results.[80, 81]

It has also been proposed that a turret be installed with its azimuth ring inclined at an angle of 15° with the horizontal (to improve aircraft performance). Again the sight will supply wrong ballistics unless some correction is made. The naive proposal of feeding $E_0 - 15°$ instead of E_0 actually gives excellent results.[82]

Consideration of the two examples above shows the need for the closest integration of armament and fire control design. Moreover, since a change in muzzle velocity and ballistic coefficient of the ammunition may call for a redesign of a sight, ordnance must also work closely with armament and fire control if production and utilization schedules are to be effective. Otherwise a sequence of modification or patching operations becomes necessary.

5.6 GYROSCOPIC SIGHTS IN FIGHTERS

5.6.1 Simplifications

Because of the radical maneuvering of a fighter during an attack, it is evident that the only principle now at hand on which a sight for a fighter can be based is that of the gyroscope. Since the guns are pointed by such maneuvering, rates relative to the gun mount do not even exist. There is no reason why the single gyro sight described in the previous section cannot be used immediately by pursuit aircraft. As a matter of fact it is possible to simplify the circuits materially. In the first instance, since the classical fighter always fires within a few degrees of its direction of motion — the difference being due to angle of attack of the guns — it follows that no allowance for trail need be made and that the ballistic coils and circuits may be deleted. Because of the diving and banking it is pointless to leave in the gravity coil and circuit since any inputs with reference to the fighter would be in elevation error. Next, it is to be expected that time of flight produced can be made quite accurate for two reasons (1) direction of fire with respect to the aircraft may be properly

[1] Thus, a sight head with one edge perpendicular to the bore axis and parallel to the azimuth plane moves so that this edge remains parallel to the azimuth plane. For this reason, a camera fixed to the guns and recording correct shooting against a pursuit curve would show the path of the target, on the composite of all frames, as an arch. (Combine this motion with Figure 8 of Chapter 4.)

neglected, and (2) calibration for the standard pursuit curve attack can be made and will hold, since this is the tracking situation with which the fighter must contend (the case of fighter versus fighter on a curved course is considered in Section 5.6.3). Altogether one expects good results from a gyro sight installed in a fighter, particularly in view of large size of the target and the high firepower of the fighter. The operational problems of tracking and ranging are still present, of course.

5.6.2 Calibration

To calibrate the fighter sight [11] suppose (1) the sight transient has decayed, (2) the target flies a straight-line course at constant speed, (3) the bullets leave in the direction in which the fighter is flying, and (4) ranging and tracking are perfect. API M8 ballistics will be used ($u_0 = 2,870$ fps and $c_b = 0.440$).

The correct lead A is then given by equation (3) of Chapter 2 and is

$$\sin A = \frac{v_T}{d} \sin \sigma = \frac{q v_T}{v_0 + v_F} \sin \sigma ,$$

where q, as usual, is $u_0/d = (v_0 + v_F)/d$. The time-of-flight multiplier t_m is obtained by inserting A in the equation of the sight. This is the meaning of calibration. We have

$$t_m = \frac{A}{\dot{\sigma} - a\dot{A}} .$$

By use of evident expressions for $\dot{\sigma}$ and $\dot{A}$ this becomes

$$\frac{t_m}{r} = \frac{q}{(v_0 - l v_F)\left(1 - a \dfrac{q v_T \cos \sigma}{v_0 + v_F}\right) + al(v_T \cos \sigma + v_F)} ,$$

where $l = q - 1$. Computations based on this formula show (1) that t_m/r is remarkably insensitive to the speed of the fighter, v_F, for $a = 0.43$ (this is fortunate since v_F is not to be an input), (2) that variations in a around 0.43 are quite irrelevant, and (3) that increase in target speed magnifies the effect of approach angle (but for fast targets one can concentrate on tail cone approaches).

Averaging with respect to those variables which cannot be used as inputs, the following table for $t_{m\ \mathrm{opt}}$ is to be used.[186]

The time of flight used by the gyro is inversely proportional to the square of the current i in the range coil (Section 5.5.3). The proportionality factor K is, in turn, a function of ρ, T, $\dot{\sigma}$, where T is the ambient temperature of the cockpit. It is determined experimentally.[186] The design problem, therefore, is to choose resistances so that $t_{m\ \mathrm{opt}}$ is as close to $K(\rho, T, \dot{\sigma})/i^2$ as possible. The details [185] need not be pursued.

TABLE 1. Average optimum time of flight for fighter gyro sight [$t_m (r, \rho)$ (seconds)].

Range r	Relative air density		
(yards)	1.0	0.6	0.3
200	0.220	0.215	0.210
400	0.465	0.445	0.430
600	0.735	0.685	0.655
800	1.045	0.945	0.885

Respectable results can be achieved by such calibrations in spite of the averaging and fitting to K/i^2. For example, for caliber 0.50 API M8 ammunition the maximum error in the plane of action is of the order of 7 milliradians.[80] (This is *not* the total Class B error.)

5.6.3 Effect of Target Course Curvature

A fighter under attack by another fighter at some small angle off tail will frequently attempt to increase the attacker's deflection and make his aiming more difficult by banking as steeply as possible to cross the attacker's bow. (The quarry may also dive or attempt to utilize his propeller wash.) For attacks in the rear hemisphere, a longer time of flight is needed against the circular target path than is required by the rectilinear path. The reason is that the tracking rate $\dot{\sigma}$ is the same at a given instant for the curved path and for the tangent to that path. But the incurving of the target calls for a greater lead. Hence t_m must increase. This effect can be demonstrated analytically.[186] The increase required of t_m can be [186, 187] from 10 to 20 per cent. But the error induced by using a t_m appropriate to rectilinear paths against curved paths rarely exceeds 6 milliradians for expected conditions.

5.6.4 Irrelevance of Angle of Attack, Skid, Slip, Offset Guns

It will be recalled from Section 3.3.1 that a fighter pilot aiming by eye must make a slight aiming allowance along the median line of his sight to account for

the angle of attack of his guns. The gyro sight includes the computation of this effect.[86] In fact, a gyro sight will give the kinematic deflection regardless of skid, slip, or guns deliberately offset at any angle, as long as a path is flown as dictated by perfect tracking and ranging. This, in itself, is a major improvement over eye shooting which must be done from a cleanly flown aircraft. No demonstration of the point is needed since a fighter in such a situation can be thought of as a special case of a turret in a bomber.

5.7 OTHER SIGHTS

5.7.1 The K-8 Sight

The K-8 electrical sight was used to a limited extent in the Martin upper turret of the B-24 during World War II. It is also of the time-of-flight — angular-rate type and, like the K-3 class, measures rates relative to the gun platform instead of relative to the air mass. However, instead of using a mechanical circuit, it computes electrically, generating appropriate voltages, utilizing attenuator circuits, and employing ultimately its computed voltage outputs to displace its sight line laterally and vertically by motors. In general, it will behave like the K-3 in that false rates will be generated when the bomber yaws, pitches, or rolls, and in that its decomposition of deflection leads to rather large errors at high elevations of fire. From the point of view of inputs and operation it is much like the K-12. A switch permits the time-of-flight calibration to be reduced by 10 per cent to counter, in a blanket fashion, pursuit curve attacks. The mechanism will not be considered in detail, but it is worth while knowing, for possible applications, that such techniques have been worked out.

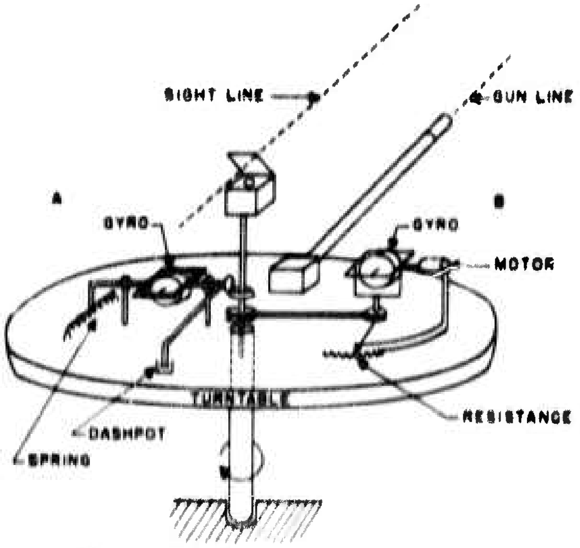


FIGURE 22. Gyro precessing mechanisms.

5.7.2 Other Methods of Gyro Control

Gyroscopes may be used in fire control in combination with forces other than eddy currents. Two different schemes[108] are shown in Figure 22 on one turntable. The mechanisms should be self-explanatory and are set up for performance in azimuth only.

5.7.3 German Gyro Sights

The major German gyroscope sights were the EZ40, EZ41, EZ42, and EZ45. The EZ40 is a single gyro sight with a mirror on the gyroscope to deflect the line of sight. (The ray from the reticle hits this mirror perpendicularly in the neutral position so that there will be no optical dip.) The EZ41 is a single gyro sight with the gyro unit remote from the sight head. The optical system is therefore actuated by motors controlled by a follow-up system. The EZ42 is a twin gyro sight installed in fighter aircraft (FW190, Me262). The two gyros are placed aft, one being mounted with its axis parallel to the longitudinal axis of the aircraft and the other with its axis parallel to the vertical axis. Friction dashpots are provided for smoothing and the gyros are further constrained by springs. Gyro deflection is picked off by tiny potentiometers and the lead is computed electrically. A servo drive, employing a very small two-phase motor, supplies motion to the optical system. (The Zeiss technique of winding variable potentiometers and certain other components of this sight are well worth keeping in mind.) The EZ45 is a remote single gyro sight. The gyro is controlled by erecting coils and its supporting case is driven by a small motor. Through a time-of-flight potentiometer the angle through which the case turns is made proportional to the lead. The sight parameter is 0.33. The position of the case is transmitted by servo to the sight head. A polarized relay of high sensitivity is used.[109]

5.8 SUMMARY

The introduction, Section 5.1, defines lead computing sights as devices which multiply the angular rate of the line joining gun to target by an appropriate time-of-flight multiplier to get an estimate of kinematic deflection. Ballistic deflection is separately obtained and combined with the kinematic deflection. The sights considered neglect the rate of change of range (and so cannot estimate future range well) but may have a suitably chosen time-of-flight multiplier to ameliorate this situation. The gunner has only indirect control over the line of sight since he controls only the guns, from which the line of sight is displaced by the mechanism.

The digression of Section 5.2 dismisses briefly early eye estimation methods based on the rate-time principle. The *Elephant*, *ABC*, and *Apparent Speed* methods attempt to make, in a mild way, a lead computing sight out of the gunner himself.

Section 5.3 considers the underlying theory common to all types of lead computing sights. Due to tracking errors the rate input must be smoothed before being used. The basic equation of these sights is discussed through the following points (1) the interpretation of smoothing as the process of taking an exponentially weighted average of all past values of the tracking rate, (2) the damping of oscillatory tracking errors, (3) the exponential decay of false leads (and slewing routines designed to minimize the decay time), (4) the amplification in going from oscillatory sight pointing errors to gun pointing errors, (5) the meaning of operational stability, (6) the delay in the sight's presentation of lead, (7) the factors affecting the choice of the sight's smoothing parameter, and (8) the analogue between aided tracking and lead computing sights. The section concludes with a discussion of time-of-flight calibration and the Class B errors of sights in general.

Section 5.4 discusses the mechanical sights of the Sperry series in some detail. The ball-cage and cam circuit that solves the problem is built up from first principles. By writing out the complete sight equations (consisting of those for the kinematic deflection components, those of the blueprint ballistics, and those governing the motion of the optical system) and the solution of that system, the causes of Class B errors are segregated into such compartments as: neglect of curvature, gun roll, feedback, and interchange of ballistics.

Section 5.5 is concerned with single-gyroscope eddy current sights. After explaining how a gyro may be used to measure kinematic deflection, the schematic details of the electromagnetic system, the optical system, and the electric circuits are given. The variation in the sight's parameter and the phenomenon of optical dip are explained. Finally, the component causes of Class B errors (calibration, optical dip, electromagnetic dip, etc.) are reviewed in some detail.

Section 5.6 points out that only sights of the gyroscopic principle can be used in fighters. The calibration of time of flight is more effective than that for a bomber's sight since there is only one direction of fire and since ballistics may be neglected. This calibration is given. The section considers briefly the problem of fighter versus fighter (curved target path) and the reliability of kinematic lead computation when the fighter's guns are not pointed in the direction of flight.

Section 5.7 supplies brief descriptions of the K-8 electrical sight and various German gyroscopic sights.

Chapter 6

CENTRAL STATION FIRE CONTROL

6.1 INTRODUCTION

6.1.1 Advantages and Disadvantages of Remote Control

Central station fire control [CFC] is an armament system in which a gunner located at a sighting station in one part of an aircraft can aim and fire guns located in a turret at some other part of that aircraft. For the B-29 airplane, for example, there are two 2 × 0.50 upper turrets, two 2 × 0.50 lower turrets,[a] and one 2 × 0.50, 1 × 20-mm tail turret. There are also five sighting stations: a pedestal station in the nose, a ringsight at middle top, two pedestal side-blister stations, and a tail pedestal station. The control of the turrets by the sighting stations is flexible—an important feature of remote gunnery. The two upper turrets are controlled by the upper station. The forward lower turret is under primary nose control and secondary blister control. The rear lower is also controlled by the two blisters. The tail turret is under primary tail control and secondary blister control. Primary control must be released before secondary control is operative.

Remote control has many advantages. Among these may be listed (1) guns and gunners may be located in the most effective positions for coverage and for vision, (2) loss of a station does not mean loss of the guns, (3) the turrets may be smaller and so afford less drag on the airplane, (4) pressurization of the airplane is simplified, and (5) the firing of the guns does not disturb the tracking.

On the other hand there are equally real disadvantages. Among these are (1) an additional correction for parallax due to the material displacement of sight line and gun line must be made, (2) torsion and bending of the aircraft structure may drastically impair harmonization, (3) relatively heavy and complex follow-up systems of control are needed,[b] and (4) the delays of the follow-up systems *may* be serious.

[a] The system is committed to upper-lower hemisphere coverage. If an attacking plane changes hemispheres the upper guns must cease fire and the lower guns take over. Should the lower guns be pointed to the wrong side, as they might well be when the airplane is under coordinated attack, the time delay in bringing them to bear in correct deflection position may be serious.

6.1.2 Elements of Central Fire Control System

The basic elements of the central station fire control system are: a sighting station, a follow-up system, a turret moved by the follow-up as dictated by the sighting station, and a computer connecting the sighting station and turret. The computer causes the gun line to differ from the sight line by an angle just large enough to compensate for the relative motion of the target during the time of flight of the bullet, to compensate for the divergence of the bullet from a straight line attributed to ballistic effects, and to compensate for parallax.

The computer receives continuous inputs of present range and lateral and vertical tracking rates from the gunner who, by keeping the target spanned, sets a potentiometer properly, and by keeping the target centered, causes two gyroscopes to set up precessional torques. The computer also receives the azimuth and elevation of the sight line automatically by selsyns at the sighting station; and, finally, receives altitude, temperature, and airspeed from a hand set operated by the navigator. Parallax and the basic ballistics of the ammunition used are built into the computer.

[b] But, neglecting follow-up equipment, the B-29 armament installation weighs 3,310 lb; whereas an installation consisting of two Martin upper turrets, two Sperry ball turrets, and one Consolidated tail turret, plus five K-8 sights would weigh 4,124 lb.

6.1.3 Restriction of Discussion to 2CH Computer

In this chapter the nature of the type 2CH computer used in this system will be studied in detail. The treatment is theoretical and schematic rather than mechanical. The important question of overall performance of the gunner-target system is not discussed. This is properly a matter of elaborate experimentation both in the air and in the laboratory. Nor are considerations in the large such as the design of formations for maximum and most efficient fire-power coverage of the attack region around the group, and the support fire problem considered.

6.2 BASIC THEORY

6.2.1 Trajectory Equations in Vector Form

The type 2CH computer (General Electric) assumes that the target moves in a straight line at constant speed during the time of flight of the projectile. It is necessary to recast the deflection theory of Chapter 2 for this situation to explain adequately the functioning of the computer. The methods of vector algebra are most appropriate.[a]

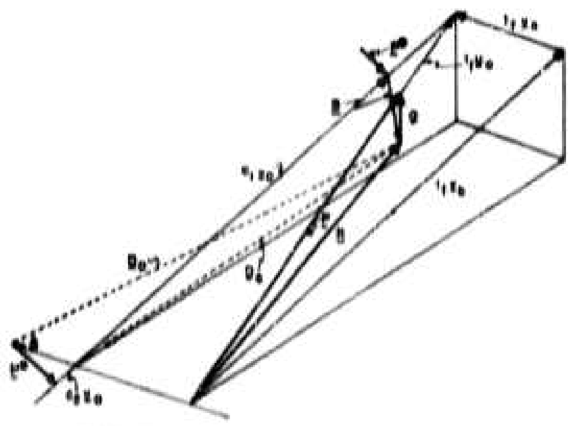


FIGURE 1. Ranges in remote gunnery.

Figure 1 shows the Slued range (distance covered in the air mass), **R**, and the corresponding Slued coordinates, **P** (measured along the line of departure to a point directly above the projectile), and **Q** (measured in a vertical plane).[a] It also shows the *two* future ranges peculiar to remote gunnery; $\mathbf{D}_g$ (the distance between gun and projectile), and $\mathbf{D}_0$ (the distance between the observer-gunner and projectile).

[a] Unit vectors carry the subscript 1.

The fundamental equation used in designing the computer may be obtained readily from the figure. Evident triangular relations are

$$\mathbf{P} = \mathbf{R} + \mathbf{Q},\ \mathbf{D}_g = \mathbf{D}_0 + \mathbf{A},\ \mathbf{u}_0 = \mathbf{v}_0 + \mathbf{v}_g,$$
$$\mathbf{D}_g = \mathbf{R} - t_f\mathbf{v}_g,$$

where t_f is the time of flight, and u_0 is the speed of departure P_0. Note that these are solely trajectory considerations. The presence or absence of a target is irrelevant. Since

$$\mathbf{u}_0 = \frac{u_0}{p}\mathbf{P} = \frac{u_0}{p}(\mathbf{R} + \mathbf{Q}),$$

the above relations combine to yield

$$\mathbf{v}_0 = \frac{u_0}{p}\mathbf{D}_0 + \frac{u_0}{p}\mathbf{A} + \frac{u_0}{p}\mathbf{Q} + l\mathbf{v}_g, \qquad (1)$$

where

$$l = \frac{u_0}{p}t_f - 1.$$

6.2.2 Angular Ballistic and Parallax Corrections

It is necessary to put ballistics and parallax clearly in view. To do so, introduce two vectors, **B** and **P***, *each perpendicular to the muzzle velocity* $\mathbf{v}_0$. The ballistic vector **B** is directed from the bore axis to the projectile. It includes both trail and gravity drop and subtends at the gun the total ballistic deflection angle Δ_b. (Minor effects such as windage jump and drift are neglected.) The parallax vector **P*** is obtained, in the first instance, as the perpendicular from the sighting station to the bore axis. The distance from the gun along the bore axis to the point of origin of **B** is $c_1\mathbf{v}_0$ where c_1 is to be determined. Similarly the distance from the gun along the bore axis to the point of termination of **P*** is $c_2\mathbf{v}_0$. Consequently

$$\mathbf{D}_0 = \mathbf{P}^* + (c_1 - c_2)\mathbf{v}_0 + \mathbf{B}. \qquad (2)$$

If, now, the vector $(c_1 - c_2)\mathbf{v}_0$ is moved parallel to itself so as to emanate from the sighting station, then **P*** may be moved to join $(c_1 - c_2)\mathbf{v}_0$ to **B**. This is a second interpretation of **P*** under which it subtends at the gun the parallax angle Δ_p. The angles Δ_b and Δ_p are called the *angular ballistic correction* and the *angular parallax correction* respectively. The

next task is to analyze the structure of $\mathbf{B}$ and $\mathbf{P}^*$. Componentially

$$\mathbf{B} = c_3\mathbf{v}_0 + c_4\mathbf{v}_G - \mathbf{Q},$$
$$\mathbf{P}^* = c_2\mathbf{v}_0 - \mathbf{A}.$$

To determine the four quantities c_1, c_2, c_3, c_4, two conditions are obtained by taking the dot product of each of the preceding two relations by $\mathbf{v}_0$. The third and fourth conditions arise if $\mathbf{B}$ and $\mathbf{P}^*$ are substituted in equation (2) and the resulting coefficients compared with those of equation (1). One finds that

$$c_1 = \frac{P^*}{u_0}\left(1 - \frac{lv_G}{v_0}\cos\gamma_G\right) - \frac{Q}{v_0}\cos Z,$$

$$c_2 = \frac{A}{v_0}\cos\gamma_A,$$

$$c_3 = \frac{Q}{v_0}\cos Z + \frac{lv_G P^*}{v_0 u_0}\cos\gamma_G,$$

$$c_4 = -l\frac{P^*}{u_0},$$

where γ_G is the angle between bore axis and platform velocity, γ_A is the angle between the vector $\mathbf{A}$ and the bore axis, and Z is the zenith angle (the angle between the vertical $\mathbf{Q}$ and the bore axis).

It is convenient for the sequel to reduce equation (2) to the scale of $\mathbf{v}_{01}$ (a *unit* vector in the direction of $\mathbf{v}_0$). Division of equation (2) by $(c_1 - c_2)v_0$ will yield the form

$$m\mathbf{D}_{01} = \mathbf{v}_{01} + \mathbf{p} + \mathbf{b}. \tag{3}$$

(We shall not stop to write out m, $\mathbf{p}$, and $\mathbf{b}$ explicitly.) The form (3) is adopted since $p = \tan \Delta_p$ and $b = \tan \Delta_b$, so that p and b themselves are excellent estimates of the two angular corrections.

6.2.3 Kinematic Deflection

As a final step in obtaining the basic equations to be mechanized, the target and its track are introduced. Since the corrections have been dismissed above, only the kinematic deflection need be deduced. If $\mathbf{v}_T$ is the target velocity and $\mathbf{r}$ is the present range (the range at the instant of fire), then *relative to the observer*

$$\mathbf{D}_0 = \mathbf{r} + (\mathbf{v}_T - \mathbf{v}_G)t_f = \mathbf{r} + \dot{\mathbf{r}}t_f,$$

which is an exact expansion, since the relative target path is straight and the target speed is constant. But $\mathbf{r} = r\mathbf{r}_1$ and $\dot{\mathbf{r}} = \dot{r}\mathbf{r}_1 + r\boldsymbol{\omega} \times \mathbf{r}_1$, where $\boldsymbol{\omega}$ is the angular tracking velocity. (This is the radial rate $\dot{r}\mathbf{r}_1$ plus the normal rate $r\boldsymbol{\omega} \times \mathbf{r}_1$ whose magnitude is $r\omega \cdot 1 \cdot \sin \pi/2$.) Using this value for $\dot{\mathbf{r}}$, we find

$$m'\mathbf{D}_{01} = \mathbf{r}_1 + nt_f\boldsymbol{\omega} \times \mathbf{r}_1 = \mathbf{r}_1 + \mathbf{\Lambda}_k, \tag{4}$$

where

$$m' = \frac{D_{01}}{r + \dot{r}t_f},$$

$$n = \frac{r}{r + \dot{r}t_f},$$

and

$$\mathbf{\Lambda}_k = nt_f\boldsymbol{\omega} \times \mathbf{r}_1. \tag{5}$$

But $\boldsymbol{\omega} \times \mathbf{r}_1$ is perpendicular to $\mathbf{r}_1$, and also to $\boldsymbol{\omega}$ since $\boldsymbol{\omega}$ is perpendicular to $\mathbf{r}_1$. Hence $\mathbf{\Lambda}_k$ is to be interpreted as the *kinematic deflection*. Finally, since $\mathbf{r}_1 \times (\boldsymbol{\omega} \times \mathbf{r}_1) = \boldsymbol{\omega}$, the tracking velocity is given by

$$\boldsymbol{\omega} = \frac{1}{nt_f}\mathbf{r}_1 \times \mathbf{\Lambda}_k. \tag{6}$$

6.2.4 Time of Flight

The time of flight t_f is, strictly, a function of $\mathbf{D}_0$, $\mathbf{v}_G$, $\mathbf{v}_{01}$, and the relative ballistic air density ρ. But v_G and ρ can be regarded as parameters, i.e., inputs set in before an attack, and present range r is to be used instead of future range. Hence, we write

$$nt_f = F(r, \mathbf{r}_{01}). \tag{7}$$

Since the vector $\mathbf{r}_{01}$ is an argument it is clear that *direction of fire* is to be taken into account by this system.

6.3 TURRET AND SIGHTING STATION

6.3.1 Follow-up System

Brief prefatory remarks on the follow-up mechanism will supply better orientation for what follows. Control consists of (1) a follow-up system, (2) a computing unit, and (3) auxiliary circuits. The last of these is rapidly dismissed as being of engineering interest only. The auxiliary circuits are: starting circuits for the rotating machinery, limit-switch circuits, firing circuits, control-transfer circuits, and the like. The follow-up system may be considered in slightly greater detail since it indicates that remote gunnery does *not* use the disturbed reticle principle of Chapter 5. Suppose that the computer has been removed. The operator moves by hand a light counter-balanced sight head. The sole job of the follow-up system is to make the gun direction $\mathbf{v}_{01}$

exactly the same as the sight line direction $\mathbf{r}_1$. (In a disturbed reticle system the operator controls $\mathbf{v}_0$ which in turn dictates $\mathbf{r}_1$. There is nothing in the concept of remote gunnery, however, that precludes the use of the disturbed reticle system.)

The follow-up system is schematized in Figure 2. The selsyn generator measures sight position by the position of its rotor and transmits this position to the selsyn control transformer which measures the difference between gun and sight positions. If a difference exists the error is sent to a vacuum-tube amplifier which builds up the signal sufficiently to control the output of a *d-c* amplidyne generator. The output of this amplidyne generator controls the *d-c* turret motor which drives the guns into parallelism

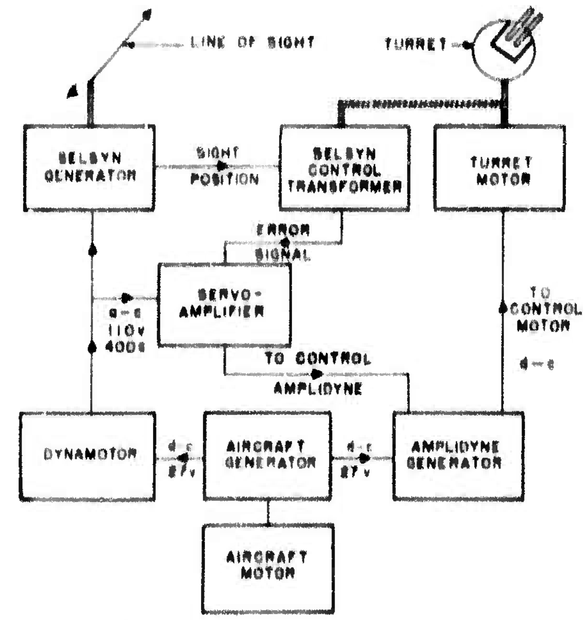


FIGURE 2. Follow-up system schematic. (Courtesy of General Electric Company.)

with the sight and causes the error signal to vanish.[a] (The peculiar advantages of the amplidyne generator are that the output voltage goes to its final value very rapidly, and that a very small field current of a few milliamperes is required to produce full voltage.) The dynamotor is simply a motor-generator set which takes dc and supplies ac to the selsyns and servoamplifier. In the actual system *two* sets of selsyns are used since a selsyn can be in error by 0.5°. The one-speed selsyns make one revolution for each revolution of the turret and so correct gross errors. The 31-speed selsyns make 31 revolutions for each revolution of the turret and refine the work of the one-speed selsyn to yield an error only $1/31$ of that of the one-speed unit.

[a] See definition of a servomechanism in footnote to Section 8.2.3.

6.4 TYPE 2CH COMPUTER

6.4.1 The Problem for the Computer

The problem put to the computer is the mechanization of equations (3), (4), (5), and (7). In operation, it is an interesting combination of gyroscopic properties, vector-mechanical constructions, and a shifting back and forth from electrical to mechanical signals. It is not calculating formulas but is effectively solving vector equations. The radical difference between the 2CH computer and those lead computing sights considered in Chapter 5 is that the 2CH is a continuously correcting system, that is, the answer at hand is continuously compared with the answer required by the inputs, and steps are taken to correct the gun position. The net result of the computer's working in conjunction with the follow-up system is to displace the guns from the line of sight by a total lead angle A, by which account is taken of the kinematic deflection, of ballistics, and of parallax. Kinematic correction pays attention to the true rate of rotation of the gun-target line, since gyroscopes are used, i.e., the rates are *not* relative to the gun mount. Ballistics, however, do depend on orientation of the guns with respect to the gun mount.

6.4.2 Rapid Review of Method of Solution

To emphasize the nature of the device as a continuous comparator, one can start with the total correction motors in discussing Figure 3. Suppose that the rotor has a certain angular displacement corresponding to some assumed total lead A. Suppose that this deflection is not the correct total lead. The output of the correction motor goes to differential selsyns which are also supplied with the azimuth and elevation of the sight line $\mathbf{r}_1$ through signals from the sighting station selsyn generators. The electrical displacement of the rotors of the differential selsyns corresponds to the *sum* of sight position and total lead, and the guns are positioned according to this sum $\mathbf{r}_1 + \mathbf{A}$ by the follow-up system.

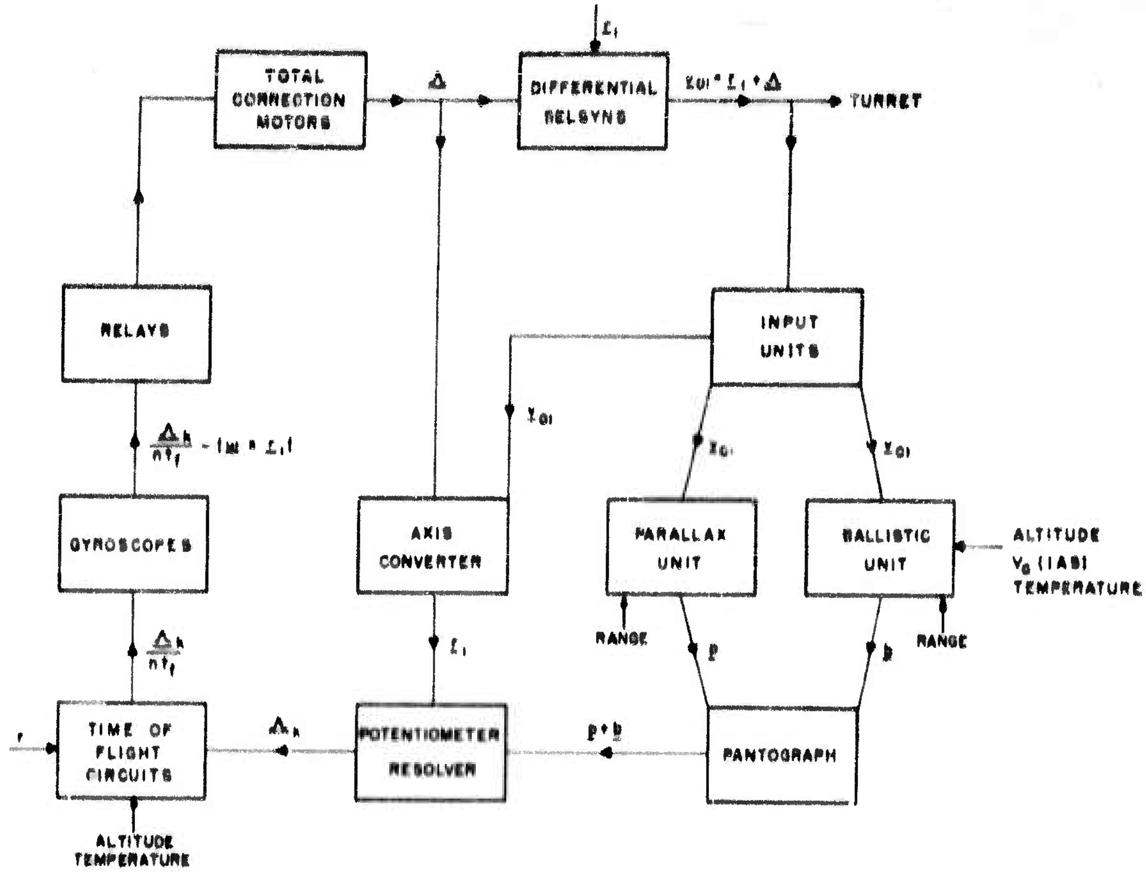


FIGURE 3. Box schematic, type 2CH computer.

The differential selsyn signals also go to input units which translate electrical values into mechanical values and send on present gun position $\mathbf{v}_{01}$ to parallax and ballistic units. These units determine the corrections $\mathbf{p}$ and $\mathbf{b}$ and transfer them to a pantograph which actually performs a vector addition of these two quantities. Thus one input to the potentiometer resolver is $\mathbf{p} + \mathbf{b}$.

Consider the other input to the resolver. The output of the correction motors goes to the axis converter which also receives gun position from the input unit. The converter combines gun position and total lead to produce by mechanical construction the unit present range vector $\mathbf{r}_1$. The potentiometer resolver now performs an operation equivalent to eliminating $\mathbf{D}_{01}$ between the equations

$$m\mathbf{D}_{01} = \mathbf{v}_{01} + \mathbf{p} + \mathbf{b}$$

and

$$m'\mathbf{D}_{01} = \mathbf{r}_1 + \mathbf{A}_k,$$

and produces as its output the assumed kinematic deflection $\mathbf{A}_k$ corresponding to the assumed total lead. It was necessary to obtain $\mathbf{A}_k$ since this value must be compared with the rate-time value from the time circuits and gyros.

The time-of-flight circuit, which has present range (and certain parameters) as input, has a total electric resistance nt_f. Consequently the current flowing in the circuit is $(1/nt_f)\mathbf{A}_k$, and it is this current that goes to the gyroscope unit. In that unit, the current produces through electromagnets a torque which opposes the precessional torque on the gyro generated by the tracking. The total torque corresponds to the difference between $\omega \times \mathbf{r}_1$ and $(1/nt_f)\mathbf{A}_k$. Depending on the sign of the difference, contacts in the gyro unit are made and one or the other of two relays trips. The relays control the total correction motors which then back up or go ahead to give an adjusted value of $\mathbf{A}$. And the cycle is repeated, or, rather, everything is happening simultaneously.

The preceding presentation used vectors as inputs and outputs. Except for the parallax and ballistic units and the pantograph, this is not quite the truth. The correction motors have azimuth and elevation components of the total lead as output since these are to be used to position the turret in azimuth and elevation. Again, the angular rates measured by the two gyros at a sighting station are vertical and lateral rates. That is, if the sight head is elevated, the "lateral" gyro is measuring the rate of rotation about an axis lying in a vertical plane and perpendicular to the line of sight while the "vertical" gyro is measuring the rate of rotation about an axis lying in the horizontal plane and perpendicular to the line of sight. Thus the outputs of the potentiometer resolver are in reality the lateral and vertical components of kinematic deflection.

6.4.3 Behavior of Specific Units

This subsection will discuss in more detail the internal nature of the component parts of the 2CH computer. Since the approach will emphasize the underlying theory, it is complementary to manuals of engineering detail.[29]

PARALLAX UNIT

This is a component in which there is effectively constructed *in miniature* a vector replica of some part of the environment. In forming such a replica

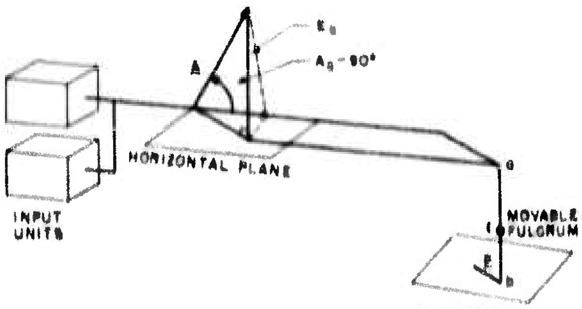


FIGURE 4. Parallax unit, schematic.

there is no need for duplicating orientation — it is relative position that is to the point. Choose, then, the vertical direction in the computer to represent the gun direction $\mathbf{v}_{G0}$. The axes of lateral and vertical gun rotation determine a horizontal plane in the computer in consequence of this choice. Only the projection of the vector **A** (which gives the distance between sighting station and gun emplacement) on the longitudinal axis of the fuselage will be used. This length is a fixed physical distance along a lever in the unit and is factory set. In order to get **p** we must first obtain the projection of **A** on the horizontal plane of the parallax unit and then modify this length according to the range. Schematically, these steps are accomplished in Figure 4. The input unit translates electric signals into gear rotations to position **A** as shown. The projection on the horizontal plane is precisely the scaled length of a line segment from the sighting station dropped on the bore axis, i.e., **P*** in Figure 1. This motion is reflected by sliding rods as corresponding motion of the point *a* (Figure 4). If the fulcrum *f* on a vertical lever (waggle-stick) is moved up and down as a function of range then the point *b* will be displaced by the angular correction **p**.

BALLISTIC UNIT

This component is very similar to the parallax unit. It differs in that the length **B** that plays the role of **A** must vary with altitude, *IAS*, and temperature. This is accomplished by gears actuated by the navigator's handset. Although the mechanization differs, the result in the plane of the pantograph is a motion of one end of another waggle-stick which represents **b** and lies in the plane of gun lateral and vertical axes just as does **p**.

PANTOGRAPH

Since **p** and **b** are now in the same plane, the pantograph can add them vectorially very readily. The device is a jointed framework of metal strips schema-

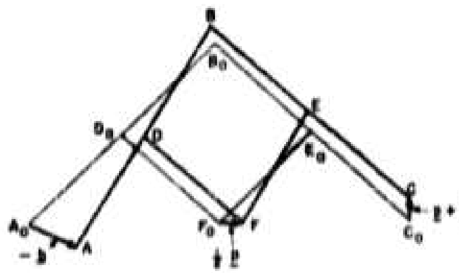


FIGURE 5. Pantograph unit, schematic.

tized in Figure 5. The parallax waggle-stick drives F and the ballistic waggle-stick drives A. Under a displacement from the original position ($A_0F_0C_0$) of amounts $A_0A = -\mathbf{b}$ and $F_0F = (\frac{1}{2})\mathbf{p}$, it is readily found from the geometry of the frame that $C_0C = \mathbf{p} + \mathbf{b}$.

Axis Converter [87]

Conceptually, this assemblage of gears and shafts has as inputs the azimuth and elevation components A_A and A_E of the assumed total lead A and the gun elevation E_G. (Actually, the inputs are three gear rotations of amounts $2A_E - 2A_A - E_G$, $E_G + 2A_A$, and E_G respectively.) As in the preceding units the vertical direction in the computer is selected to represent the gun direction $\mathbf{v}_{01}$. The horizontal plane in the computer is, then, the equivalent of the spatial plane determined by gun lateral and vertical axes. This system of three directions — gun direction and the two axes — is fixed in the unit. The job of the axis converter is to construct in correct relative position to this fixed system the corresponding system of the sight line, i.e., sight direction $\mathbf{r}_1$, and sight lateral and vertical axes. This it does by performing two rotations of the physical length corresponding to the vertical gun direction. Since the situation in the computer is only a reoriented version of the spatial picture we may revert to space for clarity. In space, rotation of a line in azimuth is represented by a

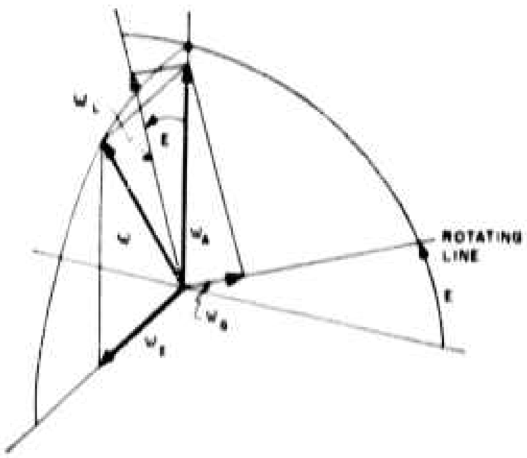


FIGURE 6. Rotational vectors.

vector $\boldsymbol{\omega}_A$ directed vertically upward. Rotation in elevation is represented by a vector $\boldsymbol{\omega}_E$ lying in the horizontal plane and perpendicular to the vertical plane through the line in question. Lateral rotation is represented by a vector $\boldsymbol{\omega}_L$ lying in the vertical plane through the line and perpendicular to that line. Vertical rotation is identical with elevation rotation so that $\omega_V = \omega_E$. The angle between ω_L and ω_A is clearly the elevation E of the rotating line. Hence $\omega_L = \omega_A \cos E$. Figure 6 illustrates the situation. ($\boldsymbol{\omega} = \boldsymbol{\omega}_A + \boldsymbol{\omega}_E$ is the *total* angular velocity, and when the decomposition $\boldsymbol{\omega} = \boldsymbol{\omega}_E + \boldsymbol{\omega}_L + \boldsymbol{\omega}_G$ is made, $\boldsymbol{\omega}_G$ is an additional and necessary angular velocity — the gun roll.) It follows from these rate relations, $\omega_V = \omega_E$, $\omega_L = \omega_A \cos E$, that, *correct to first order terms*,[a] the sight position can be obtained from the gun position by a vertical rotation of amount $-A_E$ followed by a lateral rotation of amount $-A_A \cos E_G$. This is done by gears in the axis converter. A representation of $\mathbf{r}_1$ thus appears as a mechanical displacement in a horizontal plane in the potentiometer resolver.

Potentiometer Resolver [87]

It will be recalled that the function of the resolver is to take in the sight direction and the sum of the parallax and ballistic corrections as two *vector-mechanical* inputs, and to give out *voltages* A_{kV} and A_{kL} which are the vertical and lateral components of the *assumed kinematic* deflection. As expected within the resolver is a vertical direction representing gun direction $\mathbf{v}_{01}$. The plane of the pantograph is perpendicular to this vertical direction and the output of the pantograph gives the displacement $C_0C = \mathbf{p} + \mathbf{b}$. Hence in the resolver $OC = OC_0 + C_0C = \mathbf{v}_{01} + \mathbf{p} + \mathbf{b}$ is constructed. According to equation (3) the device has physically produced the vector $m\mathbf{D}_{01}$. Another part of the resolver is rigidly affixed to the axis converter. In Figure 7, this part appears as

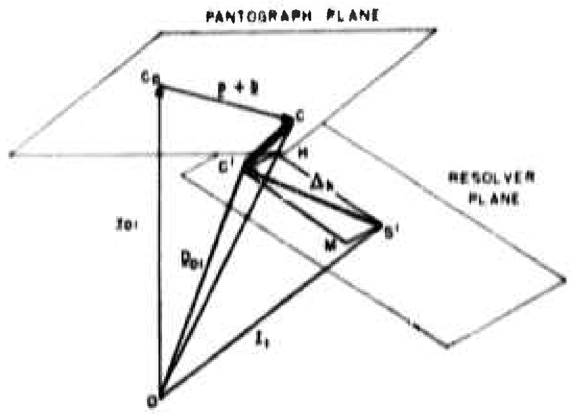


FIGURE 7. Potentiometer resolver, schematic.

OS'. At the point S' the resolver has a plane perpendicular to OS'. A waggle-stick from the point C in the pantograph plane is maintained so that $C'C$ is parallel to OS'. The vector equation $OC' = OS' + S'C'$ is translated as

$$m'\mathbf{D}_{01} = \mathbf{r}_1 + \mathbf{A}_k.$$

[a] The mechanism actually reproduces the geometrical constraints of the situation and so is not guilty of approximations.

By this frontal approach to the problem, i.e., a construction in miniature of the vector situation in space, the kinematic deflection has been produced.

It remains to obtain the lateral and vertical components of the kinematic deflection *relative to the sight line* in the form of voltages. In the resolver plane, which is perpendicular to the computer's reproduction of the sight line, $S'H$ and $S'M$ are such components. A voltage equal in magnitude to A_{KL} can be obtained by means of a variable potentiometer whose sliding contact M moves along the line $S'M$ in the lateral direction. And with some mechanical elaboration the same remarks apply to A_{KE}.

Time-of-Flight Circuits

The circuit is a combination of fixed and variable resistances such that the total resistance approximates nt_f. The schematic is shown in Figure 8. The

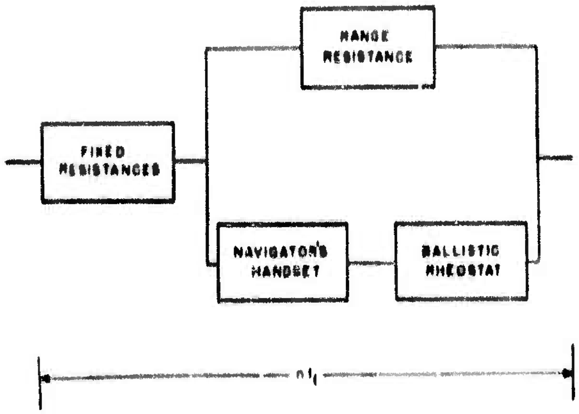


Figure 8. Time-of-flight circuit, schematic.

variable range resistance, controlled by the gunner who keeps the target framed, supplies a continuous estimate of present range. The navigator's hand set puts in altitude, indicated airspeed and temperature. The ballistic unit supplies the component of own speed in the direction of fire, which means that a good value for velocity of departure is at hand. Certainly all the appropriate variables are present to obtain an excellent value for time of flight. However, in one direction it is a question of how well a simple circuit can approximate a complicated function, and in another direction what calibrations can be made to allow for *curved* courses. It should be remembered that for straight tracks the results with respect to this second direction should be excellent since nt_f is exact by equation (4).

6.4.4 The Gyroscope Units

The gyroscope units[30] will be discussed in somewhat more detail than were the preceding components. To measure lateral and vertical tracking rates there are two gyroscopes at each sighting station. Each gyroscope consists of a flywheel and the rotor of a small, high-speed, and constant-speed electric motor running on 27 volts d-c at 10,000 revolutions per minute. The *lateral rate* unit is shown in Figure 9. The housing axis is always parallel to the

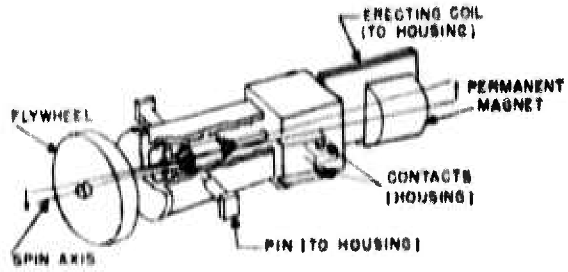


Figure 9. Gyroscope, type 2CH computer. (Courtesy of General Electric Company.)

line of sight, and the axis of the pins that support the gyro motor frame remains horizontal. The gyro spin axis may depart very slightly from parallelism with the sight line, but this departure is in *elevation* only and is limited by adjustable stops. Consequently, it will not be inferred that the gyro axis displacement measures deflection as it did in Section 5.5.2.

Under rotation about the lateral axis (this is ω_L) in Figure 9, the gyro exerts a *precessional torque* about an axis perpendicular to both the spin axis and the axis of the applied tracking torque and hence about the axis of the supporting pins. But bucking this precessional torque is an electromagnetic coupling due to the displacement of permanent magnets (which are fixed to the gyro frame) from the center of an erecting coil fixed to the housing. The current in the erecting coil is A_{KL}/nt_f, and the torque produced is proportional to this current. If this torque is not as strong as that due to precession the gyro will precess slightly until an electric contact is met. For this case the assumed A_{KL} was too small. Hence the contact causes a relay to trip which permits a current to flow in the armature of the total correction motor in a direction which will increase A_{KL}. This will strengthen the erecting coil torque and pull the gyro away from the contact. If the erecting coil torque

had been too strong initially the process would have reversed. In general, the gyro will hunt or successively under-and-over correct by going back and forth between the contacts. Back lash is deliberately introduced between the total correction motor and its output wheel to minimize hunting. Only under perfect and uniform conditions will the gyro ride with its spin axis identical with the housing axis.

It will be noticed that there is no smoothing in the precise sense of that given under "Weighted Averages" of Section 5.3.4. However, the gyro housing is attached to the sight by a form of elastic coupling damped by friction. The basic purpose is to reduce vibration but possibly a certain amount of smoothing is also introduced by such a support.

The gyroscope system introduces certain errors in the problem which can be understood by writing out the dynamical equations of the system.[36] Let $\mathbf{M}_G$ and $\mathbf{M}_F$ denote, respectively, the vector angular momenta of gyroscope and gyro motor frame. Let $-\mathbf{T}_G$ be the moment of all forces exerted by the gyroscope on the frame. (This includes reaction and bearing torques of the motor.) And let $\mathbf{T}_H$ be the moment of all forces exerted by the gyro housing on the gyro motor frame. (This includes the erecting coil torque, frictional torques, pivot reactions, and reactions when the stops are closed.) The equation for the gyro motor frame is

$$\frac{d\mathbf{M}_F}{dt} = \mathbf{T}_H - \mathbf{T}_G$$

and that for the gyro is

$$\frac{d\mathbf{M}_G}{dt} = \mathbf{T}_G$$

so that

$$\frac{d(\mathbf{M}_F + \mathbf{M}_G)}{dt} = \mathbf{T}_H$$

in a fixed coordinate system. With respect to a rotating coordinate system determined by the pin axis, the spin axis, and an axis perpendicular to these,[f] and with d/dt now measuring rates of change with respect to this rotating system, a theorem from mechanics yields

$$\frac{d(\mathbf{M}_F + \mathbf{M}_G)}{dt} + \mathbf{\Omega} \times (\mathbf{M}_F + \mathbf{M}_G) = \mathbf{T}_H$$

where $\mathbf{\Omega}$ is the vector angular velocity of the (p,s,q) coordinate system. With an obvious notation for moments of inertia, the p component of this equation is

$$(I_{Gp} + I_{Fp})\frac{d\Omega_p}{dt} + (I_{Gp} + I_{Fq} - I_{Gs} - I_{Fs})\Omega_q\Omega_s$$
$$- I_{Gs}\Omega_q\Omega_0 = T_{Hp} \quad (8)$$

where Ω_0 is gyro speed with respect to its frame. The purpose of the gyro unit can now be phrased symbolically. It is designed to approximate Ω_q (which is the tracking rate) by use of the equation

$$-I_{Gs}\Omega_q\Omega_0 = T_{Hp} \quad (9)$$

where T_{Hp} is essentially the torque exerted by the electromagnetic coupling on the spin axis. In other words, the device is neglecting the first two terms of the complete equation (8).

Consider the second term on the left of equation (8) and assume that the expression in parentheses is of the same order of magnitude as I_{Gs}. The angular rate Ω_s is precisely the equivalent of the gun-roll rate of Figure 6, i.e. $\Omega_s = \omega_A \sin E$, where A and E are the azimuth and elevation of the spin axis (which diverges very slightly from the sight axis). But ω_A cannot exceed $v \sec E/r$ where v is the velocity of the target relative to the bomber and r is the range. Hence Ω_s is large only if E is close to 90° and/or r is small. If $|E| < 77°\ 45'$, $v < 1000$ fps, and $r > 750$ ft, then $|\Omega_s|$ is 0.6 per cent of Ω_0 (which is $20{,}000\pi$ radians). The second term may be safely neglected. However this does indicate that certain restrictions on high elevation fire are inherent in the system.

Turning to the first term on the left of equation (8), we are concerned with the angular *acceleration* $d\Omega_p/dt$ about the pin axis. The lead given by equation (8) is

$$\Omega_q t_f = -\frac{T_{Hp}t_f}{I_{Gs}\Omega_0} + \frac{I_{Gp} + I_{Fp}}{I_{Gs}\Omega_0}\frac{d\Omega_p}{dt}t_f,$$

when the second term is neglected. Hence angular acceleration introduces a spurious lead (the second term on the right of the preceding equation). For *perfect tracking* of an accelerated target this can hardly exceed 8 milliradians.[36] But under normal *imperfect tracking* the operator may be making very large *oscillatory* errors in angular position. To what extent the computer filters acceleration errors is not definitely known. The obvious procedure is to make a laboratory bench test.

6.4.5 Total Correction Motors

The total correction motors make lead corrections at a constant rate of about 40 milliradians per

[f] Components in these three directions will be denoted by p, s, and q respectively.

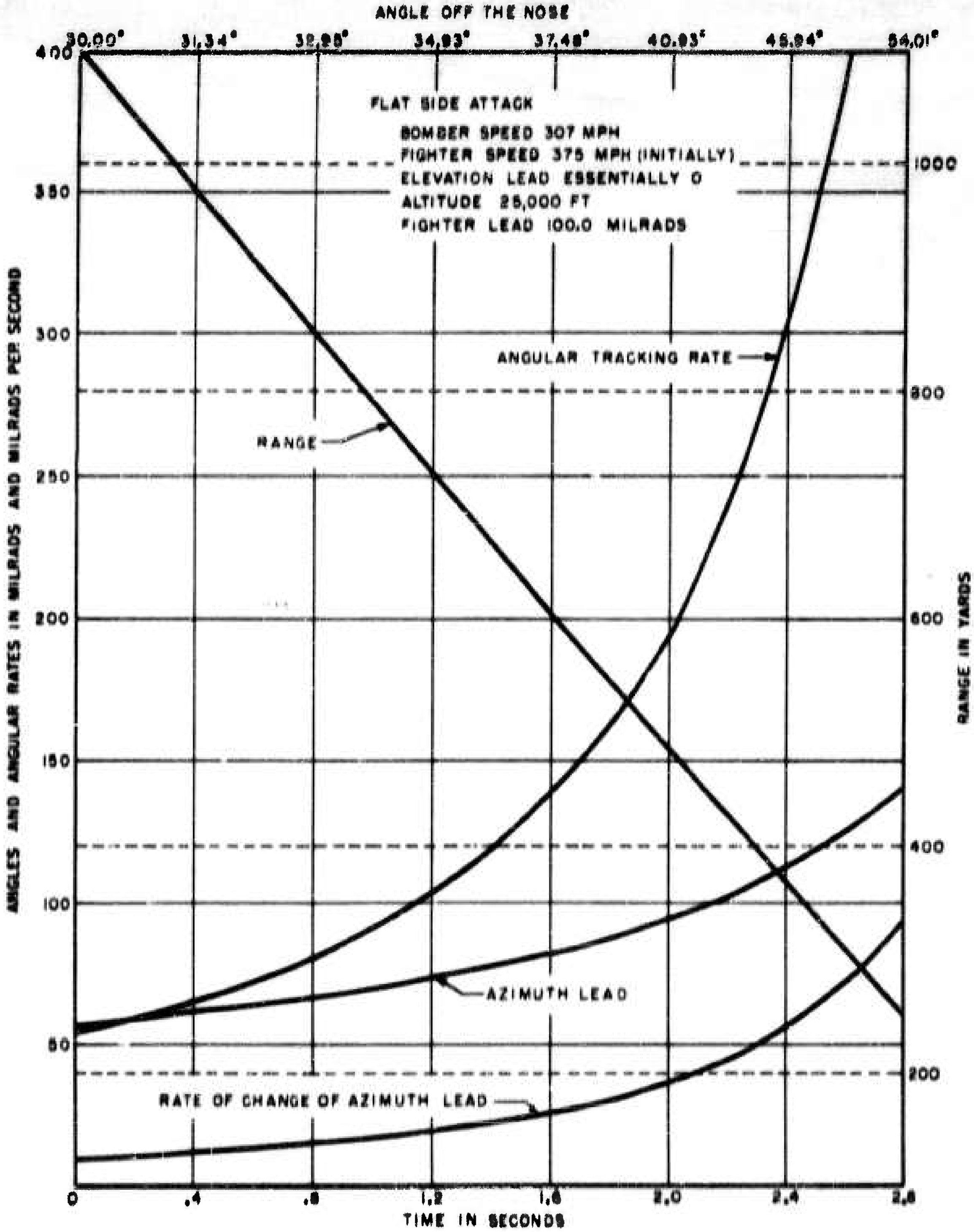


FIGURE 10. Typical frontal attack.

second. If the motor runs continuously in the same direction for 1.5 seconds, a higher speed of about 80 milliradians per second becomes operative. To illustrate the effect of the time delay caused by this motor suppose that with an initial lead of zero the gun is to be slewed in one-half second to get on and then to track a target requiring a lead of 100 milliradians. Then,[85] approximately: at $t = 0$, $A = -20$ (sight on, tracking started); at $t = 0.5$, $A = -20$ (backlash overcome); at $t = 1$, $A = 0$; at $t = 1.5$, $A = 20$ (high speed begins); at $t = 2.0$, $A = 60$; at $t = 2.5$, $A = 100$.

What seems needed is a system of variable speeds so arranged that when (and only when) the required lead correction is large a high speed will be provided.[86] This may be accomplished by using the variable pressure of the gyroscope on the stops to introduce variable resistance into the motor circuit. Dynamic braking may also be used. It should be observed that although increasing the motor speed decreases the time delay, at the same time smoothness is sacrificed and excessive hunting may occur.

To illustrate the need of high speed in getting on target quickly and of low speed still sufficiently great to keep up with lead changes, curves for a typical frontal attack are shown in Figure 10.

In addition to the variable speed proposal noted above, various other methods have been suggested.[87] Typical of these are (1) the motors run at high speed except when the trigger is pressed, (2) the shift from high to low speed when the trigger is depressed is delayed for, say, one second. It is probably worthwhile to make a rough paper assessment of expected results in assumed tactical situations under such proposals. The whole slewing operation should also receive careful attention.[88]

For purposes of reference, the motor equation may be quoted.[89,90] For a d-c motor with separately excited or permanent field

$$b\frac{dw}{dt} + \frac{a}{R}w = \frac{E}{R} - I(w),$$

where a and b are constants, R is the total resistance of the armature, brushes, and commutators; E is the applied voltage; and $I(w)$ is the current required to run the motor at the speed w, if w were constant. For frictional loading

$$I(w) = I_0 + I_1 w$$

is an adequate form, where I_0 depends on load, bearing friction, and armature hysteresis, and $I_1 w$ depends on the armature drag due to eddy currents and on viscous friction and is nearly independent of the load.

6.4.6 Range Follow-up Motor

Another inherent weakness in this computer system is that the range follow-up motor is frequently too slow. Three ranges must be considered. These are (1) the true range, (2) the range introduced by potentiometer at the sighting station, and (3) the range set into the computer by the range follow-up motor. In its steady state the motor alters computer range r (in yards) according to[91]

$$r = -110 + (r_0 + 110)e^{-0.001216kt}$$

where k is the number of revolutions per minute of the motor's output gear. The implications of this behavior can be made out by referring to Figure 10. Suppose at $t = 0$ that the true range $r_0 = 1{,}100$ yd is in both the sight and in the computer. The operator, behaving perfectly, is changing range at an almost constant rate of 320 yd per sec. The maximum rate of change of computer range, however, occurs for $k = 250$, which is the maximum motor speed. It is

$$\dot{r} = -0.304(r + 110).$$

Hence below 942 yd range the motor cannot run fast enough to keep up with the input. The gunner could suddenly reduce range to 250 yd and computer results would be just the same as under perfect ranging.

6.4.7 Calibration

In connection with the above remarks on the total correction motors and on the range follow-up motor, the effect on calibration of the time of flight may be noted. In any machine, whenever defects, unavoidable or not, call for extensive redesign or installation of new units, one attempts to save time and equipment and still achieve respectable overall results by giving artificial values to adjustable parts of the existing equipment. For example, to account for the delay in the solution time on nose attacks, the time of flight is multiplied[87] by 1.07 since leads increase on nose attacks. But calibration usually has its price. In this case performance against attacks from the rear will deteriorate since the factor should be less than unity in that hemisphere.

6.5 SUMMARY

Section 6.1 limits the discussion of this chapter to a theoretical study of central station fire control itself. The wider questions of formations, support fire, and experimental results are not considered.

Section 6.2 derives four basic equations which the computer is to mechanize. Significantly, the first three are vector equations and in large part the computer considered is a vector mechanism.

Section 6.3 discusses the nature of a follow-up system under which guns remote from an operator are made to follow his positioning of the sight line. The computer, which introduces the gun displacement from the sight line required by target motion, ballistics, and parallax is disconnected.

Section 6.4 gives first, in schematic terms, the method by which the computer solves the total deflection problem. Next the behavior of the individual components is studied in more intimate detail. Considerable attention is paid to the behavior of the gyroscope units since angular tracking accelerations can lead to appreciable errors through this system. Finally the time delay in response of the total correction motors, and the inadequate range change rate in the range follow-up motor are considered.

Chapter 7

ANALYTICAL ASPECTS OF AIRBORNE EXPERIMENTAL PROGRAMS

7.1 INTRODUCTION

7.1.1 Role of Airborne Test Programs

The closed feedback loop formed by the successive elements — target, gunner, controls, computer, guns, bullet, target — is a complicated servomechanical system. When a system is too complicated to permit an exact theoretical analysis of behavior, the common technique is to supply certain inputs and compare the actual output with a perfect output. This is the philosophy of airborne assessment tests of fire control equipment. If it is a question of choice between two fire control systems, the procedure is certainly logical, provided a proper simulation of the conditions under which the systems must perform is made, and provided that the experimental technique is considerably more accurate than the mechanism under test. If it is a question of prediction of overall performance in combat of a particular system, the attitude may still be defended, although not nearly as strongly as in the competition situation. But if it is a question of the decomposition of overall errors into componential causes, so that design improvements may be made, such assessment has minimum value, unless it is very complicated and detailed.[a]

To meet, in part, this last question, the errors made by the system under perfect operation and construction have been studied earlier in this account. The manufacturing variation from instrument to instrument and other variations caused by non-perfect construction and functioning can be studied by laboratory bench tests in which factors such as turntable speeds and temperature can be carefully controlled. More elaborate laboratory machines may use perfectly known inputs to a human gunner by means of cammed target courses and controlled gun platform motion and may measure in complete detail the behavior of each component of the system. It is this method that is most fruitful in arriving at design changes.

One cannot expect to add up componential errors deduced by the methods of the preceding paragraph to obtain the total error determined by airborne tests. The reason is twofold. In the first place, the complete system is in no sense linear. It has numerous feedbacks, lags, and unexpected gunner reactions. In the second place, the closed loop described initially contains elements which are essentially statistical. The main variables are: target behavior (a course cannot be reproduced exactly in the air), gunner behavior (intra-gunner variations may be as large as inter-gunner variations), gun mount motion (which varies with the aircraft, the weather, and the pilot, particularly under avoiding action), and instrumental variation (sights, guns, turrets as affected by wear and calibration).

But it is concluded that field tests, laboratory tests, and theoretical analysis all have an important place in the field of fire control. The important point is that they be coordinated partners in this field. The balancing and coordination of their roles is the duty of vigorous technical administration.

7.1.2 Classification of Errors by Cause and Statistics

The errors of the complete system may be classified systematically. The first decomposition[14,197] emphasizes cause:

1. *Class A errors.* Inter- and intra-instrumental variations are caused by play arising from permitted tolerances and further play introduced by loose action or worn parts. These are errors representing the failure of the instrument to operate as designed.

[a] Such a detailed assessment program was undertaken for the B-29 system by the General Electric Company.

2. *Class B errors.* As defined earlier, these are the theoretical errors of the mechanism caused by design assumptions and compromises.

3. *Operational errors.* These occur because of tracking and ranging errors, harmonization errors, and faulty presetting of flight data.

4. *Dispersive errors.* Because of mount and gun vibration, variation from round to round of the ammunition in ballistics and muzzle velocity, play in a turret, and the difficulty of holding a point of aim, the bullets of a burst spray out in a cone. Although the pattern on a section of this cone is usually elliptical,[184] the orientation of this ellipse so depends on the direction of fire and other factors that an equivalent circular pattern is taken conventionally. This assumption is augmented by supposing that the pattern is described by a circular Gaussian distribution about a mean point of impact. (See Section 1.5.2.)

It should be noted that although the pattern tends to conform to normality, the locations of individual shots are not independent. This is serial correlation and is evidently of most importance in connection with very short bursts. Roughly, if one shot has a large error of one sign, then the next round is likely to have a large error of the same sign. This is related to aim wander.[198] As the gunner tries to hold on a target, his aim traces a snakelike path. One can imagine the small pure dispersion being carried along this path as a probability cloud. We attempt to make a distinction between the dispersive serial correlation arising when a gunner tries to hold on a fixed target and the much more important tracking serial correlation. The point has not been clarified completely from the theoretical point of view. On the whole, we shall attempt to avoid it by using more naive statistical ideas.

Another classification of system errors can be made which emphasizes their statistical nature:[198]

1. *Fluctuating errors.* These errors arise from dispersion, tracking errors in angle (including any amplification of the system), and Class A errors.

2. *Quasi-steady errors.* These give a sensibly constant bias over one burst but vary from burst to burst. Typically, they are attributed to range input errors, harmonization errors, and faulty setting of flight data.

3. *Steady errors.* Given a particular situation, such as a specified point on an assigned pursuit curve, these errors yield a fixed bias. They are evidently Class B errors.

7.1.3 Nature of Modern Airborne Test

Early assessment tests were made by firing against a flag target towed past the gun mount on a parallel course at low relative speed and close range. The number of hits was counted. This procedure can be defended only on the basis that the installation *was* airborne and it *did* fire. Clearly an extrapolation of performance on such trivial courses to expected performance against an actual target course is unjustified. Furthermore, the bullets that missed are lost to analysis, since their position is not known. Thus the method is statistically inefficient.

Current methods for airborne tests are based on the dry *run* principle. That is, combat conditions are simulated as closely as possible by having a fighter plane attack the bomber. Neither side fires actual bullets.[b] Instead, by means of various accurate recording instruments, usually airborne by the aircraft participating, four basic types of data are obtained (1) the range and bearing of the target, (2) the tracking and ranging of the sight under test, (3) the actual gun pointing, and (4) the attitude, speed, and altitude of the gun-mounting aircraft. Sometimes it is desirable to obtain all or some of these data, in synchronization, for the target aircraft. To these two requirements, combat simulation and precise instrumentation, a third must be added. For a given type of tactical situation, such as a rear quarter attack, sufficient replications of the circumstance must be made to give adequate statistical value to those variables which determine the system's output but which cannot be controlled. This implies that the experiment must be designed carefully, usually by the Latin-square technique. In general, a minimum of replications of the order of 15 is needed for each system under each tactical situation.

We may now turn to some of the details of assessment programs. The three major phases are (1) instrumentation, (2) interpretation and reduction of data, and (3) measurement of effectiveness. Only a rapid survey is contemplated.

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[b] The lack of inhibition caused by gunfire is a serious flaw of the method which cannot be answered by the use of frangible bullets, since full-scale combat conditions could not then be used. Simulating recoil mechanisms may be much more to the point.

7.2 INSTRUMENTATION (AIRBORNE PROGRAMS)

7.2.1 Bearing and Range of Target and Attitude of Gun-Mounting Aircraft

To determine the correct deflection at each instant, i.e., perfect output, it is necessary to know the space path of the target relative to the gun mount as a function of time. This is normally accomplished by a determination at each instant of the range and bearing of the target with respect to the gun mount. Three 35-mm cameras with lenses of 1-in. focal length, when ganged and installed in a waist window of a heavy bomber, will cover a field of approximately 120° in azimuth and 50° in elevation.[17] The alignment is determined by photography on the ground. The lenses must be calibrated so that the magnification will be known under projection. The bearing angles relative to this installation can be determined through projection on a suitably gridded screen. There are other methods of determining bearing. The motion in azimuth and elevation of a turret which mounts a single camera and which tracks the target can be picked off the gear trains either by gears or by selsyns.[81] Dial readings are photographed. The direction of the center of the picture is known and the bearing of the (displaced) target image is readily determined. It would also be possible to use ultra-narrow-beam radar which can give angles with a probable error of 1 or 2 milliradians. Range is usually determined by measurements of the image on the film supplied by a camera of 2-in. focal length (to gain magnification at the expense of field size) clamped to the gun. This determination is usually the least accurate step in the entire scheme because of blurring of wingtips and foreshortening of the target. The use of band painted wingtips, of landing lights, and of infrared lamps has been suggested.

To reduce the relative bearings to direction in an air mass coordinate system, the yaw, pitch, and roll of the airplane must be measured. This is done by a pair of suitably gimbaled gyros which maintain the direction of their spin axes in space as the aircraft rocks about the mounting. The resulting displacements can be picked off either mechanically or by selsyns. Yaw, pitch, and roll can also be calculated after measurement of the tilt of the horizon and the position of a sunspot on some plate fixed in the aircraft.[99]

7.2.2 Astrometric Methods

The space paths of the two aircraft might well be determined by astrometric methods.[180] Suppose that two vertical cameras of 12-in. focal length are mounted on the ground at opposite ends of a suitable base line. The aircraft photographed by these cameras carries perhaps four brilliant flash units. Coincident with each flash a radio signal is sent to the ground and recorded by a chronograph with an accuracy of 1/1,000 sec. Standard precision methods are applied to measure the image positions on the two plates. With aircraft at 10,000 ft the overall probable error of position is ± 1 ft, and the probable error in velocity is $\pm \frac{1}{4}$ fps. The disadvantages of this procedure are found in the restricted sky area available to the maneuvering aircraft (this could presumably be met by employing a lattice of cameras) and in the complexity of the data reduction. Nevertheless, the flexibility of the installation in assessing almost all types of aerial problems should make it attractive to a large proving ground.

7.2.3 Synchronization

Returning to the standard airborne instruments, the gun camera used to determine range also establishes the actual gun pointing. Finally, if a camera is mounted to photograph the combining glass of the gunsight, then ranging and tracking errors are known; and if a camera photographs an instrument panel, flight data are recorded. It is evident that these four camera units, tricamera, gun, sight, instruments, must be synchronized. This is usually effected by placing a light bulb in the back of each camera which fogs the film edge in each camera at intervals dictated by a master control unit.

7.2.4 Air Mass Coordinate Technique

Somewhat simpler instrumentation is possible if an entirely different approach [98,107] to the problem of determining gun-pointing errors is made. For near pursuit curve attacks the bore axes of the bomber's guns and of the fighter's guns will be close to parallelism, since the leads taken by the duelists are equal in size and opposite in sense, to a first order approximation. (See Section 2.2.2.) Equip each aircraft with a camera kept in rigid alignment with the respective bore axes. Then the image of the opposing aircraft

may be expected on each film. Suppose, further, that the bomber is supplied with a 100 per cent own-speed mechanism which places on the film a point giving the resultant direction in the air mass that the bullet would take under the action of muzzle velocity and mount velocity. Suppose that for the fighter the angle of skid [94] and angle of attack can be determined so that the resultant instantaneous flight direction in the air mass of the fighter can be placed on the projected fighter film.

At a given instant of fire immobilize the bomber. Then on the frame corresponding to this instant the direction the bullet takes in the air mass appears as a point. If the position the fighter would reach in the air mass after the time of flight of the bullet could be plotted on the same frame, then the gun-pointing error would be known immediately. To do this, suppose that the fighter proceeded over the time of flight of the bullet at its instantaneous velocity, i.e., the curvature of its path is neglected. The crucial point of the method now appears. The plane determined by the fighter's wingtips and the bomber's camera appears on the bomber's film as a line joining the wingtips of the fighter image. This plane appears on the fighter's film as a line parallel to the bottom of the film (since the fighter's camera is so set with respect to the wingtips) and passing through the camera installation on the bomber. The plane containing the fighter's instantaneous velocity and the bomber's camera appears as a straight line on the fighter's film passing through the image of the camera installation of the bomber. Since the films of the two cameras are *nearly parallel*, the angle between this line on the fighter film and the horizontal on the fighter film may be transferred with negligible error to the bomber film with respect to the line joining the fighter's wingtips. This establishes a line on the bomber film on which the fighter's future position must lie. But the angle subtended at the immobilized bomber by the fighter motion is precisely the deflection required by the fighter's approach angle (measured on the fighter's film) and so is

$$q v_F \frac{\sin \alpha}{v_0}.$$

Consequently, the future target position is established on the chosen frame of the immobilized bomber's film record.

The method now corrects the future position by computing the fighter's normal acceleration. The true future position is plotted on a line passing through the previous future position and perpendicular to the wingtip line.

Although the procedure has real advantages from the instrumental point of view (no recording of yaw, pitch, roll is required), and from the point of view of data reduction (ballistics are not used since work is in the air mass) its overall errors are only known theoretically. It has not been carried through in actual programs as has the relative coordinate method.

7.2.5 Distant Reference Point Method

The simplest method of assessment that has been devised uses just one camera fixed to move with the gun with which it is bore-sighted.[129] It relies on the existence of distant reference points such as mountains or clouds which do not move during the assessment either absolutely or relatively. If a certain frame is chosen it may be assumed that correctly aimed bullets fired at an earlier instant, corresponding to the time of flight for the range of that frame, are hitting the target. The film is turned back by the appropriate number of frames and the fighter position, gun position, and reference points are marked on the projection screen. Turning back to the original frame, the gun error is known by superimposing the reference points, correcting original gun position for ballistics and then observing by how much the original corrected gun position fails to be on the target.

Despite the lack of accuracy, the simplicity may well recommend this method for either rapid work or pilot tests designed to show gross errors.

7.3 REDUCTION OF DATA

7.3.1 Techniques

This section will content itself with a discussion of the techniques that have been found useful in reducing the raw data obtained in airborne test programs. The computing schedules for particular applications have been written out in detail [84,98,110,177] and are not given here. The techniques are those concerned (1) with the rotations of coordinate systems, (2) with ballistic and parallax calculations, and (3) with the treatment of single-shot probabilities and of measurement errors.

7.3.2 Rotation of Coordinates

It is to be expected that three-dimensional geometry will be most important in assessment programs. Examples are (1) reduction of azimuth and elevation measured with respect to a coordinate system determined by a tricamera installation to bearings with respect to a coordinate system fixed in the aircraft that mounts those cameras (the cameras are not necessarily aligned with the aircraft),[84] (2) a similar reduction in going from bearings relative to aircraft axes to bearings relative to an air mass coordinate system (the aircraft axes are rotated in the air mass system because of yaw, pitch, and roll), and (3) a transformation of angles measured in a system relative to a camera fixed in a fighter to angles measured in a system at the fighter[86,81] and parallel to those at a bomber or at a ground target.

These examples must deal with the rotation of coordinate systems. The most elegant and painless methods of geometry are those employing matrix algebra.[84] According to these methods, if $\mathbf{x}$ is a vector in one coordinate system and $\bar{\mathbf{x}}$ the same vector in a rotated coordinate system, the two are connected by the matrix equation

$$\begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \end{bmatrix} = \begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix},$$

or, briefly,

$$\bar{\mathbf{x}} = \mathbf{S} \cdot \mathbf{x}.$$

The array S is composed of the direction cosines of the axes of the first system with respect to those of the second, and matrix multiplication according to the usual rule[a] is used.

The application of the theory is illustrated by the problem of the *stabilization of coordinates*.[81,86] In this problem, the azimuth and elevation of a target are given in a coordinate system fixed in the bomber. The bomber system has been obtained from an air mass system by a sequence of three rotations (1) a rotation in yaw, with a vertical axis, of angle Y, (2) a rotation in pitch, with a horizontal axis, of angle P, and (3) a rotation in roll, about the aircraft's longitudinal axis, of angle R. (The order is standard, and the angles are positive when the bomber makes a diving banking turn to the right.) The azimuth and elevation of the target with respect to the air mass system are required. The problem is solved by the matrix equation

$$\bar{\mathbf{x}} = \mathbf{S}_Y \mathbf{S}_P \mathbf{S}_R \mathbf{x},$$

when written in the form

$$\mathbf{S}_{-Y} \bar{\mathbf{x}} = \mathbf{S}_P \mathbf{S}_R \mathbf{x},$$

where

$$\mathbf{x} = \begin{bmatrix} \sin A \cos E \\ \cos A \cos E \\ \sin E \end{bmatrix}, \qquad \bar{\mathbf{x}} = \begin{bmatrix} \sin \bar{A} \cos \bar{E} \\ \cos \bar{A} \cos \bar{E} \\ \sin \bar{E} \end{bmatrix},$$

$$\mathbf{S}_Y = \begin{bmatrix} \cos Y & \sin Y & 0 \\ -\sin Y & \cos Y & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{S}_P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos P & \sin P \\ 0 & -\sin P & \cos P \end{bmatrix},$$

$$\mathbf{S}_R = \begin{bmatrix} \cos R & 0 & \sin R \\ 0 & 1 & 0 \\ -\sin R & 0 & \cos R \end{bmatrix}.$$

After simple calculation, one finds that

$$\sin \bar{E} = \cos P \cos E[\tan E \cos R - \sin R \sin A - \tan P \cos A],$$

$$\sin (\bar{A} - Y) = \cos E \sec \bar{E}[\cos R \sin A + \sin R \tan E].$$

In this problem the angles Y, P, and R are known from gyro readings,[87] so that these formulas are completely determinate.

In assessment programs a large number of readings must be reduced. An exact mathematical solution may be too time-consuming. Hence, problems such as the preceding one are frequently solved by mechanisms or special computers. A mechanism called a *glook*,[84] constructed with precision and consisting of rotating arcs which may be positioned to mimic the orientation of the aircraft with respect to the air mass, can be used for these reductions. A transit is provided for exact settings and markings. The most appropriate computers are based on the principle of *gnomonic* projection.[88,170] Using the center of a sphere as the center of projection and an equatorial tangent plane as the plane of projection, the great polar circles of longitude project into a family of straight lines and the small circles of latitude project into hyperbolic arcs.[b] The principle on which the use of the chart is based is simple. Imagine that a line from the sphere center to the target is fixed. Rotate the sphere and its attached planar grid

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[a] In $S'' = SS'$ the product matrix S'' is computed by the row by column rule: to find the element in the ith row and jth column of S'', multiply the elements in the ith *row* of S by the corresponding elements in the jth *column* of S' and add. Note how this rule is applied in the text to multiply a 3 × 3 matrix by a 1 × 3 matrix to obtain a 1 × 3 matrix.

[b] The U. S. Coast and Geodetic Survey has prepared plates for large, fine-scale (10-minute spacing) gnomonic charts.

at pleasure. The target line will trace a certain pattern on the chart. To be specific let the grid plane have its normal line at an elevation of 0° and an azimuth of 270°, then, if a yaw is to be removed, the sphere rotates in azimuth and the trace moves along the proper hyperbola (line of equal elevation); if a pitch is to be removed, the sphere rotates about the normal line of the grid and the trace actually describes a circular arc on the grid centered at its center; and if a roll is to be removed, the trace moves along a small circle on the sphere which projects into one hyperbolic arc of a family orthogonal to the original set. Instruments based on these and similar projection principles have been constructed.[27,28]

7.3.3 Calculation of Deflection and Parallax

The calculation of the deflection that should have been taken is carried out by direct application of elementary ideas. In particular, kinematic deflection is determined by the classic *timeback* method. If it is assumed that the gun mount is in unaccelerated motion, the range at any instant can be called a future range, and it may be supposed that a bullet fired earlier under perfect aim is just hitting the target. The dependence of time of flight on the orientation of the gun barrel is insensitive enough to bearing changes so that the azimuth and elevation of the target at the instant of supposed impact may be used. (These differ from the true bore angles by ballistic deflections only.) Entering prepared tables or using appropriate graphs for the known airspeed and ballistic density yields the time of flight. Proceeding back along the motion picture record by this length of time gives the target position at the instant of fire. The differences in stabilized azimuth and elevation of the two target positions give the components of kinematic deflection. (We need not labor the evident details of ballistic corrections, except to point out the necessity of special charts[34] and computing aids in such mass production operations.) The correct deflections may then be plotted against odd times (assessment times minus time of flight) and a curve may be smoothed through these points. The advantage of this elementary timeback idea is that full allowance is made for target acceleration. Allowance for the acceleration of the center of gravity of the gun mount is much more difficult.[116,179] To date, such allowance has been considered unnecessary since violent avoiding action has not been seriously studied.

A parallax correction in the bearing of the target must usually be made because of the appreciable displacement of gun position from that of the recording instrument. In fact, if a, b, c are the coordinates of the gun relative to the camera in the aircraft coordinate system, then the azimuth and elevation of the target relative to the turret may be called $A + \Delta A$ and $E + \Delta E$, and, when the range is r from the turret and r_0 from the camera,

$$r \sin (A + \Delta A) \cos (E + \Delta E) = r_0 \sin A \cos E - a$$
$$r \cos (A + \Delta A) \cos (E + \Delta E) = r_0 \cos A \cos E - b$$
$$r \sin (E + \Delta E) = r_0 \sin E - c.$$

From these equations the approximations

$$\Delta A = \frac{b \sin A - a \cos A}{r \cos E},$$

$$\Delta E = \frac{(a \sin A + b \cos A) \sin E - c \cos E}{r},$$

$$\frac{(\Delta A)^2}{4} \sin 2E$$

are obtained. In practical work, a and c may normally be neglected and suitable charts may be prepared from which the correction may be read. As an alternate to the formulas, the parallax correction may be made by another mechanism with an outré name — the *plaxie*.[34]

7.3.4 Effect of Errors in Measurement and Single-Shot Hit Probabilities

Indirectly, the discussion above indicates that reduction of the raw data has been brought to the point where errors in gun aim are known as functions of time over an attack. The next question is a natural one: if a single bullet is fired at a given instant with such false aim what is its probability of hitting the target? And, upon deeper consideration, how is this probability affected by the measurement error committed in stating the gun error? (Such errors in measurement must arise because of uncertainties throughout the process in observation, machine behavior, recording, and calculation.)

The first question may be answered initially on the assumptio... that there is no measurement error.[99,116] Certain additional assumptions about the target and about bullet dispersion are made before proceeding with the solution. The target is *unconditionally vul-*

nerable. For such a target the probability that a hit is lethal depends on the position of the hit but is independent of the number or location of earlier hits. Abundant evidence, both theoretical and empirical in character, indicates that this is a valid assumption to make about fighter aircraft. It is recommended further,[98] that the typical fighter target be replaced by a sphere of diameter 5 ft with a uniform vulnerability factor of 0.40. This means that if a bullet hits the previously undestroyed sphere at any point the probability that it will be lethal is 0.40. Turning to the bullet, angular displacements may be measured in a plane of action (the plane containing target and gun-mount velocity at a given instant) and perpendicular to this plane. Thus, if a perfect bullet with no dispersion were fired with a given aiming error, the relative angular separation from target center to the bullet when both are at equal range may be called $(x) = (x_1, x_2)$. The gun error generated by the test is (x). An actual bullet fired with gun error (x) will have a relative angular separation from the target center, when both are at the same range, of $(x + b) = (x_1 + b_1, x_2 + b_2)$, where (b) is a measure of dispersion. (Strictly, the actual bullet is assumed to have the muzzle velocity and initial yaw of the perfect bullet for reasons of range matching.) It is supposed that (b) is distributed normally with a mean of $(0,0)$ and a variance of $(\sigma_b^2) = (\sigma_{b_1}^2, \sigma_{b_2}^2)$. The quantity (b) includes the effect of firing at a fixed target as well as the excess in also disturbance in combat over the disturbance in a test. The variance (σ_b^2) is called the *gun-mount* variance.

With these assumptions it is evident that the actual bullet will hit the spherical target if

$$(x_1 + b_1)^2 + (x_2 + b_2)^2 < \rho^2,$$

where ρ is the angular radius of the target at the instant when the bullet is at the same range. In other words, the probability that the bullet will hit the target is the volume, under the two-dimensional normal dispersion surface, erected on a circular base in the (b_1, b_2) plane of radius ρ and center $(-x_1, -x_2)$. The single-shot lethal probability is, consequently,

$$p = 0.4 \iint_T \frac{1}{2\pi\sigma_{b_1}\sigma_{b_2}} e^{-(u_1-x_1)^2/2\sigma_{b_1}^2 - (u_2-x_2)^2/2\sigma_{b_2}^2} du_1 du_2$$

$$= 0.4G(x_1, x_2; \rho; \sigma_{b_1}, \sigma_{b_2}),$$

where T is defined by

$$u_1^2 + u_2^2 < \rho^2.$$

The first question, as to the single-shot probability, is answered completely if this calculation is refined to permit variations in muzzle velocity and initial yaw.[91] The refinement is accomplished by setting

$$b = c + b',$$

where c is *quasi-steady* and b' is *fluctuating* (in the sense of the second classification of errors of Section 7.1.2).

The effect[98] of a measurement variance (σ_m^2) may now be introduced. Instead of knowing (x), one knows only (z), where

$$z_1 = x_1 + y_1$$
$$z_2 = x_2 + y_2.$$

The total measurement error (y) may, in its turn, be written as a sum of a quasi-steady part[a] (h) and a fluctuating part (m). It is the fluctuating part which has the variance (σ_m^2). The problem is to estimate the lethal probability p as adequately as possible using (z) and (σ_m^2). The estimate of p selected is

$$p' = 0.4G(z_1, z_2; \rho; \sigma_1, \sigma_2),$$

where

$$\sigma_1^2 = \sigma_{b_1}^2 - \sigma_{m_1}^2$$
$$\sigma_2^2 = \sigma_{b_2}^2 - \sigma_{m_2}^2.$$

(The *subtraction* of variances should be carefully noted.) The sense in which p' is an adequate estimate of p is that its weighted average over the universe of possible (m) (or its expected value with respect to (m)),

$$\frac{0.4}{2\pi\sigma_{m_1}\sigma_{m_2}} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-m_1^2/2\sigma_{m_1}^2 - m_2^2/2\sigma_{m_2}^2} \cdot$$

$$G(x_1 + h_1 + m_1, x_2 + h_2 + m_2; \rho; \sigma_1, \sigma_2) dm_1 dm_2,$$

is precisely equal to

$$0.4G(x_1 + h_1, x_2 + h_2; \rho; \sigma_{b_1}, \sigma_{b_2}).$$

The result, and with it, the subtraction of variances come about as follows. Starting with the idea that the weighted average of $G(z_1, z_2; \rho; \sigma_1, \sigma_2)$ should be $G(x_1, x_2; \rho; \sigma_{b_1}, \sigma_{b_2})$, when h is put equal to zero, the identity

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{1}{2\pi\sigma_{m_1}\sigma_{m_2}} e^{-m_1^2/2\sigma_{m_1}^2 - m_2^2/2\sigma_{m_2}^2} \cdot \frac{1}{2\pi\sigma_1\sigma_2} \cdot$$

$$e^{-(u_1-x_1-m_1)^2/2\sigma_1^2 - (u_2-x_2-m_2)^2/2\sigma_2^2} dm_1 dm_2$$

$$= \frac{1}{2\pi\sqrt{\sigma_{m_1}^2 + \sigma_1^2}\sqrt{\sigma_{m_2}^2 + \sigma_2^2}} \cdot$$

$$e^{-(u_1-x_1)^2/2(\sigma_{m_1}^2+\sigma_1^2) - (u_2-x_2)^2/2(\sigma_{m_2}^2+\sigma_2^2)}$$

clearly requires $\sigma_b^2 = \sigma_m^2 + \sigma^2$, which yields the σ^2 described above.

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[a] It will not be inferred that the quasi-steady part of the measurement error has any relation to the quasi-steady gun-pointing error. The same remark applies to the fluctuating part.

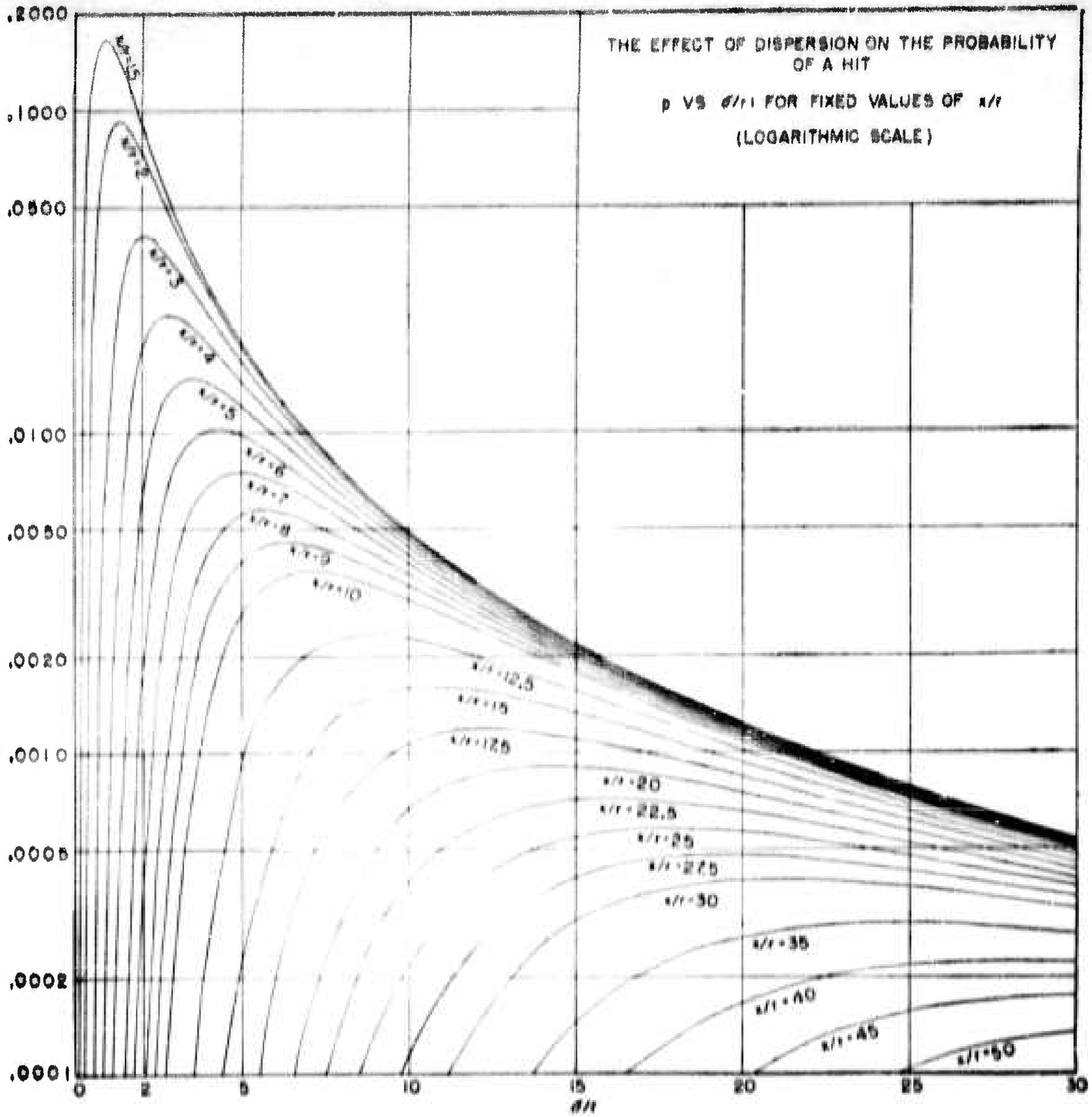

FIGURE 1. Single-shot hit probabilities.

In a poor experiment it is conceivable that $\sigma_m^2 > \sigma_h^2$. It can be shown analytically [illegible] that the above solution exists uniquely when and only when $\sigma_m^2 < \sigma_h^2$.

In practical work it is not usually necessary to evaluate the double integral defining p'. The assumption commonly made is that $\sigma_1 = \sigma_2 = \sigma$. Then, if the distance $\zeta = \sqrt{x_1^2 + x_2^2}$ is used, the probability p' can be expressed as a function of ρ/σ and ζ/σ. Based on tables made for scatter bombing,[illegible] charts have been prepared [illegible] which permit one to read p' (or rather $p'/0.1$) directly. Two of these charts appear here as Figures 1 and 2.

7.4 MEASURES OF EFFECTIVENESS

7.4.1 Recommended Measure and Others

If an objective of an airborne assessment program is to rank two or more fire control systems, some method of compressing single-shot probabilities,

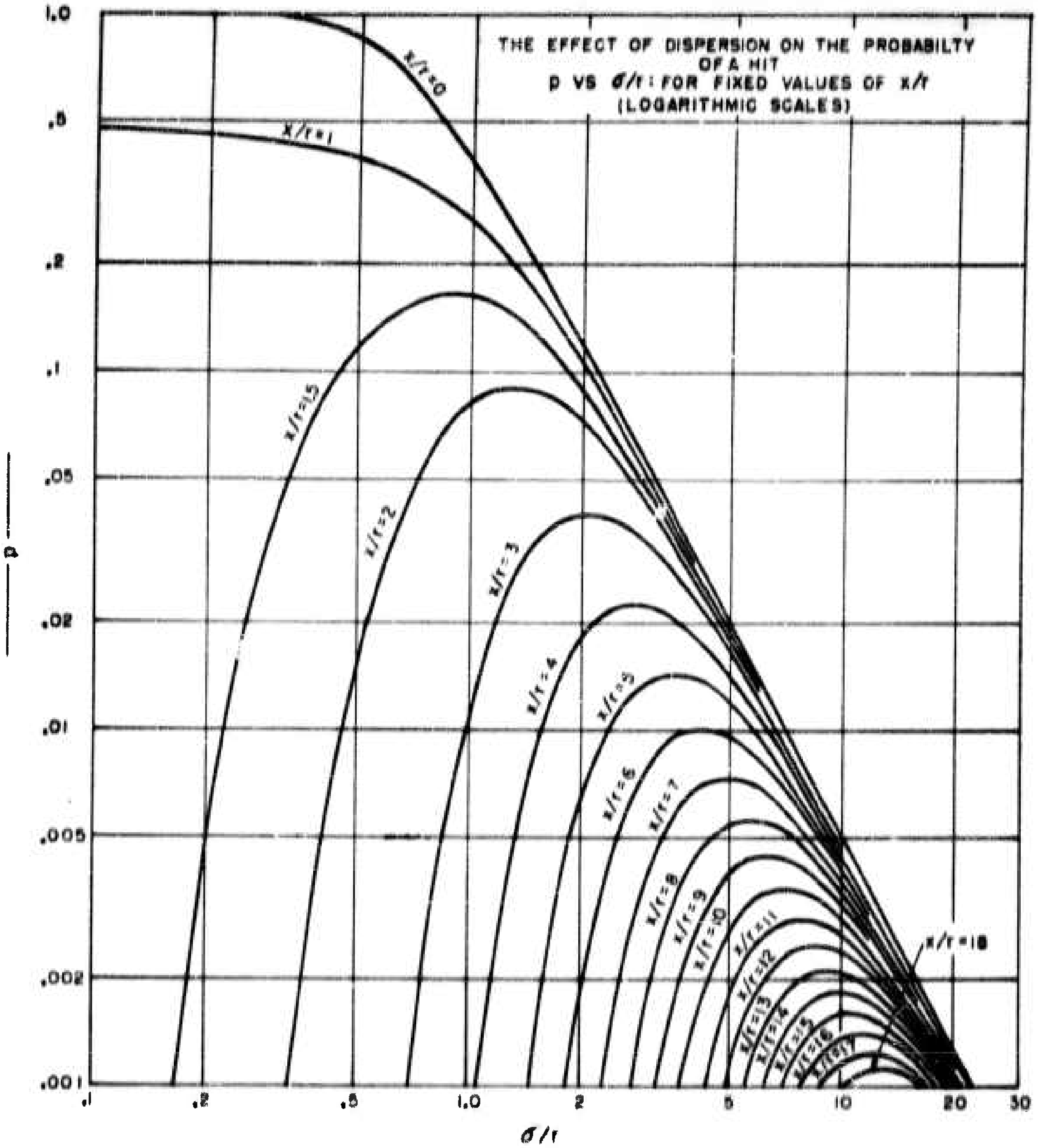

FIGURE 2. Single-shot hit probabilities.

which are given as functions of time over particular attack courses, should be devised. One such compression is the generation of the average probability of *at least one* lethal hit for a properly stratified set of particular attacks of the same general type. Proper stratification means that the replications faithfully permit the variables of gunner, installation, attack path variations, and the like to express themselves. This criterion of effectiveness[18] may better be expressed as the *expected proportion of engagements of a given type which will be successful* for a system. Success is the destruction of the target, the survival of the gun mount, or some combination of both.

Other measures of effectiveness have been pro-

posed. One might determine the expected number of hits on the target per engagement. Two objections may be raised. First, since the target is unconditionally vulnerable, there is no cumulative effect of nonlethal hits. Second, because of serial correlation in the gun-pointing errors during the attack, the number of lethal hits cannot be converted into a destruction probability.

A common British measure has been the index I defined by the expression

$$I = 100 \frac{\text{(Bullet density on target, using the system)}}{\text{(Standard bullet density)}}.$$

The standard bullet density is the density at the target range of the same number of bullets (as in the numerator) distributed uniformly over a circle 1° in diameter. With the aid of I, determined by experimental procedures, hit expectations and the probability of success in an engagement have been calculated.[100] This measure, and others [118] have been carefully reviewed.[130]

Although the discussion here gives insufficient detail to make it evident, the problem of measures of effectiveness is sophisticated. One can be led to speculations about the relative worth of systems whose performances as a function of long- and short-range invert. One can attempt to predict expected losses and gains in combat, e.g., fighters shot down per bomber lost, similar to those determined after the event from an analysis of combat records.[108] It is likely that these subtleties are inappropriate in the current state of the art. But the attempt to relate a a particular technical branch of air warfare to the overall situation may well be prosecuted in the future.

7.4.2 Expected Proportion of Successful Engagements

Turning now to the first criterion, the expected proportion of engagements which will be successful, suppose that shots are fired at times $t_1, t_2, t_3 \cdots$ during a particular run. The probability that the target will survive the first i shots is

$$s(t_i) = [1 - p'(t_1)][1 - p'(t_2)] \cdots [1 - p'(t_i)].$$

Consequently, the probability that the target will be destroyed during the first i shots is

$$d(t_i) = 1 - s(t_i),$$

provided that the bomber is invulnerable in the sense that it can fire those i shots. Since we wish to work with a type of tactical situation rather than a particular attack of that type, the expected value, $D(t_i)$, of $d(t_i)$ is required. One may think of $D(t_i)$ as referring to an engagement involving a particular type of attack and type of fire control, but with a random selection of fighter pilots, gunners, equipment specimens, weather, and like variables.

There are two ways of estimating $D(t)$. The direct method determines $d(t)$ for each attack of the type and forms an average of the resulting values. The statistical method attempts to describe the universe of gun errors, made during all attacks of the type, by certain parameters. $D(t)$ is to be estimated from these parameters. For example, it may be assumed that gun errors can be described as a two-dimensional Gaussian distribution with the parameters: mean traverse error, mean elevation error, variance of traverse error, variance of elevation error, correlation coefficient of traverse and elevation errors. The adequacy of this particular description may be questioned on the ground that the effect of serial correlation in aim wander during a representative attack has been ignored. If, however, the universe of gun errors can be correctly described, then the statistical method is superior to the direct method in that it is statistically efficient, extracting the maximum amount of information from the available data.

The value of $D(t)$ as a measure of effectiveness does not depend on the assumption of bomber invulnerability. If A and B are two fire control systems, for which $D_A(t)$ and $D_B(t)$ are known, and if $D_A(t)$ is greater than $D_B(t)$ for all values of t, then: A is superior to B in the sense that A will enjoy a greater proportion of successes than B whether success is defined as the destruction of a target by an actual (vulnerable) bomber or as the survival of an actual (vulnerable) bomber. The theorem requires detailed proof [98] which will not be reproduced.

7.5 OPTIMUM DISPERSION

7.5.1 Expectation of Optimum Pattern Size

In aerial gunnery, dispersion is essential if hits are to be scored. The reason is that, with these weapons, a burst is fired with a certain bias in aim. The mean point of impact [MPI] is displaced from the center of the small target and, when the MPI is off target, zero dispersion means that no hits are obtained. With the aiming errors of current systems the MPI is

rarely on target for more than a fraction of a second. In fact, for systems in which the MPI can be kept on target for an appreciable length of time, it is desirable to renounce the principle of rapid fire of small caliber projectiles in favor of slower fire of larger caliber ones. The vulnerable subarea of the target increases with caliber up to the point of complete structural failure caused by a hit on any part of the target.

In the opposite direction, excessive dispersion is undesirable. Even with a perfectly aimed gun the spacing could be so large between bullets of the burst that the small target may escape. Large dispersions are used in warfare only against large targets.

Evidently, one looks for an optimum size of the dispersion pattern. The determination of this size demands a knowledge obtained from experimental programs of the universe of gun-pointing errors, although it is possible to attempt to use indirect combat information such as rounds fired per enemy aircraft destroyed in an analysis.[108] Such a procedure is of doubtful efficiency since, not knowing how the gun-pointing errors vary and how they are serially correlated with range, it must assume a fixed target size and a constant aim error (or one that varies according to some supposed distribution). To optimize the dispersion with assurance one needs precise gun-pointing information and should use the technique of Section 7.4.2 to determine the average destruction probability.

7.5.2 Nose Attacks on B-29

The determination of optimum dispersion size according to this schedule appears attractive in connection with nose attacks on the B-29,[50] since extremely accurate data are available from tests on the testing engine of the War Research Laboratory of the University of Texas.[f] The attacks were both pure and aerodynamic pursuit courses. Four modifications of the nose-sighting stations with twelve replications of each of four attacks furnished 192 samples of the type of tactical situation in question. Six gunners were used.

The numerical values of the variances (see Section 7.4.2) are: $\sigma_b^2 = 2.58$ (after tests at the AAF Central School for Flexible Gunnery at Laredo, Texas); $\sigma_{m_1}^2 = 1.16$, $\sigma_{m_2}^2 = 0.41$ (estimates of machine accuracy at the University of Texas.) We shall take $\sigma = \sqrt{\sigma_1\sigma_2} = \sqrt{\sigma_b^2 - \sigma_{m_1}^2}\sqrt{\sigma_b^2 - \sigma_{m_2}^2} = 1.33$ milliradians. The results of a calculation of D (the average of all destruction probabilities with t set equal to the duration of the attack) as a function of σ_b, by the methods of Section 7.4.2, is shown in Figure 3. The

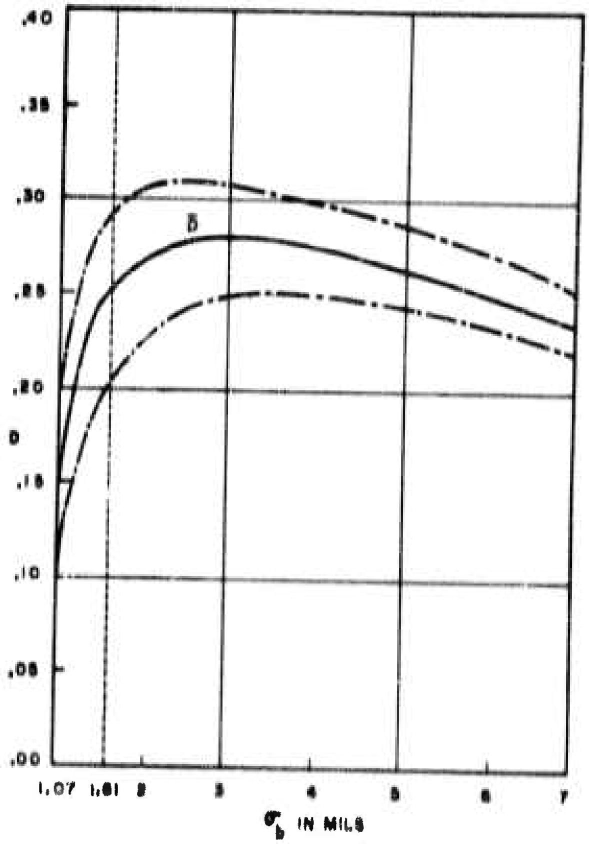

FIGURE 3. Average probability of at least one lethal hit.

figure also gives a 0.95 fiducial zone. This means that the probability is 0.95 that D lies in the interval shown for any given σ_b.

It is concluded from the figure that the optimum dispersion is $\sigma_b = 3$ milliradians. However, the actual value $\sigma_b = 1.61$ corresponds to a destruction probability of 0.25 which is only 0.03 below the maximum probability. About all that can be said definitely is that σ_b should not be reduced below its present value.

[f] In this machine an actual gunner operates the sighting mechanism against a programmed target whose motion is very realistic. The machine records continuously the gun errors in traverse and elevation. It, of course, has preknowledge of the correct values. As a major advantage, replications are possible in which only gunner and sight specimen are variables. Even platform motion is programmed.

7.5.3 Theoretical Solution for Fighters

In spite of the emphasis of this chapter on the necessity of precise experimental results, there are certain advantages to an entirely theoretical approach to these problems. The average assessment program is complicated and costly in matériel and personnel. Its conclusions may be reached too late and even then may be subject to censure on grounds of experimental design or accuracy. In the theoretical treatment the variables are always under control. The relative effects of variation in different parameters may be readily studied. Such a study is also modest enough to state not what will happen but what the range of results will be for some range of variability in a parameter whose values cannot be known exactly. When its results point in a direction independent of such ranges of variability, it is conclusive and an experimental study is obviated. Finally, the theoretical approach may lead to economy and efficiency in experimental programs by dictating exactly what details need an empirical foundation.

Some of these advantages are illustrated by a study of the performance of a fighter aircraft (an F6F) directed by a gyro gunsight (the Mark 23).[38] The fighter attacks a target flying straight and level at constant speed. To determine the basic position and orientation of the fighter at each instant, it is assumed first that a perfect pursuit curve is flown. To get single-shot probabilities this orientation is perturbed by the ranging error assumptions (1) perfect range, (2) consistent under range by 10 per cent, (3) consistent over range by 10 per cent, and (4) varying from 25 per cent under range at 800 yd to 25 per cent over range at 200 yd; and by the tracking assumptions of (1) perfect tracking, and (2) tracking with a circular normal distribution of errors so that the pipper is within 3 milliradians of the target during 50 per cent of the time. (Such ranging and tracking assumptions may come from rapid preliminary experimentation or even from subjective estimates by operators.) In addition to these operational errors, Class B errors will occur because of the particular calibration of time of flight in the sight. In fact, if A^* is the correct lead, the sight's time-of-flight multiplier should be

$$t_m^* = \frac{A^*}{\dot{\theta} - \dot{A}},$$

where θ is the approach angle measured at the bomber. (This is in agreement with the calibration ideas of Section 5.3.5.) At a given altitude, temperature, range, and angular rate, the sight will actually use a value t_m according to the calibration of the range circuit. Then the error in lead is approximated by

$$\epsilon_A = (t_m - t_m^*) \frac{v_B \sin (\theta - A^*)}{r},$$

which is the error in time of flight multiplied by the angular rate of the gun-target line for a correctly flown fighter. This expression for the error includes the effect of lag in lead computation since t_m^* was chosen with attention to the equation of the sight. The remaining sources of Class B error, such as optical and electromagnetic dips, and feedbacks, are ignored. The performance of the fighter really appears through t_m in the above expression.

The method of computing single-shot probabilities is relatively straightforward. The value of ϵ_A gives the displacement of the mean point of impact from the target. This mean point of impact is surrounded by a circular normal distribution whose variance is the sum of those due to dispersion and aim wander. To get this sum it is assumed that a good value of σ_D (arising as a result of ground firings of a fighter-mounted caliber .50 machine gun using API M8 ammunition) is 1.2 milliradians. Next, the gun pointing is an amplified form of the sight line wander (see Section 5.3.4, "Amplification of Tracking Errors") which depends on the frequency. An average value of 1.5 is selected arbitrarily for the amplification factor. Consequently, the variance due to gun wander is $[(10)(0.425)]^2$, where 0.425 converts the diameter of a circle containing 50 per cent of the random gun directions into σ for this distribution of directions. Finally, the pattern is narrowed in the air because of the forward motion of the fighter (see Section 4.5.3) by a factor $v_0/(v_0 + v_F)$. If $v_F = 200$ yd per sec and $v_0 = 950.7$ yd per sec, the shrinking factor is 0.82. The shrunk sum of variances is 10.85 milliradians squared. The target is taken to be a small bomber represented by a circle of radius 4 yd with a uniform vulnerability factor of 0.3. The single-shot probabilities may now be read from Figures 1 and 2.

In fighter fire a complication arises because of the harmonization of a battery of guns. A typical U. S. Navy procedure is to boresight the six wing guns to converge horizontally and vertically with the line of sight at a range of 300 yd. Strictly, one should there-

fore calculate probabilities for each gun separately or at least for the middle port gun and the middle starboard gun. When this is done, paying proper attention to parallax, boresighting, and aircraft banking, it is found that the probabilities for the various guns do differ appreciably. In summarizing calculations the joint effect of all guns firing together and continuously is given by totalizing the effects of the individual guns.

The single-shot probabilities may be processed into the probability of at least one lethal hit (as a function of time along the course) by the methods of Section 7.4.2. For an attack, made at 374 knots against a target at 195 knots, starting at 900 yd on the beam and terminating 3.3 sec later at a range of 200 yd and an angle off of about 40°, the results of the calculation are given in Table 1.

Section 7.3 considers the reduction of the raw data. Techniques rather than schedules are given. A recurring problem of rotation of coordinate systems is handled generically either by matrix methods, or by mimicking mechanism, or by gnomonic computer. The calculation of deflection uses the timeback method in which a given frame represents impact by a bullet fired earlier. Formulas for the parallax correction required by the displacement of guns and camera are given. The final technique is considered in detail. This is the problem of reducing gun pointing errors to the probability that a single round fired with such an error would be lethal to a fighter target in the sense that the fighter could not return to its base. It is also emphasized at this point that measurement errors of the experiment lead to a measurement variance which must be *subtracted*

TABLE 1. Probability of at least one lethal hit as a function of time × 100.

Time (seconds)	No range error			Range 10 per cent too small			Range 10 per cent too large			Range from −25 per cent to +25 per cent		
	$a = 0.08$	$a = 3.20$	$a = 5$	$a = 0.08$	$a = 3.20$	$a = 5$	$a = 0.08$	$a = 3.20$	$a = 5$	$a = 0.08$	$a = 3.20$	$a = 5$
0.6	50	33	24	0	0	1	0	3	3	0	0	0
1.2	74	64	50	0	0	1	0	3	8	0	0	0
1.8	74	73	66	0	0	1	0	24	31	0	1	6
2.4	74	77	70	0	0	1	45	85	75	71	67	58
3.3	74	78	82	0	0	1	100	100	98	100	100	99

7.6 SUMMARY

The introduction, Section 7.1, points out that the basic purpose of airborne assessment programs is the *ranking* of different fire control systems. To arrive at changes in design, a combination of analytical methods and controlled tests on laboratory machines is necessary. The errors of a fire control system are first classified by cause: mechanism, Class B, operational, and dispersive; and then by statistical nature: fluctuating, quasi-steady, and steady. Current assessment programs use simulated attack situations with camera recording to get at the overall errors of systems.

Section 7.2 discusses the methods by which the path in space of a target may be determined. Instrumentations are: three fixed cameras, plus a range camera, plus gyro records of platform motions; a deflectometer which records the turret position in azimuth and elevation; long focal length cameras fixed on the ground with data reduced by astrometric methods; a camera in the gun mount and also one in the target; and, simply, one camera which relies on fixed and distant reference points in the landscape.

from the dispersion variance before the single-shot probability is computed.

Section 7.4 is concerned with a still further reduction, the generation of a measure of effectiveness from the single-shot probabilities. The measure proposed is: the expected proportion of engagements of a given type which will be successful in the sense that the target is shot down, or the bomber survives, or some combination of these occurs. This measure is the evolutionary result of much previous thought.

Section 7.5 discusses the choice of an optimum size for the dispersion pattern in relation to aiming errors. Using experimental data of a precise nature, the situation for nose attacks on a B-29 is explored by the schedule of the previous section. Using purely theoretical methods, the situation for a fighter equipped with a gyro gunsight is discussed. For the first case, pattern size might be increased slightly over its present value, but the change in probability of at least one lethal hit would be only from 0.25 to 0.28. For the second case, it is shown that errors in ranging greatly overshadow variations in dispersion.

Chapter 8

NEW DEVELOPMENTS

8.1 INTRODUCTION

This chapter does not predict future developments. Instead it describes in compact form those systems and ideas which were being seriously considered in the closing months of World War II as suitable for the next stage of that war. As a result, the ideas are, on the whole, limited in perspective to the air tactics then current. The fire control systems discussed are concerned with doing a better job than existing equipment. But it is the same job. For example, no real step is taken to solve the problem of an ultra-speed target, nor are ranges of combat extended materially. Under new tactics, only rather slight variations in existing fighters are considered.

These remarks are critical rather than censorious. Failing the introduction of unforeseen and radical principles, the systems of the future will probably lean heavily on the ideas of this chapter. Consequently, these ideas must be explored still further in order to gain an adequate knowledge of theoretical and mechanical limitations. The best case in point is radar gun laying, which involves both the theory of tracking and the facts of mechanical responses.

One may hope that the next phase in fire control development will consist of a fresh reexamination of the problem, which will result in a system combining the best of the new ideas and which will avoid the known errors of existing mechanizations. The theoretical design of such a system should precede its construction, for it is a curious fact that many machines are built and then analyzed, with patchwork as the usual result.

Fire control design has lagged tactical circumstances. If the broad outlines of future combat circumstances are made available to this design, even at the early and tentative stages of tactical expectation, this need not be true.

8.2 STABILIZED SIGHT SYSTEMS

8.2.1 Nature and Types of Stabilization

Of the various mutations that have occurred in the development of airborne fire control, stabilization is perhaps as important as was the introduction of power turrets. Stabilization permits the gunner to concentrate on his real job, tracking the smooth target motion. The normal irregular motions of the gun platform, which make this a difficult task under the usual arrangement, no longer disturb the gunner's line of sight. In principle, any motions *not called for by the gunner* through his handlebars are detected by gyroscopes which immediately send impulses to the turret motor which, in turn, acts to neutralize the perturbation.

Either *position* or *rate* stabilization may be used. In position stabilization a free gyroscope is employed. By the first basic principle of gyroscopic behavior, i.e., maintenance of a fixed direction in space, motion of the sight line away from the gyro axis is detected and corrected when the handlebars are in neutral. In tracking, a connection between handlebars and gyro is made so that the gyro will precess when moved by the handlebars. It is the differential signal, between handlebar motion and total motion, which is detected and eliminated by the stabilization. In rate stabilization, the gyro is constrained. When the line of sight moves, taking the gyro with it, the gyro responds with a precessional torque (kick). The magnitude of this torque is proportional to the angular rate of turn of the gyro. If the sight line starts to move, under external influences, the precessional torque of the gyro is measured and the turret motors restore the sight line to its original position very quickly. When tracking, the gyros must be prevented from stabilizing out the intended motion. Since the gyro is constrained, no major precession occurs. The kick due to the handlebar motion is neutralized by suitably calculated impulses from the handlebars. Thus, this part of the kick does not act to restore the turret to its original position.

It will be appreciated that there is nothing inherent in stabilization which will improve the mechanism on which lead computation is based. Instead, one expects the improvement in inputs to cause the computer to perform better. The stabilization circuit may be regarded as an improved analogue of the B-29 follow-up system under which the guns follow

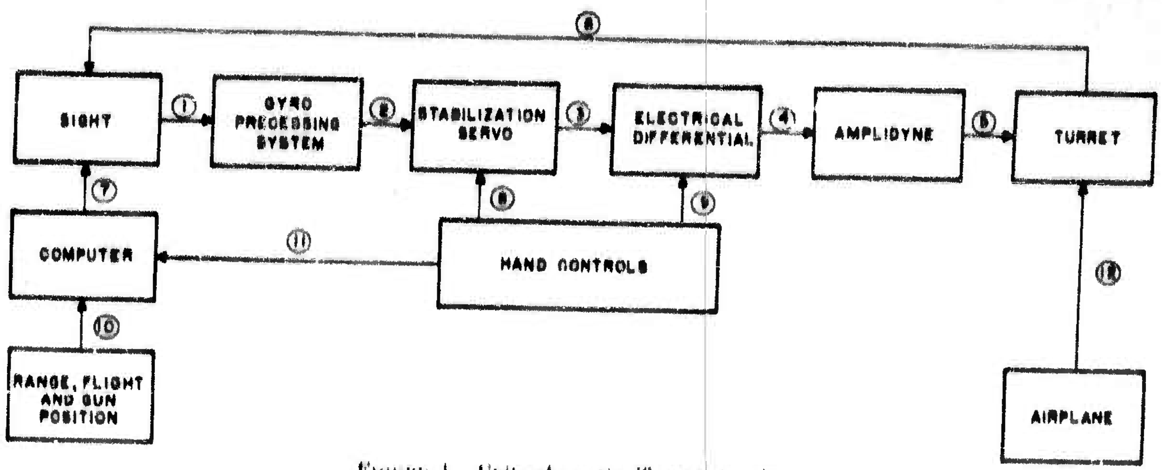

Figure 1. Unit schematic, Type S-4 sight.

the sight line. The lead computation is an additional function in each case.

8.2.2 Fairchild S-4 System

An excellent example of a rate stabilized sighting system is the Fairchild S-4 (or S-3) installation[220] for inhabited turrets. The stabilization features are the same for both types. The S-4 is ballistically superior in that it will function for all types of Type 5 projectiles in a speed range for which the three-halves power drag law is valid. One need only insert muzzle velocity and ballistic coefficient. A ballistic mechanism then effectively solves the Siacci differential equations for the trajectory, instead of translating special ballistic tables calculated by this method and referring to particular ammunition. (This is an excellent example of how proper fire control design can anticipate ordnance changes and thus avoid lengthy and expensive modification in the future.) The S-4 will be discussed. The theory developed will also cover both hydraulic and electric turret mechanisms.

There are two major circuits of the system. One operates the *azimuth* turret motor and controls the *lateral* lead. The other operates the guns in *elevation* and controls the *vertical* lead. Each of these circuits employs a captured gyroscope which has a dual role. It enters into the determination of the kinematic lead of its component and, independently of this, stabilizes the line of sight in its component. The unit schematic of the S-4 is shown in Figure 1.

8.2.3 Circuit Components of the S-4

The system may be best understood by writing out the differential equations of the circuit.[100] It suffices here to consider the situation in one component. The general picture involves the interaction of two components and is necessarily complicated. The basic assumption of this treatment is that the inertia of the various moving elements may be neglected. This is a major assumption since successful operation depends on the relative speed of response of gyros, servomotors,[a] and the amplidyne turret.

Let a gyroscope whose spin axis lies in a horizontal plane be rotated about an axis perpendicular to that plane, i.e. let it track in azimuth only. Then the gyroscope develops an internal torque whose axis is perpendicular to the spin axis and to the axis of rotation and so also lies in the horizontal plane. The magnitude of this torque is $I\Omega d\alpha/dt$, where I is the moment of inertia of the rotor, Ω is the angular speed of the rotor, and α is the angle at any instant between the line of sight (the spin axis) and some direction fixed in space, i.e. the direction of a star. If the gyro is so mounted that the spin axis can move only in a plane vertical to the horizontal it will precess in this vertical plane by an angle θ. If its motion in the vertical plane is inhibited by a damping torque proportional to velocity, $Dd\theta/dt$, and a spring torque proportional to displacement, $S\theta$, then

$$D\frac{d\theta}{dt} + S\theta = I\Omega\frac{d\alpha}{dt},$$

since all these torques have the same instantaneous axis in the horizontal plane.

[a] A servomechanism is a power control device in which a weak input is amplified to control a strong output. This output, in turn, feeds back to combine with and modify the original input.

The displacement angle θ can be transformed into an output voltage $K_1\theta$ by means of an E magnet. This device is illustrated in Figure 2. When an a-c voltage is impressed on the primary the output from the secondary is zero, when the gyro is undeflected, since the output arms are wound in series opposition. Upon deflection, more voltage is induced in one arm than in the other, and the arrangement is such that the imbalance is proportional to the deflection.

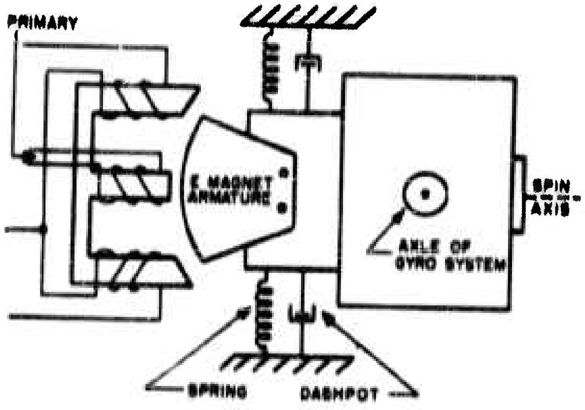

FIGURE 2. Gyro E magnet unit. (Courtesy of Fairchild Instrument Co.)

The voltage from the hand control potentiometer is $K_9\mu$, where μ is the positional angle of that potentiometer. Because of the interconnection (8) (see Figure 1), this voltage is subtracted from the total voltage (2) $K_1\theta$. Hence the input to the stabilization servo is $e_1 = K_1\theta - K_9\mu$. The output voltage (3) e_3 of the servo is tapped off by a moving potentiometer arm so that

$$\frac{de_3}{dt} = K_8 e_1.$$

The inputs to the electrical differential (two potentiometers in parallel) are (3) e_3 and (9) $K_4\mu$ from the hand control potentiometer. Hence the output (4) is a voltage e_4 such that

$$e_4 = K_4\mu - e_3.$$

Assuming that the output voltage (5) of the amplidyne is proportional to the input voltage, the angular rate given to the guns by the turret motor is proportional to this amplidyne output voltage. But the angle which the guns make with the longitudinal axis of the aircraft is $\gamma - \alpha$ where γ and α are the positional angles of gun and longitudinal axis with respect to the original reference line in space. (The angle α is the unwanted angle of yaw.) Consequently,

$$\frac{d(\gamma - \alpha)}{dt} = K_5 e_4.$$

Up to this point nothing has been said about the total lead angle Λ. Since the sight head is affixed to the turret, the sight line will follow roughly the gun line. The small displacement of sight line from gun line to give deflection is accomplished by a sight servomotor whose mechanical output gives a rate to the sight line relative to the sight head which is proportional to its input voltage. Now the computer produces two voltages representative of the time-of-flight multiplier t_m and the ballistic deflection Λ_b. And in a steady state condition the rate of rotation of the sight line is proportional to the control handle potentiometer position, or

$$\frac{d\sigma}{dt} = K_7\mu.$$

Hence introduce as the sight servomotor voltage input $K_7t_m\mu - \Lambda_b - \Lambda$. The output of the sight-line motor is a *rate of change* of sight line relative to the sight head (i.e., relative to the gun line), and this rate $d(\gamma - \sigma)/dt = d\Lambda/dt$ is proportional to the voltage input. Hence

$$K_8\frac{d\Lambda}{dt} = K_7t_m\mu - \Lambda_b - \Lambda\,.$$

It follows immediately that there is a delay in lead computation for this system. (See the analysis of Section 5.3.4 "Delay in Lead Computation.") Only if Λ is *constant* and *equal to* $K_7t_m\mu - \Lambda_b$ will the servomotor stop.

This account rather conceals the role of the gyro in measuring angular rate. Actually the whole system acts to keep the input voltage e_1 to the stabilization servo zero, or to make $K_1\theta = K_9\mu$, this being the essence of stabilization. In other words the voltage μ being used in determining lead *is* really measured by the precessional torque of the gyro.

6.2.4 Complete Circuit Equations of the S-4

The preceding basic circuit equations are combined most expeditiously by placing $d/dt = p$ and treating p as an algebraic quantity. There result

$$p^2(\gamma - \alpha) = -\frac{I\Omega K_1K_3K_5}{Dp + S}\,p\alpha + K_5(K_3K_9 + K_4p)\mu \quad (1)$$

$$\left(p^2 + \frac{I\Omega K_1 K_8 K_5 p}{Dp + S}\right)\gamma = p^2\alpha + \frac{I\Omega K_1 K_8 K_5 p}{Dp + S} \cdot \frac{K_7 t_m \mu - \Lambda_h}{1 + K_6 p} + K_5(K_8 K_a + K_4 p)\mu \quad (2)$$

$$\left(p^2 + \frac{I\Omega K_1 K_8 K_5 p}{Dp + S}\right)\sigma = p^2\alpha - p^2 \frac{K_7 t_m \mu - \Lambda_h}{1 + K_6 p} + K_5(K_8 K_a + K_4 p)\mu. \quad (3)$$

Since the yaw α of the aircraft enters the equation as $d^2\alpha/dt^2$, it follows that the S-4 stabilizes completely only *nonaccelerated* motions. Again, a rather bizarre form of aided tracking is present since by equation (2) neither gun position nor gun rate is simply proportional to handlebar position.

To complete this phase of the discussion a prime advantage of this system should be emphasized. The notion of time constant or decay of false leads does not apply except in the generalized sense of the time for unwanted transients to vanish. The point has not been made out in detail theoretically. But from the standpoint of actual operation, the gunner seems to have about as much control over his line of sight as he would have with a fixed sight. The disturbed reticle behavior is not in evidence.

9.2.5 Class B Errors of the S-4

Little can and will be said about the Class B errors of the S-4. It is evident that lag in lead computation exists and that this lead is itself based on first order theory [101] and does not allow for curvature. Calibration is necessary. There is also a small error committed in computing vertical lead [102] (of the gun-roll type) because the vertical gyro measures an angular rate about an axis that is not perpendicular to the line of sight. This second order error is given by $1000 A_L^2 \tan E$ milliradians, where A_L is the lateral lead in radians and E is the elevation.[103]

9.2.6 Sperry S-8B System

An example of a position stabilized system is the Sperry S-8B sight. For purposes of exposition the discussion will again be restricted to the azimuth plane. The appropriate sight schematic is given in Figure 3. Various angles needed in the development are shown in Figure 4.

The angle α is the yaw of the airplane, and μ is interpreted as the displacement of the handlebar controls. The optical system is so designed that

$$\sigma - \theta = \tfrac{1}{6}\mu. \quad (4)$$

(Recalling Sections 5.3.3 and 5.5.4, we immediately expect to find equations resembling those for simple lead computing sights with a sight parameter of 1/6). The gunner has direct control over the relative position of sight and gyro lines. The combining glass is controlled by the handlebars and the stabilization mirror by both the gyroscope and the handlebars.

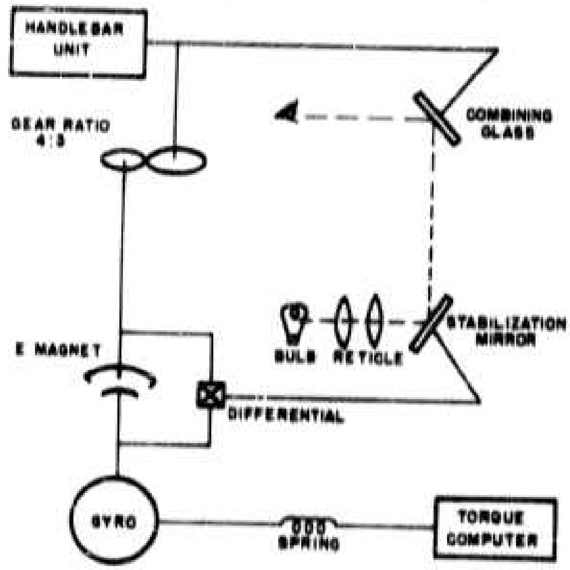

FIGURE 3. Unit schematic, Type S-8B sight.

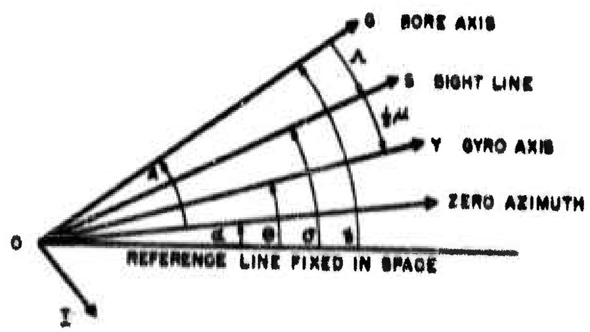

FIGURE 4. Angles in azimuth.

The gyroscope is universally gimbaled. It is precessed in azimuth by a torque caused by a spring wound around an axis lying in the horizontal plane and making a 90° angle with the bore axis. This torque T is equivalent to a force $T \sec(\gamma - \theta)$ applied perpendicularly to the azimuth plane at a

point unit distance from 0 out along the gyro spin axis. Hence, as in Section 5.5.2, the precessional rate is

$$\dot{\theta} = \frac{T \sec(\gamma - \theta)}{I\Omega}. \tag{5}$$

It is the function of the torque computer to make

$$T = \frac{I\Omega}{t_m}\lambda\left[\cos\left(\frac{4\mu}{3}\right)\right]\left[g\left(\frac{4\mu}{3}\right)\right], \tag{6}$$

where λ is given by

$$\mu = \lambda - A_b$$

(A_b being the ballistic deflection) and where t_m is a computed time of flight. The function $g(4\mu/3)$ has the form

$$g\left(\frac{4\mu}{3}\right) = 1 + a_2\left|\frac{4\mu}{3}\right|^2 + a_4\left|\frac{4\mu}{3}\right|^4$$

where[104, 107]

$$a_2 = 0.0836200$$
$$a_4 = 0.11107$$

when $4\mu/3$ is in radians. From its form, it is evident that g is a correction function. Ideally, the gyro should precess with an angular velocity

$$\dot{\theta} = \frac{\gamma - \theta}{t_m}.$$

(See Section 5.5.2 and remember that, here, $\gamma - \theta = \frac{1}{3}(\gamma - \sigma)$, and $\dot{\theta} = \frac{1}{3}\dot{\sigma}$.) For tracking in general, i.e., not simply in azimuth, this would require the torque T to depend on the azimuth and elevation of the gyro axis. To simplify the mechanism, it was decided to make T depend only on azimuth. Hence a correction function was chosen to minimize the error due to this design, and to distribute the remaining error as symmetrically as possible about the bore axis.

Suppose now that a target is being tracked and that the gun mount is yawing. The output of the gyro is the angle of deflection $\gamma - \theta$. This output is *first* used to displace the stabilization mirror through this angle. At the same time the handlebar motion displaces the stabilization mirror by an angle $4\mu/3$. (See the gear-differential circuit of Figure 3.) Consequently if the mirror is rotated through $\frac{1}{2}[\gamma - \theta - 4\mu/3]$ and the combining glass is rotated through $\mu/2$ (no gear reduction) it follows that the line of sight will be deflected by $\frac{1}{3}\mu$ according to the doubling principle of mirror rotation (Section 5.5.4). The output of the gyro is used *secondly* to displace the armature of the E magnet (see Figure 2). But the gunner has rotated the transformer of the E magnet by $4\mu/3$. Hence the angle at the E magnet is $4\mu/3 - (\gamma - \theta)$ which is $\mu - A$ by Figure 4. This is the *error signal*. For small displacements, the E-magnet voltage is proportional to this error signal. Turret servos are activated to move the turret in such a direction as to decrease this error.

To see why we want $\mu = A$, differentiate $\sigma - \theta = \frac{1}{3}\mu$ and use the relation

$$\dot{\theta} = \frac{\mu + A_b}{t_m^*},$$

in which

$$t_m^* = t_m\frac{\cos(\gamma - \theta)}{\cos\left(\frac{4\mu}{3}\right)g\left(\frac{4\mu}{3}\right)},$$

to obtain

$$\tfrac{1}{3}t_m^*\dot{\mu} + \mu = t_m^*\dot{\sigma} - A_b. \tag{7}$$

If $\epsilon = \mu - A$ is the error, we have

$$\tfrac{1}{3}t_m^*\dot{A} + A = t_m^*\dot{\sigma} - A_b - \tfrac{1}{3}t_m^*\dot{\epsilon} - \epsilon.$$

If A is to be a smoothed value of $t_m^*\dot{\sigma} - A_b$, as it should be, ϵ must be kept at zero.

It follows that the accuracy of the lead obtained depends very heavily on the characteristics of the turret servo. Instead of subjecting the servo system to a component analysis as was done in Sections 8.2.3 and 8.2.4, one may proceed experimentally. This has the advantage of including inertia effects in the system. It has the disadvantage that the constants and form must be obtained as suitable averages for an appreciable sample. This is not possible during design and construction of pilot models. To obtain the *transit function* $H(p)$ of the system (p is d/dt) the definition

$$A = H(p)(\mu - A), \tag{8}$$

is used, and, typically, it is found that

$$H(p) = \frac{a_2p^2 + a_1p + 1}{b_4p^4 + b_3p^3 + b_2p^2 + b_1p},$$

where $a_2 = 1.161 \times 10^{-2}$ sec^2, $a_1 = 0.404$ sec, $b_4 = 3.84 \times 10^{-5}$ sec^4, $b_3 = 2.93 \times 10^{-4}$ sec^3, $b_2 = 2.47 \times 10^{-2}$ sec^2, $b_1 = 9.51 \times 10^{-3}$ sec.

Taking $H(p) = P(p)/Q(p)$, where P and Q are polynomials, manipulation yields[104] the lead equation

$$S(p)[P(p) + Q(p)]A = P(p)[t_m^*p\sigma - A_b] - S(p)\cdot Q(p)\sigma + S(p)Q(p)\alpha, \tag{9}$$

where $S(p) = \frac{1}{3}t_m^*p + 1$, and t_m^* is held constant.

This is the analogue of equation (2), or equation (3), for this sight.

8.2.7 Errors Caused by Aircraft Accelerations in the S-8B

The presence of the term involving the yaw of the gun mount, α, in equation (9) is an error. How serious it is depends on the servo system. Typical calculations show that if the gun mount is making a flat turn at a rate of 150 milliradians per second the error will be less than 1.5 milliradians. Suppose, however, that the aircraft is yawing harmonically so that

$$\alpha = A \cos 2\pi ft.$$

After transients have decayed the gun error, ϵ_a settles down to

$$\epsilon_a = B \cos (2\pi ft + \lambda).$$

The amplification B/A is tabulated below.[904] The operator $H(p)$ was not used. The particular results

TABLE 1. Amplification of aircraft motion.

Frequency, f (cycles per second)	Amplification B/A 1st experiment	2nd experiment
0.1	0.012	0.012
0.2	0.031	0.036
0.5	0.100	0.147
1.0	0.647	0.807
1.5	1.300	1.580
2.0	2.400	2.308
3.0	2.480	2.377
5.0	1.283	1.000

of two experiments were employed in the calculations. The steady-state error would be obtained otherwise from

$$[P(p) + Q(p)]\epsilon_a = Q(p)\alpha.$$

To get the transient error one solves

$$[P(p) + Q(p)]\epsilon_a = 0,$$

and finds that the *time constant* is 0.4 sec, i.e. by that time the *transient* will be less than $1/e$ of its initial value.

The Class B errors of the system will not be pursued further.[907]

8.2.8 Summary of Advantages of the S-8B

In summary, the main advantages of the S-8 system are (1) lead control (the handlebars control the lead and not the turret velocity — the disturbed reticle principle is inoperative), (2) *variable* aided tracking[908] (neglecting A_b, from equation (7), $\dot{\sigma} = \frac{1}{3}\dot{\mu} + (1/t_m^*)\mu$, so that velocity control of the line of sight is $(1/t_m^*)\mu$, and $\frac{1}{3}\mu$ is the positional control of the line of sight — recalling Section 5.3.4, "Aided Tracking"), (3) improved operational stability (if the handlebars are given a jerk of size μ_0 and then held at that value, the sight line responds with an equally sudden jerk in the same direction of size $\mu_0/3$ and then climbs along a line of slope $1/t_m^*$), and — of course — (4) full stabilization of the line of sight.

8.3 RADAR

8.3.1 Desirability and Requirements

When the range and positional inputs to a fire control system are in serious error, it is only by a fortuitous combination of those errors that the gun will be pointed properly. Repeated tests show the real difficulty experienced by a human gunner when he attempts, as simultaneous operations, framing, tracking, and triggering. This suggests that radar inputs be used to accomplish automatically one or two of these operations. Since framing is generally much poorer and more biased than tracking (it being difficult to frame, for example, when the target is not centered) and since radar ranging involves the simplest type of radar equipment, it seems desirable to use radar range input for seen fire. In the case of unseen fire it is essential, of course, that both range and rate be supplied by radar.

Obvious requirements for airborne radar gunnery aids are lightness and noninterference with aircraft speed. Ranging equipment weighs approximately 125 lb and is quite compact (the complete transmitter-receiver unit weighs 28 lb and fits in a can 15 in. long and 9 in. in diameter).[108] The antenna external to the aircraft is roughly equivalent to another machine gun barrel, and is, in fact, clamped to a barrel. The increased drag will cause a speed reduction of only a few miles per hour. (For purposes of comparison, open bomb-bay doors cause a speed reduction of the order of 34 mph for the B-29 airplane.)[108]

Turning to radar which tracks in angle, it might be presumed that an ultra-narrow beam should be used. The angular beam width is directly proportional to wavelength and inversely proportional to

the antenna aperture. High frequencies[b] would then be needed. Narrow beams also have the advantages of giving greater concentration of power (echo energy falls off as the *fourth* power of range) and of being less susceptible to interference (jamming).[149] However, ultra-narrow beams (12 milliradians) have not been used. Instead beam widths of 30° have been employed, relying on proper antenna design for accuracy. For example, a dish antenna can do a conical scan of angle 1.5° about an axis which keeps pointing at the target. It is also possible to employ lobe-switching in the antenna to achieve accuracy in determining the direction of a target.

The presentation of the radar output is important. In general, it is desirable to avoid visual presentation since this requires an operator who may make errors in utilizing this presentation. Even with automatic presentation it is not desirable to consider the radar as purely ancillary, with a shaft rotation or voltage output fed into a standard computer. Instead the original design should make provision for radar as an integral part of the system. Important simplifications may then be feasible.[150]

8.3.2 Airborne Range Only[c]

An *Airborne Range Only* [ARO] set is the AN/APG-5 (*Army-Navy Airborne Pulse Gun Laying*). APG-5 operates in the S band and has a beam width of 30°. Accurate pointing of the antenna at the target is not required. The effective tracking range is of the order of 1,500 yd. The range presented has a probable error of ± 20 yd. The error is probably quasi-steady. That is to say, during one run it might overrange steadily at 10 yd, but during the next run it might underrange steadily at 20 yd. Such performance is excellent, if it can be adhered to under field conditions of calibration and maintenance.

APG-5 searches automatically in range, sweeping from 200 to 2,000 yd twice each second. Upon encountering a target it locks on that target and tracks in range automatically. The gunner must still track in angle and trigger. Because of ground reflection the present unit is sometimes not dependable under an altitude of 8,000 ft.

8.3.3 Range and Angle

Airborne Gun Sights [AGS] AN/APG-8 (inhabited turrets) and AN/APG-15 (remote turrets) supply not only range but also azimuth and elevation. The system weighs approximately 150 lb. It has the same range accuracy and angular coverage as does APG-5, but employs the conical scan described above. However, tracking in angle is not as accurate as is visual tracking. For tail warning and defense, point-blank fire is used. The presentation is a dot on a small cathode-ray tube (C scope), which must be kept centered by the gunner. A damping circuit is employed to minimize "jiggle." Precisely as in the case of lead computing sights, a lag in presentation is a penalty. With a time constant of $\frac{1}{4}$ sec (which is considered short) this lag is of the order of $\frac{1}{8}$ sec.

8.3.4 Airborne Gun Laying

Airborne Gun Laying [AGL] sets AN/APG-1, AN/APG-2, AN/APG-3, and AN/APG-16 are designed for *fully* automatic ranging and tracking in azimuth and in elevation. APG-1 and APG-2 weigh about 400 lb and operate in the S band. APG-3 and APG-16 weigh about 250 lb and are well up in the X band. Little is available in the current literature concerning tracking errors in angle.[d] Estimates of accuracy have been as optimistic as 5 milliradians but this depends somewhat on the angular rate. APG-16 is engineered for use with the Sperry S-8 sight. The antenna is a conical scanner 9 in. in diameter mounted in a ball 18 in. in diameter which is fastened rigidly to the outside of the turret[106] (e.g., the Sperry A-17 ball turret in the tail of the B-32). The ball contains motors and gears which spin and direct the antenna relative to the ball. When tracking, the axis of the conical scan points directly at the target. The motion of this axis is the vector sum of turret motion and motion with respect to the ball. It follows that the relative displacement of the antenna within the ball is the lead, since the guns are boresighted with the turret, and so is an output of the S-8 system.

8.3.5 Instability of a Radar-Gyro-Sight System

In illustration of new difficulties that may arise when radar is used in conjunction with a sighting system, consider the stability problem[107,108] In the

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[b] Nomenclature for bands is: P, 225 to 390 mc; L, 390 to 1,550 mc; S, 1,550 to 5,200 mc; X, 5,200 to 11,000 mc; K, over 11,000 mc.

[c] Further information on the sets discussed will be found in the Summary Technical Report of Division 14, Volume 2.

[d] The Armament Laboratory of Wright Field is conducting tests on this point.

British *Automatic Gun Laying Turrets* [AGLT] Mark I. (With a director system such behavior as that discussed below does not occur.) The direction of the target is shown to the gunner as a collimated scope spot. A (single) gyro gunsight is used to predict the lead, and the guns are to be moved so as to keep the moving reticle of the sight on the synthetic radar target. Range is fed automatically. The radar has, however, an appreciable lag (stickiness) and the synthetic target is governed by the equation

$$(sp + 1)(\gamma - \rho) = \gamma - \tau,$$

where s is a damping constant, p is the operator d/dt, and γ, ρ, τ are respectively the angles that the gun line, radar target line, and actual target line make with some fixed reference line. The equation of the sight is familiar from Chapter 5. Neglecting ballistics, it is

$$[(1 + a)t_m p + 1](\gamma - \sigma) = t_m p\gamma,$$

where σ is the angle made by the sight line with the reference line.

Suppose now that the target is initially astern and that the guns suddenly start to move (under some aiming disturbance) according to $\gamma = \gamma_0 t$, where γ_0 is a constant. If the gunner does nothing and the target remains astern, then for unit gun motion and $s = 0.25$, $a = 0.43$, $t_m = 1$, the resulting angular motions are shown in Figure 5. The gunner, since the

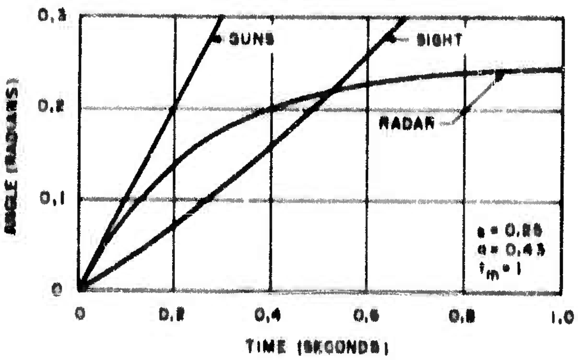

FIGURE 5. Radar and sight responses: AGLT Mark I. (Figure from British source: Armament Department, R.A.E.)

sight seems to be falling behind the target, will accelerate the guns in the wrong direction. This drags along the radar curve — which has the same initial slope as the gun line — and so aggravates the situation. If the gunner could actually keep his sight line on the radar target perfectly, then $\rho = \sigma$, and the two equations above yield, for constant t_m,

$$[-t_m s p^2 + a t_m p + 1]\gamma = [(1 + a)t_m p + 1]\tau.$$

The transient term is damped and the system is stable only if the roots of

$$-t_m s x^2 + a t_m x + 1 = 0$$

have negative real parts. This means that $t_m s$ must be negative, which is not possible. One could obtain neutrality by making s negligibly small but this is not possible since radar spot jitter must be damped.

It is clear that the difficulty arises since the initial slope of the radar curve is equal to that of the gun curve and the initial slope of the sight curve is only $a/(1 + a)$ times that of the gun curve. It seems plausible to decrease the slope of the radar curve by first reflecting the target spot from the mirror of another gyro unit.[200] If the equation of the auxiliary gyro is combined with the two original equations it is readily seen that a parameter can be chosen to assure stability of the resulting third order differential equation.

It is also possible[197] to reduce the slope of the radar curve below that of the sight curve by an electric circuit such as that shown in Figure 6, in

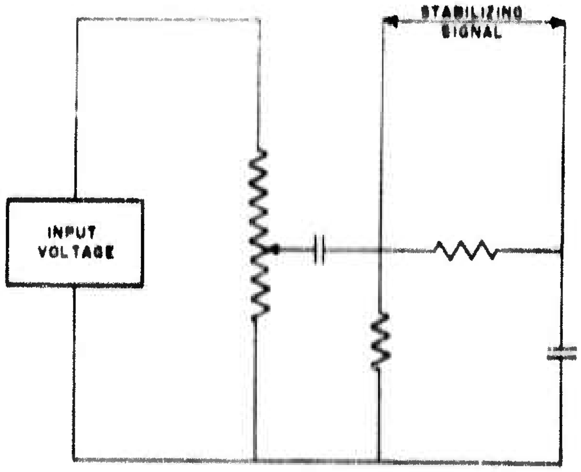

FIGURE 6. Stabilizing circuit: AGLT Mark I. (Figure from British source: Armament Department, R.A.E.)

which the input voltage is proportional to displacement of the gun relative to the platform. The stabilizing signal is

$$\frac{Pp^2}{(1 + Qp)(1 + Rp)}(\gamma - \alpha)$$

where α is the directional angle of the gun platform

relative to the datum line, and P, Q, and R are circuit constants. The position of the radar target is given by

$$\rho = \frac{sp\gamma}{1 + sp} + \frac{\tau}{1 + sp} + \frac{Pp^2(\gamma - \alpha)}{(1 + Qp)(1 + Rp)}.$$

If this equation is combined with the sight equation, ρ being placed equal to σ, a fourth order differential equation arises whose stability can be established.

8.4 NEW CONCEPTS

8.4.1 Sight Parameter as a Function of Time

One of the early proposals for improving the performance of lead computing sights (angular rate by time-of-flight multiplier) suggested that the sight parameter, a, be varied as a function of present range, r.[107] This work assumed that the target followed a straight line course and that the time-of-flight multiplier, t_m, was the time of flight over present range, t_p. (This antedated the calibration concept.) According to Section 5.3.4, "Delay in Lead Computation," for positive a, the lag in lead computation of the smoothing circuit may just balance the error in $t_{p\omega}$ if a is chosen properly. An analysis up to first order terms shows that a must be given by the expression

$$a = \frac{r'}{r' + c}\,\frac{t_p r' - r}{2t_p r' - r},$$

where the constant c is to be chosen somewhat larger than the expected target speed, and $r' = dr/dt_p$.

From the point of view of present day airborne fire control this refinement has little weight for three reasons: (1) against curved courses approximating pursuit curves $t_{p\omega}$ is in error by about 10 per cent, a cannot be chosen to affect this materially, and the calibration concept effectively does the same job without the necessity of awkward mechanizations of variable a; (2) present range input is too rough to be efficient in determining r'; and (3) any improvement in accuracy is completely masked by much larger operational errors.

It is evident that such circuit modifications should await a renewed study of the fire control problem with all elements in proper focus. At present, the idea is much more pertinent to antiaircraft fire.

8.4.2 Exponential Spot Sights

A second early proposal[108,132] for improving lead computing was much bolder in concept. The equation connecting sight angle and gun angle is

$$(1 + a)t_m\dot{\sigma} + \sigma = \gamma + at_m\dot{\gamma}.$$

Suppose that the reticle pattern consists of a large number of spots which are *not fixed* but which have *relative motion* with respect to the reticle center. Let σ_n be the angle between a line of sight to one of these moving spots and the datum line. Hold this spot on target. Now σ_n will satisfy the equation

$$(1 + a)t_m\dot{\sigma}_n + \sigma_n = \gamma + at_m\dot{\gamma}$$

if

$$\sigma_n - \sigma = k_n e^{-t/(1+a)t_m},$$

which means that the spot must approach the reticle center exponentially. Continuously rotating reticle discs which when superposed will give spots behaving in this way are shown in Figure 7.

Suppose now that the guns are slewed rapidly around to get on a target moving at a constant rate γ_0. With the usual sight the reticle would be left behind and it would take an appreciable time for the gunner to get on target. The whole point of this device is to eliminate this waste time. To illustrate this, let the guns be in correct lead position and moving at the correct rate, but suppose that the reticle center has not yet come in correct lead position. There will be, however, a spot centered on the target which seems fixed and toward which spots on the right and on the left are converging. Analytically, let

$\gamma = \gamma_0(t + t_m)$, $\sigma = -\sigma_0$ and $\tau = 0$ when $t = 0$.

Then, for constant t_m,

$$\sigma = \gamma_0 t - \sigma_0 e^{-t/(1+a)t_m}$$

$$\sigma_n = \gamma_0 t + (k_n - \sigma_0)e^{-t/(1+a)t_m}.$$

Consequently the spot n^* for which $k_{n^*} = \sigma_0$ is (and stays) in correct lead position since $\gamma - \sigma_{n^*} = \gamma_0 t_m$. Furthermore, since

$$\sigma_n - \sigma_{n^*} = (k_n - \sigma_0)e^{-t/(1+a)t_m},$$

$$\frac{d(\sigma_n - \sigma_{n^*})}{dt} \quad \begin{matrix} < 0 & k_n > \sigma_0 \\ > 0 & k_n < \sigma_0 \end{matrix}$$

the remaining spots converge on n^* as noted. This is not the whole story (t_m was held constant and the field may not be wide enough) but it does demonstrate the value of the idea. It is not clear, however, what method would be used to get a framing circle to move suitably. This ingenious idea may be

assessed, perhaps, as a patching-up of existing gear. But the principle of the rotating reticle may well be kept in mind.

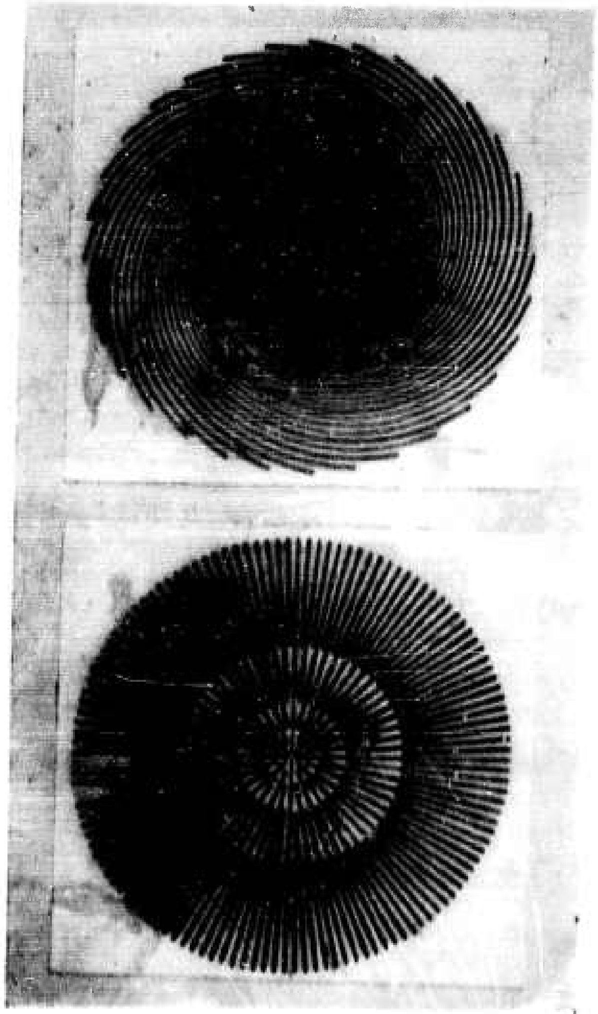

FIGURE 7. Exponential spot reticle.

8.4.3 Aided Tracking on the Line of Sight

A frequently neglected point in the design of equipment is ease of control (which affects the efficiency of operation). This phase of the design involves the experimental study of such details as inertia, friction, and cramping, using gunner subjects. It may involve more subtle matters. As an example of the latter, it has been proposed that *aided tracking on the line of sight* be used in conjunction with a single gyro sight.[100,180]

Suppose that the voltage derived from the handlebar position is fed not only to the turret motors, but also to the sight after it has been differentiated (see footnote to Section 8.4.4). Superimpose on the voltage source for the *trail coil circuits* (Section 5.5.5, "Electromagnetic System") this additional voltage $\dot{\mu}$. Then the magnetic center toward which the gyro axis processes is shifted through an angle

$$\beta = k\dot{\mu}.$$

The gyro precessional rate is now given by

$$(1+a)t_m\dot{\theta} = \gamma - \theta + \beta.$$

Since $\theta = \gamma - (1+a)\lambda$, it follows that

$$t_m\dot{\gamma} = t_m(1+a)\dot{\lambda} + \lambda + \frac{\beta}{1+a}.$$

Next, if the yaw of the gun mount is α, then for a velocity controlled turret the absolute gun position γ is determined by

$$\dot{\gamma} - \dot{\alpha} = B\mu,$$

where μ is the displacement of the handlebar controls.[a] Using this relation to eliminate γ, the lead equation is

$$t_m(1+a)\dot{\lambda} + \lambda = t_m\dot{\alpha} - \frac{k\dot{\mu}}{1+a} + t_m B\mu.$$

The sight line equation is, putting $p = d/dt$,

$$[t_m(1+a)p+1]\sigma = [at_m p+1]\alpha + \left[\frac{1}{p} + at_m\right]B\mu + \frac{k}{1+a}p\mu.$$

(The term involving $d\alpha/dt$ can be construed as an error source, because of it, motions of the gun mount interfere somewhat with the correct positioning of the sight line.) *Neglecting aircraft motion*, give a sudden jerk $\Delta\mu$ to the handlebars. The response $\Delta\sigma$ of the sight line is inferred from the preceding equation by supposing that the jerk is an instantaneous matter so that $p\sigma$ and $p\mu$ are infinite. Hence divide by p and then let $p \to \infty$. The result is

$$\Delta\sigma = \frac{k}{t_m(1+a)^2}\Delta\mu.$$

This shows the direct (or positional) tracking effect of the coupling of handlebars to sight. On the other hand, the sudden change in angular velocity of the sight line, $\Delta(p\sigma)$, caused by a sudden change in handlebar position $d\mu$ can be obtained by solving for

[a] The *constant* B is not always a constant for all turrets. It may be, for example, $B + A|\mu|$.

$p\sigma$ and neglecting $B\mu/p$ and $kp\mu/(1 + a)$, i.e., the integral of $B\mu$, which is $B\mu/p$, is taken over an interval of zero length, and the derivative of μ is zero at both the beginning and end of the step-jump in μ. Hence

$$\Delta(p\sigma) = \frac{a}{1 + a}\,\Delta(B\mu) - \frac{k}{t_m^2(1 + a)^2}\,\Delta\mu.$$

This shows the velocity tracking effect. For constant B, if

$$k > Ba(1 + a)^2 t_m^2,$$

which could happen for a too large value of k or a too small value of t_m, the velocity response will be negative and poor tracking will undoubtedly result.

It is recommended[130] that circuits such as these be studied with the aid of *simulating electronic circuits* in which various parameters may be varied.[1]

8.4.4 Correction Mechanisms for Target Course Curvature

A constantly recurring motif throughout previous chapters has been the serious effect on the deflection put out by computing sights attributable to the neglect of course curvature. Mechanisms have been proposed[133] which will take this into account, not by the naive expedient of flipping a switch from 100 to 90 per cent of present time of flight in going from rectilinear to curved target paths.

By equation (11) of Chapter 2 the correct kinematic deflection is given quite closely in terms of the angular momentum, $M = r^2\omega$, of a target of unit mass. This deflection may be written

$$A_k = \frac{qr\omega}{v_0}\left[1 + \frac{t_f}{2}\frac{d(\ln M)}{dt}\right].$$

The classical lead computing sight contents itself with

$$A_k = \frac{qr\omega}{v_0}.$$

Even this value might well be corrected by utilizing the rate of change of present range, assuming good input of range. The point of the present discussion, however, is to indicate a mechanism which can be combined with an ordinary gyro sight to yield the angular momentum correction.

Use the single gyro from a K-15 sight, without the ballistic coils. The deflection of the mirror of this gyro is made proportional to M by feeding a current to the range coils through a nonlinear potentiometer (r^2) whose position is determined by present range r. The light reflected from this gyro mirror is to come through a reticle of such variable transparency that the intensity of passing light at any point is proportional to the logarithm of the distance from the center. After reflection from the gyro mirror the light hits an opaque screen with a small central hole behind which is a photocell. Since only that part of the reticle yielding light proportional to $\ln M$ hits the photocell, the voltage output of the cell is proportional to $\ln M$. This voltage is amplified linearly and fed into a circuit containing a condenser, which differentiates $\ln M$,[a] and containing a range potentiometer which introduces $t_f/2$. By means of a servo, a rotation proportional to $\ln[1 + (t_f/2)(d\ln M/dt)]$ can be introduced in one side of a differential. The input to the other side is proportional to $\ln r$. The differential's output (the sum) may then be sent to the range potentiometer of the usual gyro sight, and the correction has been effected.

The gain in accuracy is most significant when this correction is made. The effect of roughness in input of r and ω is, nevertheless, still open to question.

It is also possible to utilize variable speed drive computers (ball-cage integrators) to apply this correction to the stabilized S-3 sight.[134]

8.4.5 An Own-Speed Plus Rate Mechanization

Lead computing sights have decomposed deflection into kinematic and ballistic components. Other decompositions and consequent mechanizations are possible.[48] To keep one such new idea clear, various parts of the proposal (corrections for target and gun platform accelerations, and use of range rate) will not be considered.

A basic lead equation of gunnery is equation (1) of Chapter 2. It is

$$\sin A = \frac{v_G}{v_0}\sin\tau - q\frac{v_T}{v_0}\sin\alpha = \lambda_0 - \lambda_T.$$

The mechanization suggested uses a standard own-speed sight (the 0 computer) to mechanize v_G sin

[1] Such studies were made, for example, at the Marcellus Hartley Laboratory of Columbia University under J. R. Russell.

[a] If the input voltage is in series with a small resistance and a condenser of large impedance, the voltage drop across the resistance is approximately $RCdE/dt$. This small voltage may be fed to an amplifier.

r/v_0 with no percentage correction. It uses a rate computer to deal with $v_T \sin \alpha/v_0$ (the T computer). Since target speed and approach angle are not known, the T computer proceeds in what seems to be a circular fashion by accepting as its input, in difference, $v_G \sin \tau/v_0$, and $r\omega/v_0$ (as determined by ranging and gyro-tracking); and yielding, through a smoothing circuit

$$a(r)\dot{\lambda}_T + \lambda_T = v_G \sin \tau/v_0 - r\omega/v_0,$$

an angle λ_T. (The smoothing circuit selected suggests that a be a function of r to give less damping at greater ranges for which $v_T \sin \alpha/v_0$ is relatively constant.) This output λ_T must be combined with the ballistic factor q. For projectiles of proper shape and in that velocity range for which the three-halves power drag law holds,

$$q = \frac{1}{1 - \dfrac{b\rho P}{c_0\sqrt{u_0}}},$$

where ρ is the relative ballistic air density, and

$b = 0.00323$ (when the unit of length is yards),
$b = 0.00186$ (when the unit of length is feet).

The actual air range covered by the bullet can, in the absence of useful range rate, be approximated by

$$P = cr,$$

where c is a constant. For approach angles that do not exceed 30°, this is very effective. The velocity of departure, u_0, is given adequately by

$$\sqrt{u_0} = \sqrt{v_0}\left(1 + \frac{v_{G(po)}}{2v_0}\right),$$

where $v_{G(po)}$ is the component of mount velocity along the *present range* line.

It is the above quantity q that is to be computed with precision. Through it, i.e., through this decomposition, ballistics are automatically taken into account, and no separate trail ballistic circuits are needed. (The time of flight, t_f, which is normally determined by lead computing sights, will enter here only in making the small curvature correction. See equation (7) of Chapter 2. The simple estimate $t_f = r/v_0$ should prove satisfactory.)

This own-speed rate system has real advantages when used against curves approximating pursuit curves flown against one's own bomber or against a neighboring bomber in the formation. The reason is that only a small part of the total lead is subjected to the smoothing circuit. Consequently, percentage errors of the same size as those of a normal rate-time sight now generate a small absolute error. Against forward hemisphere attacks, the advantage is even more marked. The initial error will be small and at close ranges when the required deflection builds up rapidly — primarily because of the rapid change in own-speed deflection — gun pointing should be quite good in virtue of the presence of the O computer.

9.4.6 General Approach to the Tracking Equation

The study of a complete fire control system demands a knowledge of the *tracking equation* which connects handlebar motion to sight line motion. In the case of manual operation by a human gunner a theoretical description of the complete system is difficult since the gunner's responses cannot be described exactly — he is an imperfect servomechanism. Consequently, up to the present, resort has been made to experimental and statistical procedures rather than theoretical analysis. It is evident that the advent of automatic inputs will render mathematical methods more attractive. But the elements of a general theory are useful even in the case of manual operation [88] for which only probable behavior on the part of the operator may be assumed.

To illustrate the methods of the theory, consider aided tracking combined with a lead computing sight. The gun position, γ, is connected to the handlebar angle, μ, by

$$\dot{\gamma} = A\dot{\mu} + B\mu.$$

The sight equation relates gun angle to the sight line angle, σ, by

$$at_m\dot{\gamma} + \gamma = (1 + a)t_m\dot{\sigma} + \sigma.$$

If γ is eliminated between these two equations, then

$$(1 + a)t_m\ddot{\sigma} + [1 + (1 + a)\dot{t}_m]\dot{\sigma} = Aat_m\ddot{\mu} + [A(1 + a\dot{t}_m) + Bat_m]\dot{\mu} + B[1 + a\dot{t}_m]\mu.$$

It will be noted that t_m is permitted to vary with time. The solution of this equation may be written in the form [88]

$$\sigma(t) = \frac{Aa}{1 + a}\mu(t) + \int_{t_0}^{t} K(t,t')\mu(t')dt',$$

where

$$K(t,s) = B + \left[\frac{A}{(1 + a)^2 t_m(s)} - \frac{B}{1 + a}\right]e^{-\tau(t,s)/1+a},$$

$$\tau(t,s) = \int_s^t \frac{dt'}{t_m(t')}.$$

The artificial time, τ, moves faster than t if $t_m < 1$.

More generally, suppose that the gun-handlebar equation is

$$R\gamma = R'\mu,$$

and that the gunsight equation is

$$P\gamma = Q\sigma,$$

where R, R', P, and Q are polynomials in $p = d/dt$ whose coefficients depend on t_m. If t_m is held constant, the tracking equation is

$$RQ\sigma = PR'\mu.$$

The tracking equation with t_m variable is not obtained so readily. Since, however, the change in t_m due to ranging has but little effect on the nature of the tracking, this equation may be used to infer various properties of the tracking performance. For example, there are various solutions of the tracking equation depending on the initial handlebar motion. If the difference of some pair of these solutions does not approach zero, the tracking is *essentially unstable*. (The operator could track a target at rest by handlebar motion not approaching a position at rest.) The difference between any pair of solutions must satisfy the equation

$$PR'\mu = 0.$$

This has a solution not approaching zero if and only if PR' has a nonnegative root. (For example, for aided tracking with a lead computing sight, the roots of PR' are $-B/A$ and $-1/at_m$. Stability is the normal situation.) The general equation can also be used to study hunting and the response to handlebar jerks.

The most useful form, here, of the solution of the tracking equation is given by operational methods employing the convolution integral.[68] The reason is that the resulting solution expresses $\sigma(t)$ as a weighted average of earlier values of $\mu(t)$. This was illustrated explicitly in the preceding paragraph. The solution of the tracking equation with constant coefficients is

$$\sigma(t) = C\mu(t) + \int_{t_0}^{t} K(t - t')\mu(t')dt' \quad (t > t_0) .$$

Further details and interpretation will not be given. In summary, the point to be made is that in future problems of overall system design and system response, methods of this nature will be exploited. These general methods will doubtless require extension to nonlinear systems, only special examples of which have been discussed.[43]

8.5 NEW TACTICS

8.5.1 Offset Guns in Conventional Fighters

The conventional fighter can mount a heavy forward firing battery at minimum cost in installation weight and balance. The penalty is that the entire aircraft must be aimed and must fly a predictable course, the pursuit curve, if a succession of hits are to be scored on a bomber target. At the other end of of the scale is a turreted fighter which may fire as flexibly as its target. The P-61 is an example. This type is penalized not only by increased weight and size, and crew, but also by all the difficulties in taking deflection experienced by the defending bomber. A middle course appeared feasible during the closing months of World War II. If *fixed* guns are installed at a large angle to the thrust axis, installational weight is not significantly increased, the modification is possible for many existing fighters, and there is no increase in crew. The following questions must be answered. (1) What are the advantages? (2) What is the aiming problem with an offset installation? (3) What are the disadvantages? Considerable study has been devoted to these points.[42,126,130,164,178]

8.5.2 Attack by Pacing Behind and Below

An attack is advantageous to a fighter if his attack path is nearly rectilinear, so that coordinated banked flying under conditions of high load is not required, if the deflection required is small and does not change rapidly, and if continuous fire may be put into the bomber until destruction ensues.

The simplest method of gaining these advantages with an offset installation is to mount the guns pointing upward and forward in a vertical plane at such an angle that, for *pacing* flight below and behind, and over a considerable range interval, the backward curve of the trajectory caused by trail is sensibly balanced by the forward and downward curve attributed to gravity and by the parallax correction caused by separation of guns and sight. The feasibility of this depends on the ballistics of the ammunition employed. For example,[42] using API M8 caliber .50, an offset angle of 30°, and a parallax distance of 2 yd, the total lead required is: 4 milliradians (±1) for a speed of 175 mph, 6 milliradians (±1) for a speed of 250 mph, 8 milliradians (±2) for

a speed of 325 mph. These negligible variations correspond to an interval in range of from 100 to 400 yd. Given an offset angle and a range interval it is necessary to design special ammunition to achieve constancy of lead. Then the attacker need not determine speed and range with precision. With unsuitable ammunition, for example, typical Japanese 20-mm ammunition (v_0 = 2,358, c_0 = 0.240) fired at an offset of 45° with a parallax of 2 yd, the required deflection increases from 13 to 28 milliradians (at 250 mph) as the fixed range at which the attack is made changes from 200 to 500 yd, and from 18 to 42 milliradians (at 325 mph) over the same range interval. The attack is still quite possible, but if it is not made at a precalculated speed and range, the pilot must lay off deflection.

8.5.3 Attack on Collision Course

A second possibility is to attack on a collision course. The attacker crabs in on a path which in relative motion is a straight line terminating at the bomber. If the guns are offset so as to lie slightly forward of this relative path (to allow for trail) then continuous fire is possible with but little change in the required small deflection. (During World War II the German Air Force occasionally tried skidding attacks with conventional fighters.[b] Evidently this is equivalent to mounting guns at a small offset. Without the aid of a gyroscopic lead computing sight, it is difficult to fly.) Using the notation of Figure 4 of Chapter 3 and formula (4) of Chapter 2, the (lead) angle between the relative path and the bore axis must be $1000 l v_F \sin \delta / v_0$, where $v_F \sin \delta = v_B \sin \theta$ is the collision course condition. The term $l = q - 1$ is an almost linear function of $\rho P / c_0$. Consequently, the larger the ballistic coefficient is, the smaller the spread in l over a given range interval will be, and the more efficient a fixed average offset from the relative path will be.

Two methods of making this attack are possible. Suppose first that the sight is offset by an angle δ and the guns by an angle $\delta - l v_F \sin \delta / v_0$ radians where l is an appropriate average. Use a *fixed* fighter speed and fly so as to keep the target centered in the offset sight. Then for a value of θ different from the optimum given by $v_F \sin \delta = v_B \sin \theta$, the fighter's path will curve (quite gently) and certain aiming errors will occur. As a numerical example,[184] if v_F = 440 fps, v_B = 321 fps, altitude = 20,000 ft, and if API M8 ammunition is used, then, with sight offset 45° 15′ and gun offset 45°, the angle off can vary from 70° to 120° off nose, and the aiming error will vary only from a maximum of 0.7 milliradians behind a (point) target to a maximum of 5.8 milliradians ahead of it. As a second method, suppose that the pilot keeps the target centered in his offset sight but adjusts his speed according to $v_F = v_B \sin \theta / \sin \delta$. In this case a true rectilinear course is flown and the error will again be slight.

Tactically, the accuracy of the fighter's fire should not depend *critically* on initial range and bearing. Allowance for error must be permitted. Similarly, a variation in the calculated speeds of fighter or target must not cause the accuracy to deteriorate too much.

8.5.4 Discussion of Offset Gun Attacks

The discussion above indicated that specialized attacks of this nature are possible. But there are serious disadvantages. If a fighter armed with offset guns attempts to make anything like a pursuit curve attack in a slant plane, his required motion will be a twisted space curve with curious torsion, which probably cannot be flown. In particular, a fighter without a forward battery can hardly attack another fighter. Even against a bomber, the arguments of Sections 8.5.2 and 8.5.3 are predicated on a bomber that maintains a straight and level course. Rather mild evasive action can cause the attack to abort. Finally, assuming the bomber does cooperate by flying a rectilinear course, the fighter is an equally easy shot for any of the fire control systems of Chapter 5 which have well adjusted, i.e., variable, trail allowances.

In view of these disadvantages, it appears that these attacks are most promising when made by a night fighter. For this case, the parallel course attack of Section 8.5.2 is probably best. The defensive radar tail-cone search can be avoided, and, if provision is made for both upward and downward firing guns,[178, 180] proper advantage of night sky conditions may be taken. These attacks would also be dangerous to a bomber which has stripped its armament to tail-cone defense to obtain speed and additional pay load, if it is assumed that in so doing the fighters could only make tail-cone attacks for reasons of head and buffeting. It may also be kept in mind that the paths of Section 8.5.3 are collision courses

[b] Interrogation of Th. W. Schmidt at Coburg, Germany, E. W. Paxson, June 5–8, 1945.

and so may be as attractive to a guided or self-guided missile as is a homing course.

8.5.5 Upward Barrage Fire

Turning from conventional machine gun armament in fighters, upward barrage fire at close range has been proposed.[1] The installation consists of 40 barrels, each containing but one round, mounted to fire sensibly upward. The attack is to be from the front and initiated by a dive down and across the bomber's bow. Upon pull-up below in the same vertical plane the tubes are ripple-fired automatically by acoustic pickup from the bomber's propellers or by optical pickup of the bomber's shadow. The range of fire is to be 50 to 100 m, which means that the violent motion described on the attacker's part is necessary if he is to arrive unscathed at the firing point. Reliance is placed on a linear pattern approximately 80 meters long, rather than on a precise estimate of deflection.

There are certain technical difficulties. To avoid high recoil a low muzzle velocity of the order of the firing aircraft's speed is to be used. Consequently, large caliber (2.0 to 2.5 cm), high-explosive, and fused projectiles must be used. The initial yaw is of the order of 40°. For a spin stabilized projectile (rifling of 30 calibers per revolution) this yaw does not damp sufficiently to assure an angle of impact favorable to the functioning of the fuse. A fin stabilized projectile is more satisfactory.[21]

8.6 SUMMARY

Section 8.1 limits the chapter to those ideas in hand during the closing months of World War II. The next phase in fire control development should be a judicious blend of principles now known to be essential. The plea on behalf of the designer is made that the shape of expected tactical situations be made available.

Section 8.2 deals with systems that stabilize the gunner's line of sight by removing perturbations caused by aircraft motion. The theory of the rate-stabilized Fairchild S-4 system is brought to the point of deducing the complete circuit equations. The position-stabilized Sperry S-8B sight is discussed similarly although the circuit response equation is obtained experimentally rather than analytically. Stabilization does not extend to aircraft accelerations, and the time constants of follow-up circuits must be carefully considered.

Section 8.3 discusses the use of radar in supplying automatic inputs, in range and in bearing, to a fire control computer. Conditions of instability in tracking are mentioned briefly.

Section 8.4 lists as new concepts (1) the variation in the sight parameter as a function of range, (2) a mechanism designed to cheat partially the transient time in a lead computing sight, (3) the use of aided tracking on the line of sight, (4) an ancillary circuit for lead computing sights which gives a correction for target course curvature through the rate of change of the angular momentum of the target about the gun, (5) the mechanization of a decomposition of lead into own-speed allowance and target motion allowance, rather than into kinematic and ballistic deflections, and (6) the elements of a general theory of system response to control movements.

Section 8.5 considers as new tactics (1) the use of a fixed offset gun in a fighter attacking on a pacing course, (2) the use of such an installation on a collision course attack, and (3) upward barrage fire at close range.

No account was given of air-to-air rocketry or air-to-air bombing. One must also look elsewhere for information on guided and semi-guided missiles.

[1] Interrogation of Doetsch, Blenk, Rossmann, Schlüter, Hackemann, at the Luftfahrtforschungsanstalt, Braunschweig, Germany. E. W. Paxson, June 28–29, 1945.

PART II

ROCKETRY

Chapter 9

FIRE CONTROL FOR AIRBORNE ROCKETS

9.1 INTRODUCTION

In the spring of 1944, the program of the Applied Mathematics Panel in *rocketry* was initiated by a request from Division 7, NDRC, for AMP's cooperation in the development of the best possible computing sights for rocket firing, where the term *best* was to be interpreted as applying to practicability and availability in point of time, as well as to accuracy. There were subsequent requests from Division 7 and from the Navy for assistance with various aspects of the rocket sighting problem. As a result of these requests AMP's activities in this field fell into three categories.

First, there was a general study of what sighting methods are feasible. For rockets, the essential problems involved in this question have to do with ballistic formulas,[16,17] attack angle and skid,[15,16] the effect of wind and target motion, how these various factors affect each proposed sighting method, and how tracking affects and is affected by them.[7,20] Two AMP reports[1,2] consider general aspects of the problem, give formulas relating to the quantities involved, and study in some detail various sighting methods. This chapter is based largely on the material in the first of these reports[1] which is considered fundamental to any future work concerned with the development of computing sights for rocket firing. A brief discussion is presented here of the principal ballistic questions involved in the problem; a simplified treatment of kinematic lead is given; and certain of the questions involved in the design of a computing mechanism are set forth.

Second, several specific proposals for rocket sights were made and studied.[8,10] For example, a range-finding method using the Mark 23 gyro sight was studied;[12] a miniature rocket sight called PARS, consisting of a mechanical addition to an altimeter with an electrical connection to a sight head, was proposed[8,10] and was scheduled for test. A brief description of the characteristics of this sight is given in the Division 7 Summary Technical Report.[4] AMP also participated in the program for the development of a *pilot's universal sighting system* [PUSS],[11,16,28-31] reported in Division 7 Summary Technical Report; and made related studies of toss rocketry[18,24] and of toss bombing in the presence of wind.[21,22,26-27]

Third, a method was formulated for determining whether pilots who are attempting to fly a diving course at a target skid badly enough to cause a substantial error in rocket fire.[3,22] For further information concerning this and other detailed questions studied by AMP, the reader is referred to the reports listed in the bibliography at the end of this volume.

One of the principal advantages of rockets over shells is that no heavy gun or gun mount is needed in rocket firing, since the recoil is taken up by the ejected gases. For stability, the rockets considered here are constructed with fins near the tail. These fin stabilized rockets, fired forward from aircraft, have become an important weapon of warfare. Because of their relative newness as an aircraft weapon, the problem of how they may best be aimed is not yet fully understood, and no very satisfactory computing sight was in service at the termination of hostilities. The problem is considerably more complicated than that of aiming bullets. In the first place, due to the presence of the fins, the rocket picks up most of its velocity in the direction of motion of the airplane in the surrounding air mass; this may differ from any fixed direction in the airplane by a considerable amount. Moreover, to determine this direction is a difficult job. Secondly, except at short ranges, the gravity drop must be accounted for; it can be many degrees at 3,000- to 4,000-yd range. This requires some (direct or indirect) knowledge of the range;[6,9] and there is no simple known instrument for use in a fighter plane which gives the range. A third consideration which affects the whole character of the problem is the temperature of the propellant grain which is dependent upon the initial

temperature (determined by conditions of storage) and is affected by the altitude at which the airplane may have been traveling for some time before the attack. Finally, the projectile takes normally two to ten seconds to reach the target. By this time the target may have moved a considerable distance relative to the air mass. On the other hand, since the ammunition dispersion amounts to only a few mils, and since most targets for aircraft rockets are rather small — tanks, locomotives, gun emplacements, and small shipping, for example — it would be of great advantage to obtain an accurate solution of the sighting problem.

9.2 FIN STABILIZED ROCKETS

Rockets are normally held under the wings of the aircraft. When fired, they accelerate, increasing their velocity to a maximum of usually 800 to 1,500 fps, depending mainly on the type of rocket. Burning time for most rockets is about a second; it depends greatly on the temperature of the propellant, being shorter at higher temperatures. After burning, the rockets slow down again, due to drag.

The launchers commonly used now are *zero length launchers.* The rocket is suspended at two points, and becomes free after it has traveled an inch or less. Since it is already moving through the air mass with the velocity of the aircraft, the air acts strongly on the fins, turning it into the wind as soon as it leaves the launcher if it is not already pointed in that direction. Angular momentum carries it beyond this point, so that its direction oscillates with decreasing amplitude about the direction of its vector velocity.

For sighting purposes, one may neglect these oscillations, provided that one determines their overall effect on the general direction of the path. This is accomplished by defining an *effective launcher line,* along which the rocket is supposed to start out.

9.2.1 The Effective Launcher Line

The effective launcher line is a line a certain fraction f of the way from the launcher line (the line of the rocket at firing) to the velocity vector relative to the air mass of the airplane firing the rocket.

The value of f is commonly between 0.8 and 0.90. For the 5-in. HVAR rocket and in the vertical plane, f is given approximately by the formula

$$f = 1.02 - \frac{2190 + 22.2T}{v_i^2}, \tag{1}$$

where T is the propellant temperature in degrees Fahrenheit and v_i is the indicated airspeed of the launching airplane in yards per second. Thus f depends on indicated airspeed and propellant temperature, but not on dive angle. Numerical values of f are given in a report[8] which we shall hereafter describe as the "tables."

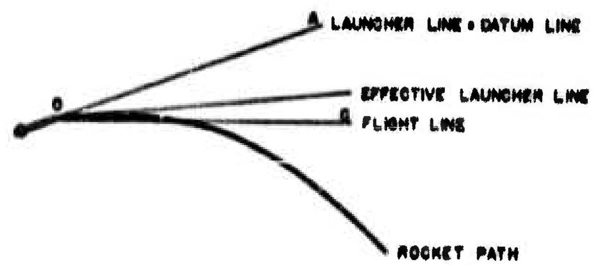

FIGURE 1. Effective launcher line.

The f factor in the lateral plane need not equal the factor in the vertical plane, but probably is close to it. Accordingly, the effect of skid (motion perpendicular to the plane of symmetry of the aircraft) is important: Suppose, for example, that the velocity vector of the aircraft points 20 mils to the right of its plane of symmetry. Then the rocket will turn $20f$ mils into the wind. If the target were fixed and if the plane of symmetry of the aircraft contained the target, this would lead to an error of $20f$ mils, which is of the order of 18 mils.

The purpose of defining an effective launcher line and the f factor is this: In our analysis the effective launcher line is taken as the initial direction of the trajectory. A knowledge of this effective initial direction is necessary to permit the determination of the manner of flying the launching aircraft which will be most likely to achieve a hit.

If a long launcher (rail or tube) were used, or if the rocket propellant were faster burning, f would be smaller. For a bomb, $f = 1$. For a machine gun bullet, the initial velocity of departure is the vector resultant of the velocity of the airplane acting in the instantaneous direction of motion and the velocity provided by the propellant, acting along the bore axis (launcher line); thus f is obtainable from a vector diagram.

In the case of rockets, the effective launcher line gives us the effective initial direction of the trajectory. The trajectory itself, relative to its effective initial direction, is then given by the angular gravity drop and time of flight, which we now discuss.

9.2.2 The Angular Gravity Drop λ_g; the Time of Flight t_f

If the rocket had its full velocity at the start, it would follow approximately a parabolic path, and the angular drop λ_g due to gravity from the effective launcher direction to the rocket would be approximately proportional to the time the rocket had traveled to any given point, and hence to the range r to that point: $\lambda_g = B_1 r$. However, the slow speed of

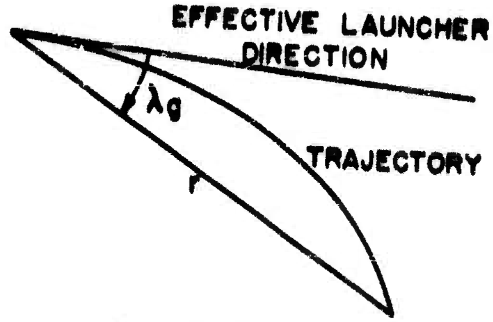

FIGURE 2. Gravity drop.

the rocket in the first part of its trajectory lets it fall more per unit distance; it also points down more, and hence picks up speed in a somewhat downward direction. The net result is that the drop λ_g is a certain amount B_0 more than the above amount $B_1 r$. Both constants B_0 and B_1 are approximately proportional to the cosine of the dive angle γ from the horizontal to the effective launcher direction in the vertical plane. Accordingly, we may write

$$\lambda_g = (A_0 + A_1 r) \cos \gamma \text{ in mils,} \qquad (2)$$

as a good approximation, where A_0 and A_1 are constants. For the 5-in. HVAR, A_0 is about 15 mils, and A_1 about 10 mils per thousand yards; they depend on the plane's airspeed and the propellant temperature.

Because of the slow speed at the start of burning, the time of flight t_f, at fairly short ranges, is not proportional to the range but is nearer a constant plus a multiple of the range. At longer ranges, when the speed has died down again, the time of flight is approximately proportional to the range. A quadratic function of range with coefficients depending slightly on own speed gives t_f to within 0.1 sec.[18]

Numerical values of gravity drop λ_g, angle of fall, and time of flight t_f are given in the "tables."[8] At $r = 3{,}000$ yd, for the 5-in. HVAR rocket, λ_g may be as much as 100 mils; t_f, as much as 7 sec. Formulas for these quantities in terms of range to target at impact, dive angle (omitted in the case of time of flight), airspeed of launching airplane and propellant temperature are available.[18]

9.2.3 Average and Instantaneous Projectile Speeds

The *average speed* $\bar{V}$ is obtained by dividing the range r by the time of flight t_f. Some values of $\bar{V}$ for the 5-in. HVAR, at an own speed of 300 knots and a propellant temperature of 70 F, are as follows.

r	500	1,000	1,500	2,000	3,000	4,000 yd
$\bar{V}$	435	500	518	510	508	400 yd per sec

Thus 500 yd per sec is a good round value. At lower temperatures, it takes longer for the rocket to burn and hence pick up its maximum speed; hence the flight times are longer, and the average speeds are less.

A formula giving roughly the variation of $\bar{V}$ with the plane's airspeed v_{kn} in knots or v in yards per second is:

$$\bar{V} = 500 + 0.45(v_{kn} - 300) = 500 + 0.8(v - 169).$$

The variation with propellant temperature is not large, and not worth taking account of, considering the inaccuracy already present.

The *instantaneous projectile speed*, for instance at $v = 300$ knots, $T = 70$ F, is 600 yd per sec at 500 yd range, 503 yd per sec at 2,000 yd range, and 400 yd per sec at 4,000 yd range.

9.2.4 Dispersion

The dispersion of actual rockets about their average position (all being fired under the same conditions) is only a few mils. The radius of the 50 per cent cone may be of the order of 7 mils.

9.3 AERODYNAMIC ASPECTS

It will be assumed that the launching aircraft is a fighter plane. Unless otherwise specified units will be yards, seconds, and radians. One radian is equal to one thousand mils.

9.3.1 Skid, Angle of Attack, Dive Angle, Other Angles

Let $\mathbf{v}^*$ be the vector velocity relative to the air mass of the aircraft at an instant. We break $\mathbf{v}^*$ into

two components: $\mathbf{v}$ *in the plane of symmetry of the aircraft and* $\mathbf{v}_s$ *perpendicular thereto:*

$$\mathbf{v}^* = \mathbf{v} + \mathbf{v}_s.$$

The vector $\mathbf{v}_s$ is the *skid*. We use the term true airspeed for the magnitude v of the vector $\mathbf{v}$. (Since skid is small, v is sensibly equal to v^*, the magnitude of the vector $\mathbf{v}^*$.) The skid speed is v_s, the magnitude of the vector $\mathbf{v}_s$. Since v_s is small compared to v, the angular amount of skid is v_s/v radians.

In the plane of symmetry of the aircraft there is a direction in which the rockets are held prior to their launching. This direction is known as the *launcher line* or *datum line*. (In general, the datum line is merely a line of reference in the plane of symmetry of the aircraft. We assume for simplicity that the launcher line, datum line, and thrust axis are the same. If these lines differ appreciably, the necessary adjustments in our analysis can and should be made.)

The *angle of attack* α is the angle from the datum line to the aircraft's velocity $\mathbf{v}$ in its plane of symmetry. We adopt the convention in our diagrams that an angle in the clockwise direction is positive. Thus the angles α_0, α and $f\alpha$ in Figure 3 are negative and the other indicated angles are positive.

Consider the plane of symmetry of the aircraft, and suppose that it is vertical. Let δ be the angle from the horizontal to the datum line. If there were no gravity acting, the aircraft (in steady flight) would move approximately along a hypothetical line in its plane of symmetry; this line is called the *zero lift line*. The zero lift line is at some angle α_0 from the datum line; being above the datum line in the figure, α_0 is negative. Due to the influence of gravity, the flight direction is at some angle $\alpha - \alpha_0$ below the zero lift line. The lifting power of the wing is approximately proportional to the square of the airspeed; it is also proportional to $\alpha - \alpha_0$ up to a certain point. Hence $\alpha - \alpha_0$ is inversely proportional to the square of the airspeed. The appropriate speed here is the effective airspeed v_i depending on the density of the air, rather than true airspeed, v. The airspeed meter feels v_i rather than v, so v_i is called the *indicated airspeed*. In steady horizontal flight, we may write, therefore,

$$\alpha - \alpha_0 = \frac{b}{v_i^2}g, \tag{3}$$

where g, the acceleration due to gravity, is 10.7 yd per sec² and b is a constant of the dimension of distance. For combat planes, b is commonly between 80 and 85 yd.

For altitudes h less than 11,000 ft, the relation between indicated airspeed v_i and true airspeed v is

$$v_i/v = e^{-1.48\times10^{-5}h},$$

where h is measured in feet. The indicated airspeed is less than the true because the air pressure is reduced by altitude.

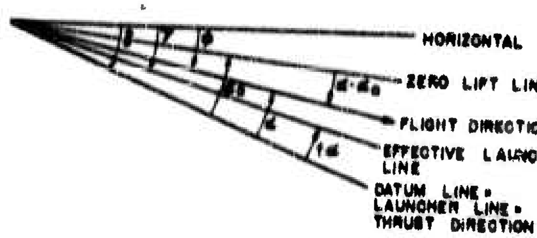

FIGURE 3. Angles associated with rocket trajectory.

The *flight angle* ϕ is the angle from the horizontal to the line of flight. The *dive angle* γ of the rocket is the angle from the horizontal to the effective launcher line.

In a steady climb or dive, only the component $g\cos\delta$ of gravity need be considered. (Perhaps some angle δ' different from δ should be used here; it is difficult to determine exactly what angle fits best, but the final results would probably not differ by very much.) If the airplane is nosing up or down, there is centripetal acceleration a_N in the direction normal to the flight path to be taken into account. If R is the radius of curvature of the flight path, and $\dot\phi = d\phi/dt$ is the time rate of turn of the flight path, then, since $\dot\phi = v/R$,

$$a_N = \frac{v^2}{R} = v\dot\phi. \tag{4}$$

Thus, in general, g in equation (3) should be replaced by $g\cos\delta - a_N$. Further, since the lift required on the airplane is proportional to its weight W, b is proportional to W. Hence equation (3) now becomes

$$\alpha = \alpha_0 + \frac{b}{v_i^2}(g\cos\delta - v\dot\phi), \quad b = b'W, \tag{5}$$

where b' is a constant. This relation is the *attack angle formula*, the attack angle α being the angle from the datum line to the flight line.

Observe that we are really interested in the effective attack angle of the rocket, the angle from the datum line to the effective launcher line of the rocket. We take this to be $f\alpha$, where α is given by equation (5) *with constants* α_0 *and* b *properly determined for the particular plane and rocket* by experimental firings.

Typical values for a fighter plane are $\alpha_0 = -40$ mils and $b = 75$ yd.

We may rewrite equation (5) by using the relation $\alpha = \phi - \delta$ which is clear from Figure 3. Then we obtain

$$\frac{bv}{v_t^2}\dot{\phi} + \phi = \delta + \bar{\alpha}\,, \tag{6}$$

where

$$\bar{\alpha} = \alpha_0 + \frac{bg\cos\delta}{v_t^2}\,. \tag{7}$$

The time constant

$$T = \frac{bv}{v_t^2}\,\text{sec}$$

of equation (6) is of the order of 0.5 sec, since $b \sim 75$ yd and $v \sim v_t \sim 150$ yd per sec.

9.3.2 Response of Flight Angle ϕ to a New Fixed Direction of Datum Line

Suppose that the pilot pulls the nose of the aircraft up or down until a fixed sight is on the target and then holds it there. He will then fly so as to keep δ constant after a certain instant. In this case equation (6) implies that the flight path tends toward the final direction

$$\phi = \delta + \alpha$$

and moves six-tenths of the way from its initial to final direction in T sec, since $e^{-1} = 0.4$ approximately.

If the plane is banked, the attack angle formula equation (5) may be applied in the plane of symmetry of the aircraft, except that $g\cos\delta$ must be replaced by $g\cos\delta\cos\beta$, if β is the angle of bank (measured about the thrust axis).

Now consider motion in the lateral plane, first assuming the plane is not banked but is perhaps skidding. If the flight line is not in the plane of symmetry, there will be a cross-wind force, tending to bring it back. If ϕ and δ now denote angles in the lateral plane from a fixed direction to the flight direction and to the datum line, respectively, it may be shown that

$$T'\dot{\phi} + \phi = \delta, \tag{8}$$

the time constant T' in this case being, at least for motion in a horizontal plane without banking,

$$T' = \frac{Wv/g}{\text{Thr} + 0.2\rho v^2 S}\,. \tag{9}$$

Here Thr is the thrust, ρ the air density, and S the wing area. For the P-47D, for example, at 5,000 ft altitude, we may take $W = 14{,}600$ lb, $S = 300$ ft^2, Thr $= 1{,}830$ lb, $v = 345$ mph $= 300$ knots $= 507$ ft per sec, $\rho = 0.0021$ slugs per ft^3; these give, using $g = 32.2$ ft per sec^2,

$$T' = \frac{230{,}000}{1{,}830 + 32{,}400} = 6.72\ \text{sec}.$$

Thus the flight direction approaches the thrust direction very slowly; it would take 7 sec for it to go six-tenths of the way from any initial direction, assuming the thrust direction held fixed. Stated in other words, *any skid the plane may have disappears very slowly*, unless it is eliminated by changing the bank to bring the flight path around.

The lateral motion of the aircraft when there is bank but no skid has been studied[16] but will not be considered here.

9.3.3 Tracking with a Fixed Sight

The problem of tracking may be looked at from two points of view: (1) How well can the pilot keep the sight line on the target? (2) How well can he keep the flight direction (or better, the effective launcher direction) where required? If a perfect computing sight were used, solving (1) would solve (2). At the present time, sights do not correct for skid, though they may correct for most of the attack angle; it is therefore necessary for the pilot, as far as possible, to avoid skidding. In many cases, wind and target motion are not corrected for; the pilot may make partial correction by pointing his sight at some other point than the target. This problem is not one of tracking. A computing sight may have a sight line which moves relative to the airplane in a manner dependent on various inputs, in particular, on the manner in which the plane is being flown. The problem of tracking can then only be studied when this sight line motion is known. Here, we shall consider only the problem of holding a fixed sight on the target, and discuss briefly the effect on the flight path.

Suppose first that the fixed sight is approximately along the thrust axis. Consider what motion of the sight line will result from various manipulations of the airplane controls. Moving the stick back or forward will raise or lower the nose, thus raising or lowering the sight line; thus the pilot has easy control over up and down motions. Similarly, use of the rudder will move the sight line to right or left. However, this will not result in a coordinated turn, but will introduce skid. If the pilot sees that the target is to the right of the sight line, for instance, the proper way of bringing the sight line over — and the only

way of bringing the flight direction into agreement with the sight line — is to bank, using the rudder to keep the ball centered, that is, keep the total force in the plane of symmetry. Then the lift vector, instead of being vertically up, will have a component to the right, and the airplane will start turning to the right, at the proper rate.

Tracking in azimuth is thus a more complicated procedure than tracking in elevation. However, for combat pilots the operations needed in controlling the plane are almost automatic. Recalling the results of the preceding section, we may sum up as follows. A pilot can track a ground target fairly well. If the sight line is held close to the target for a couple of seconds, the flight direction, in elevation, will differ from the sight line by not very much more than the steady state attack angle. If the airplane has been flown very carefully so that no serious skid has entered, the flight direction is close to the sight direction in azimuth.

On a calm day, the sight line can be held on a target to within 3 to 5 mils. On a rough day, or under bad conditions (for instance, in too steep a dive or at too great airspeed), the sight line may waver 10 to 20 mils from the target. Whether the error introduced by skid is likely to exceed these figures is not yet known, but seems very probable.

Suppose next that the pilot is tracking with a fixed depressed sight. If, for example, there is a computer holding the sight line in the plane of symmetry below the datum line by the required amount of gravity drop for a rocket, the sight feels like a fixed depressed sight as far as tracking is concerned. The discussion above applies to the present case, with one important change: If the pilot wishes to move the sight line to the right, for example, and hence banks to the right, the immediate result is to carry the sight line in the wrong direction, i.e., to the left. For the airplane tends to roll approximately about the thrust direction, and the sight line, being below this direction, moves in a direction opposite to the direction of banking. The magnitude of this effect has been studied.[10] The time t_R that it takes for the sight line to get back to its starting point is indicated by the table given below.[18] The assumed conditions are v = 100 yd per sec, T = 70 F, dive angle zero and 5-in. HVAR rocket.

r	t_R (approximate values)
1000 yd	0.47 sec
2000 yd	0.70 sec
3000 yd	1.07 sec

There are two things which alter these values somewhat. First, the proper lead from the datum or thrust line to the sight line differs from λ_g by $f\alpha$, α being the attack angle; α is generally negative, which would reduce the values of t_R above. Next, if a firing course is being followed (i.e. a course such that a rocket released at any instant will hit the target), the airplane's course is curved downward, which has the effect of reducing the lift L; this would increase the values of t_R somewhat. At any rate, it takes a good fraction of a second for the sight line to come back to where it started from and get going in the right direction, which means that tracking cannot be easy in azimuth.

The above discussion assumed that the turn was perfectly coordinated to avoid skid, and that the plane was banked suddenly. It should be clear in any case that the pilot's problem of bringing the sight line where he wishes it and at the same time of avoiding skid is far from easy.

When the airplane is banked to the right by the amount β, the lead λ in elevation becomes a lead $\lambda \cos \beta$ in elevation and $\lambda \sin \beta$ in azimuth. This helps take care of azimuth target motion.[18]

9.3 AIMING PROBLEM

The *lead* is the displacement from the datum line to that line of sight which is most likely to achieve a hit on the target. The lead is compounded of four elements:

1. The displacement from the datum line to the effective launcher direction, $f\alpha$.
2. The gravity drop λ_g.
3. The parallax correction.
4. The kinematic lead, that is, the displacement which is necessary to compensate for the target motion relative to the air mass.

We shall consider now the lead in the plane of symmetry of the aircraft.

9.3.1 Lead in Vertical Plane

We suppose that the launching aircraft is flying without bank; and we consider the lead in the plane of symmetry of the aircraft, which is vertical.

We have already discussed in Section 9.2.1 the contribution $f\alpha$ to the lead which is due to the displacement from the datum line to the effective launcher line. We have also discussed in Section 9.2.2 the contribution λ_g to the lead due to gravity drop.

Strictly speaking the gravity drop λ_g should be computed for the range R to the position of the target at impact rather than the range r to the position of the target at firing (see Section 9.4.3 below).

The parallax correction is of the order of

$$\frac{2}{r}\text{ radians} = \frac{2000}{r}\text{ mils},$$

where r is the range to the target in yards. This is due to the fact that the sight is about 2 yd above the rocket launcher, so that the sight line must be pointed down an extra angle $2/r$ radians, as is clear

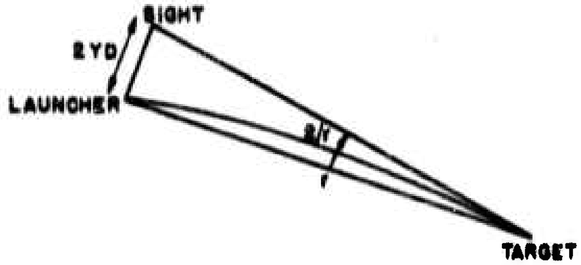

FIGURE 4. Parallax correction.

from Figure 4. Thus in the case of no wind or target motion the formula for total lead λ is

$$\lambda = \lambda_g + f\alpha + \frac{2}{r}.$$

When wind or target motion is present, the kinematic lead λ_K must be added to this expression to obtain the total lead λ.

9.4.2 Kinematic Lead in Vertical Plane

We shall suppose that the air mass containing the trajectory of the rocket moves with a constant velocity. We consider all velocities relative to this air mass rather than relative to the earth. The reason for this is that the behavior of the rocket is determined by the air mass. What the earth is doing relative to the air mass or the target or the launching aircraft does not enter into the problem.

We denote the target velocity by v_c. (If the target happens to be fixed relative to the earth, v_c is the wind velocity, oppositely directed. In all cases v_c is to be the target velocity relative to the air mass.)

We are considering a vertical plane and we assume that the target velocity v_c is in the horizontal direction in that plane. Let σ be the angle from the horizontal to the line of sight at the firing instant. Setting aside the launcher line, gravity drop, and parallax effects, the kinematic lead λ_K can be obtained by

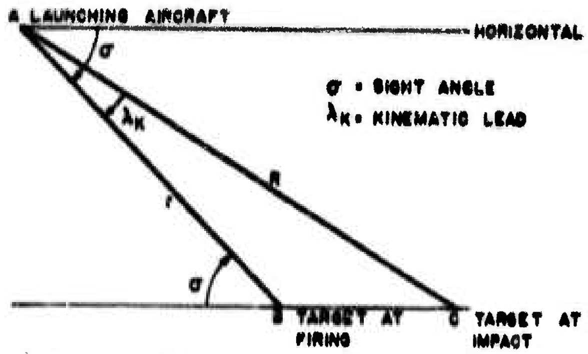

FIGURE 5. Figure for computing kinematic lead.

applying the law of sines to the triangle ABC of Figure 5. Thus

$$\frac{\sin\lambda_K}{\sin\sigma} = \frac{BC}{AC} = \frac{v_c}{V},$$

where V is the average speed of the rocket. Since λ_K is small, we replace $\sin\lambda_K$ by λ_K and obtain the expression

$$\lambda_K = \frac{v_c}{V}\sin\sigma \tag{10}$$

for the kinematic lead. Here v_c is positive if the target is moving away from the launching aircraft.

The formula for λ_K may also be obtained by considering the target motion across the line of sight; its rate is $v_c \sin\sigma$. The lead must be such that the rocket's average rate across the line of sight $V\sin\lambda_K$ equals that of the target.

Thus the total lead (that is, the angle from the datum line to the correct line of sight) is

$$\lambda = f\alpha + \lambda_g + \frac{2}{r} + \frac{v_c}{V}\sin\sigma \quad \text{radians}. \tag{11}$$

9.4.3 Approximation for Range R to Impact Point

The range R to the impact point is given approximately by the formula

$$R \approx r\left(1+\frac{v_c}{V}\cos\sigma\right). \tag{12}$$

This can be seen from Figure 5 by dropping a perpendicular from B onto the side AC or by expressing the length of the side r by the law of cosines, applying the binomial expansion to the resulting expres-

sion, neglecting terms of order higher than v_t/V and replacing $\cos(\sigma - \lambda_K)$ by $\cos \sigma$.

9.4.4 Kinematic Lead in More General Situations

A discussion of the kinematic lead in azimuth is available.[17] It is shown that, at least in certain cases, the sight line need only be moved in azimuth through 80 or 90 per cent of the angle that the flight path must be moved through, because of the effect of bank. It should be noted that the indeterminacy of skid, which affects azimuth errors with any existing type of sight, makes it seem undesirable to add much complexity to a computer for the sake of that dimension.

Kinematic lead in the plane of symmetry has also been studied, taking into account kinematic effects on the launcher line term $f\alpha$ and the gravity drop λ_g. It has been shown that under normal circumstances the simplified treatment of kinematic lead that we have given in Section 9.4.2 is sound.

9.5 DESIGN OF SIGHTING MECHANISM; GENERAL CONSIDERATIONS

The problem to be considered is the following.

A computing sight is desired which is simple to build, install, and maintain, is as easy for the pilot to use as possible, does not restrict his tactics, takes a minimum of time to use, and gets the rocket as near the target as possible under all conditions.

It is important to get as full and clear a picture of the entire problem as possible. If there is too much concentration on one aspect, one is likely to discover that a mechanism has been built which will do one part of the job beautifully, but fails badly on another part. For example, gravity drop being the most obvious quantity to be taken account of, and the simplest one to study theoretically, there is a great tendency to propose sights which correct for gravity drop but do nothing about other quantities, such as target motion. One cannot hope to solve the problem completely in any simple manner. The more fully one tries to achieve the aims of accuracy and tactical freedom, the more complicated the apparatus is likely to become, not to mention the probable increase in weight and space taken. One should probably strive for some kind of compromise, sacrificing some points that seem of less importance to keep the mechanism within reasonable bounds.

The essential difficulty of the problem is apparent from the following consideration. A much used form of attack is to fly in toward the target at tree top level, remaining hidden as long as possible; at the last instant, one rises up, puts the sight on the target, fires, and pulls away. One cannot hope to get an accurate determination of range in such an attack; hence, unless one fires from quite close range (which, as a matter of fact, is usually done), one cannot expect an accurate value of the gravity drop.

Another fundamental difficulty is that caused by the tendency of an airplane to skid. It seems fairly certain that skidding of the order of 20 mils is hard to avoid. The seriousness of the resulting error has been discussed in Sections 9.2.1 and 9.3.2. If one is doing lead computing with a gyro sight, there is likely to be some correction for skid; perhaps a third of the error will be removed.

9.5.1 Possibility of Simple Computers

It has been common practice either to aim rockets by eye, or to strive to attain a standard set of conditions (of dive angle, airspeed and range), and fire, using a fixed lead set in beforehand. It must be realized that with long training and practice, pilots can become fairly accurate in their firing. A good tennis player can return a ball fairly closely to where he wants it; it would be quite difficult to construct a computer that would move to the right position and hit the ball in the right way to make it do the same thing.

If a simple computer is designed to do part of the job and let the pilot do the rest, it must be designed so as not to upset the pilot's normal reactions to the situation; for example, it must not replace a train of habitual thought by a set of mental numerical calculations.

Suppose first no computer is used. Pilots can be trained to dive at a given dive angle without very large errors; it is more difficult for a pilot to decide when he is at a given range. Probably distances at the target can be estimated much better than the range to the target.

The simplest kind of a computer that would be a real help would probably use altitude above the target (from altimeter and hand set target altitude), dive angle (from an accelerometer, to be used while a fixed sight is held on the target[18]), and indicated airspeed to compute gravity drop and attack angle, and feed this in some manner into the sight line.

If there is a gyro lead computing sight in the aircraft (for guns), it may be used with a large constant time-of-flight setting to give a good average value of gravity drop and attack angle (for a not too large range of dive angle), and to take some account of wind and target motion (see Section 9.6.4 below).

9.5.2 Comparative Importance of Different Components of Lead

We now consider what must be done to obtain a fairly complete solution of the rocket sighting problem. We shall study the problem mainly in the vertical plane, since the principal difficulties (except for skid) are encountered there.

The formula for the total required lead is equation (11). For present purposes, we may omit the small parallax term $2/r$. Thus we consider:

$$\lambda = \lambda_g + f\alpha + \frac{v_r \sin \sigma}{V}. \qquad (13)$$

The angle λ is the angle from the datum line of the plane measured downward to the sight line. Thus λ is positive when the sight line is below the datum line. (Writing the formula this way does not mean that the datum line has to be used as the reference direction. In terms of another direction, constant or variable relative to the aircraft, the required formula may be deduced from equation (13).)

A rough estimate of the sizes of the three terms is important in determining how careful one must be in taking care of each of them.

The *gravity drop* term λ_g is roughly 10 mils, plus 15 to 20 mils for each thousand yards of horizontal distance to the target. (See Section 9.2.2 and Figure 6 below.) Thus if one wishes to fire from any horizontal distance ranging from 3,000 yd in (with the 5-in. HVAR), λ_g will vary roughly from 65 to 20 mils, a variation of 45 mils.

The *attack angle* term $f\alpha$ depends primarily on the airspeed and dive angle. Suppose the likely ranges of airspeed are from 240 knots = 135 yd per sec to 380 knots = 214 yd per sec, and of dive angle, from 20° to 60°. Taking the F6F aircraft, with $h = 71.3$ yd and $\alpha = -0.043$ radians, and the 5-in. HVAR, propellant temperature 70 F, we find from equation (5) (setting $\phi = 0$), and equation (1) (or from the "tables" [8])

$\alpha = -3.6$ mils, $f = 0.82$, at 240 knots, 20° dive,
$\alpha = -34.6$ mils, $f = 0.94$, at 380 knots, 60° dive.

Thus $f\alpha$ varies about from -3 to -34 mils, a 31 mil variation.

The *kinematic lead* $v_r \sin \sigma / V$ may be very large for fast targets on windy days, and negligible for fixed targets on very calm days. Let us take a 30-mph target motion relative to the earth, with an equal wind in the opposite direction, as a typical value of v_r; then $v_r = 60$ mph $= 29.3$ yd per sec. If $\sigma = 60°$, $v_r \sin \sigma = 25.4$ yd per sec. Taking $V = 500$ yd per sec (see Section 9.2.3) gives

$$\frac{v_r \sin \sigma}{V} = 51 \text{ mils}.$$

Since the kinematic lead may be in any direction, we have a possible variation of 100 mils in its value. Thus *on windy days, with moving targets, the kinematic lead can be the most important item to be taken care of; on calm days, with targets at rest, it is relatively unimportant.*

9.5.3 Graphs of Lead

To help the reader's feeling for the dependence of lead on range and dive angle, we give some graphs of lead as required on a *firing course*, that is, a course at each instant of which the rocket may effectively be launched. In each graph, contour lines of constant lead (in mils) are plotted on a polar diagram in which the radius vector is the range r and the polar angle is the sight angle σ. Thus the graph coordinates are similar to actual conditions. All graphs are for the 5-in. HVAR, propellant temperature 70 F, and for plane characteristics $\alpha_0 = -0.043$, $h = 71.3$, as for the F6F aircraft.

Figure 6 shows the dependence of $\lambda_g + 2/r$ on range and dive angle, other quantities being fixed. It shows how, to a reasonable approximation, the required lead is a function of horizontal range $x = r \cos \sigma$, if x is not small. At shortest ranges, the parallax correction $2/r$ has a considerable effect in curving the contour lines.

If the computer is to compute gravity drop, parallax, and launcher line term, at a given airspeed, it must produce the leads indicated in Figure 7. The fact that angle of attack α depends on dive angle has the effect of tipping the contours considerably.

Figure 8 would not be used in a computer containing an airspeed input. The most likely airspeeds during a dive will be higher for steeper dive angles; this will decrease the lead required in the steeper dives. Typical indicated airspeeds are 300 knots for a dive

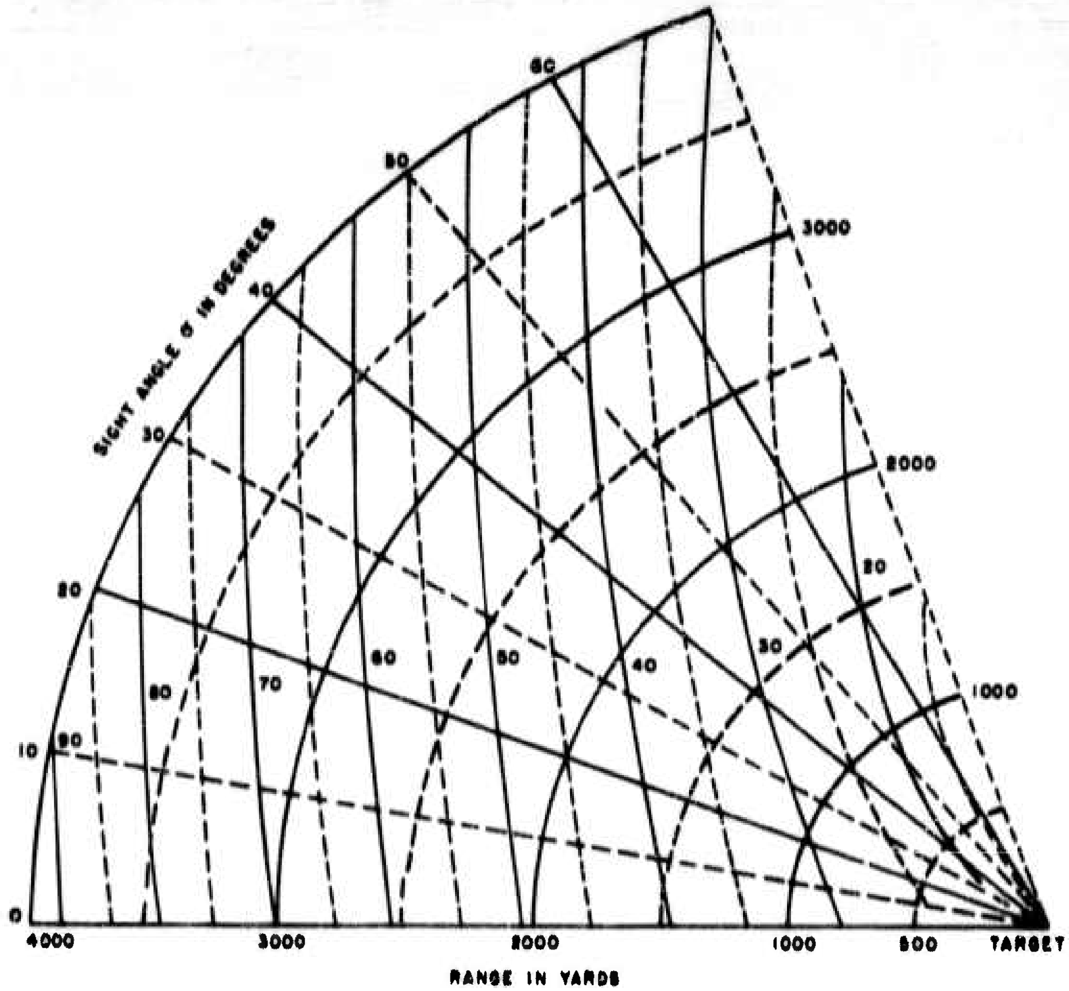

FIGURE 6. Gravity drop and parallax (fixed airspeed). Firing course. Contours of $\lambda g + 2/r$ in mils. 5.0" HVAR. $v = 300$ knots, $T = 70$ F.

angle $\delta = 30°$, and 375 knots for $\delta = 60°$. In the graph, we have taken

$$v_s = 225 + 2.5\delta,$$

where δ is measured in degrees and v_s in knots. Accordingly the graph shows appropriate values of λ at each range and dive angle.

9.5.4 Computing Kinematic Lead

The pilot's ideal would be to put the pip on the target, then fire at once, or wait as long as he likes before firing. In other words, he would like: (a) an instantaneous solution of the problem, so that the moment the sight line is on the target, the correct lead is obtained, and (b) a continuous solution, so that the lead continues to be correct as long as he holds the sight line on the target — he is following a firing course.

The first thing to be noted is that an instantaneous solution of the problem, including kinematic lead, is impossible, if information from outside sources about wind and target motion is not used. This fact is simply demonstrated by the following example. Suppose the pilot flies out of a cloud, and finds he is aiming at a tank on a bridge. He fires at once, hoping to destroy both tank and bridge. By the time the projectile arrives on the ground, the tank and bridge are some distance apart; the rocket cannot be expected to blow them both up.

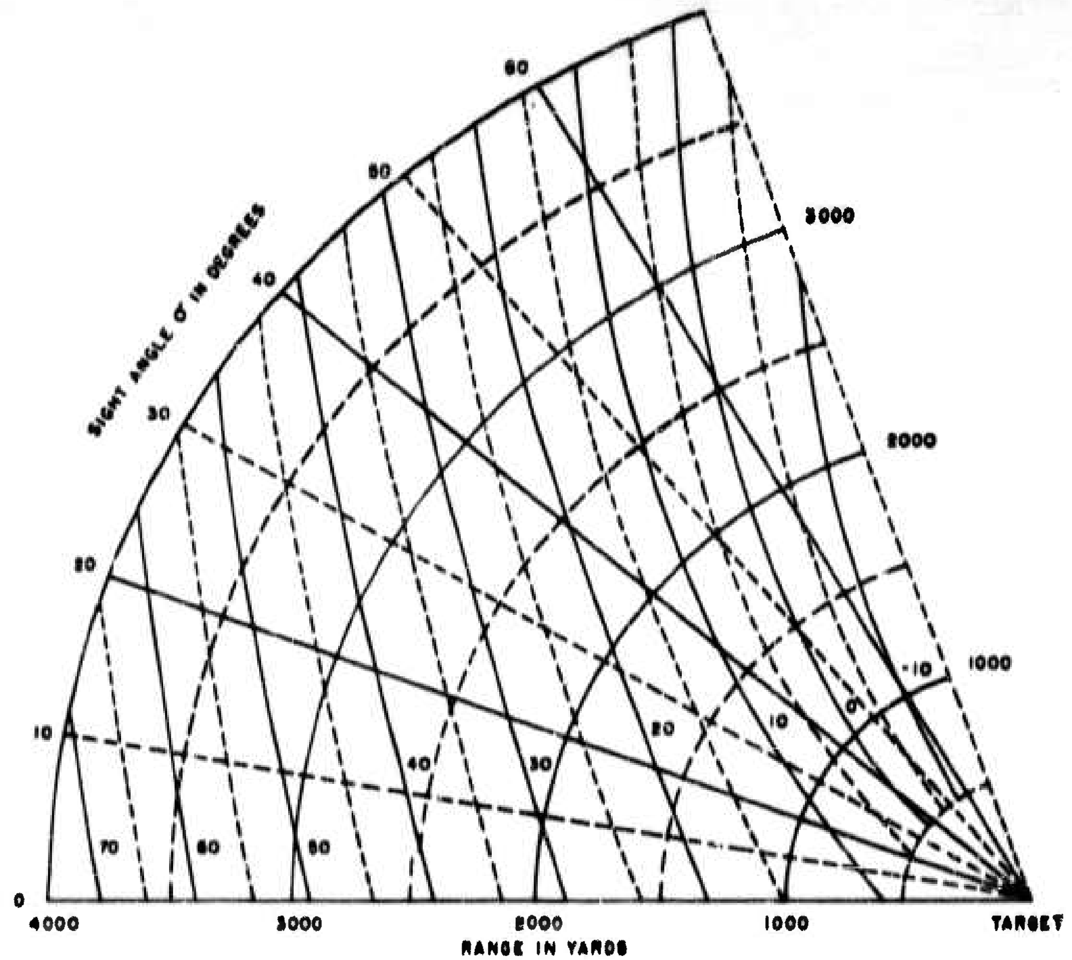

FIGURE 7. Total lead (no wind or target motion), fixed indicated airspeed. Firing course. Contours of λ in mils. $\lambda = \lambda_g + f_a + 2/r$. 5.0" HVAR, F6F, v_i = 300 knots, T = 70 F.

The mere fact that kinematic lead is due to changing conditions — rate of change of position of the target relative to the air mass surrounding the plane — shows that it can be determined only by noting the change over a period of time. If it is assumed that the pilot's only contact with the target is visual, and use of this information is made by holding a sight line on the target, it is seen that the sight must be on the target during a definite period of time if kinematic lead is to be determined.

Since a sight line cannot be held perfectly on the target, the rate of motion of the sight line at a given moment is apt to be quite different from the angular rate of the target direction at that moment. Hence the sight line will have to be held on the target long enough so that some kind of average sight rate will be near the actual target rate.

Let us say the kinematic lead is to be determined within 5 mils, and tracking errors of the order of 3 mils may be expected. We wish to find out what lower limit this puts on the tracking time. Suppose, for instance, that the target rate is determined by comparing the target direction at two instants t_1 and t_2. If the sight line is, say, 3 mils off in one direction at the start and 2 mils off in the other direction at the end, there is an error of 5 mils over the time interval $\Delta t = t_2 - t_1$, making an error of

$$\Delta \dot{\sigma}_k = \frac{0.005}{\Delta t} \text{ radians per second}$$

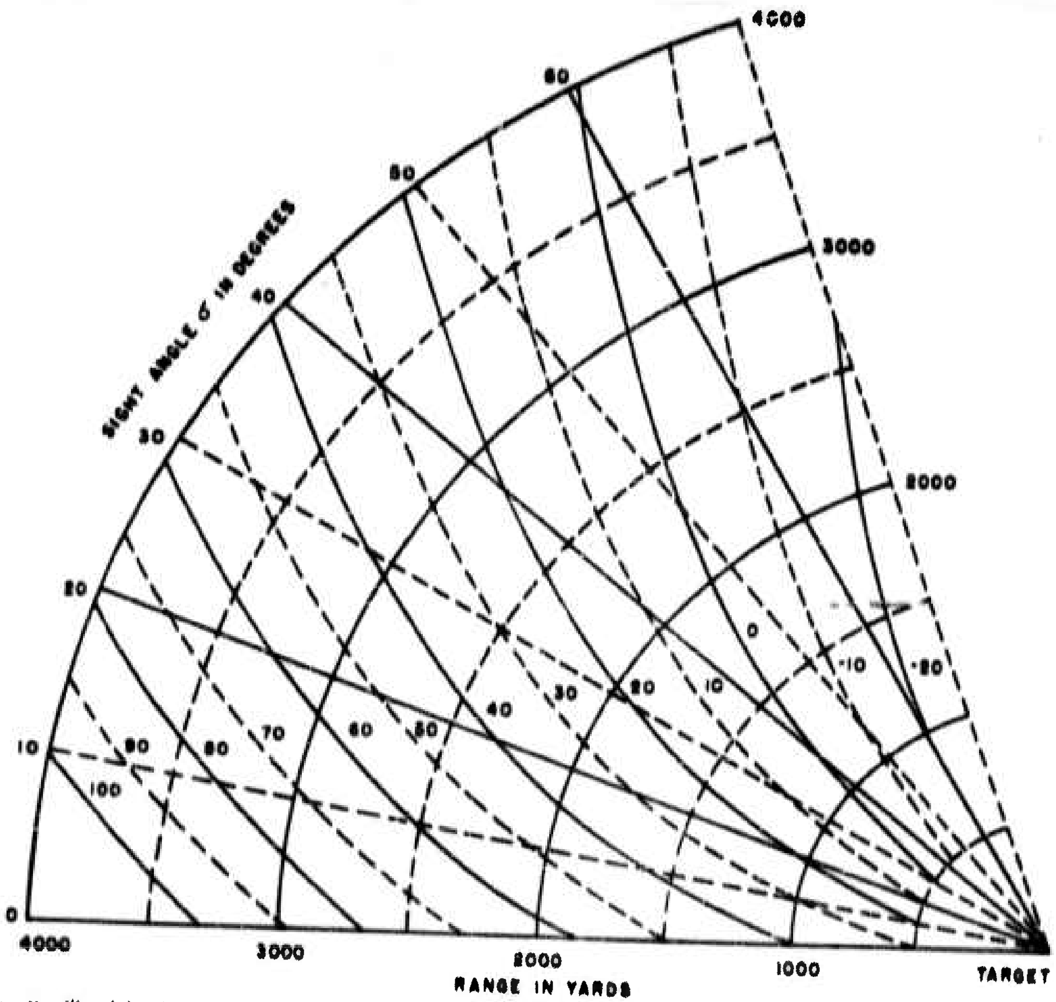

FIGURE 8. Total lead, variable airspeed (no wind or target motion). Firing course — variable velocity. Contours of λ in mils. $\lambda = \lambda g + f\alpha + 2/v$. 5.0" HVAR, FGF. v, variable, $T = 70$ F.

in the rate $\dot{\sigma}_k$ of the sight line, which is attributed to kinematic lead. Now the sight rate $\dot{\sigma}_k$ due to wind and target motion, and the kinematic lead λ_k, are given by

$$\dot{\sigma}_k = \frac{v_t \sin \sigma}{r}, \qquad \lambda_k = \frac{v_t \sin \sigma}{V} = \frac{r}{V}\dot{\sigma}_k;$$

hence the error $\Delta\dot{\sigma}_k$ in $\dot{\sigma}_k$ causes an error $\Delta\lambda_k$ in kinematic lead given by

$$\Delta\lambda_k = \frac{r}{V}\Delta\dot{\sigma}_k = \frac{r}{500}\,\frac{0.005}{\Delta t} = \frac{r}{1000}\,\frac{0.01}{\Delta t}.$$

(This assumes a fixed sight is being held on the target. The figures would not be very different if the sight line were being moved to take account of kinematic lead.) Hence, if we wish $\Delta\lambda_k < 0.005$, we must have

$$\Delta t = \frac{0.01}{\Delta\lambda_k}\,\frac{r}{1000} > 2\,\frac{r}{1000}.$$

Thus at 2,000 yd range, we must take at least 4 sec for tracking. *Note that this is entirely independent of what sort of computer is used.*

The above was based on the assumption that the computer started functioning at the instant t_0. Since the computer cannot know at what instant the sight line has settled on the target, this requires an operation on the part of the pilot (say the pressing of a button) to start the computation of kinematic lead. Of course, there might be some automatic way of

starting the computation at approximately the right moment; for example, it might start functioning as soon as the angular rate has gotten below a certain value.

Probably a lagged value of the angular rate [19] will be used rather than the angular change over the time taken. Then momentary oscillations of the sight line may have a less harmful effect; but also there may be difficulty with the transient term. If the computing is started too soon, the angular rate due to the sight line getting onto the target will give a false picture of kinematic lead unless tracking is continued long enough for this to die out. In any case, it is unlikely that the kinematic lead can be calculated better than to about 5 or more mils in 4 sec if the range is over 2,000 yd.

9.5.5 Sighting Procedures

We study here the relation between what the computer needs in order to compute the lead and what the pilot must do to enable the computer to work correctly. We saw in the last section that the sight must be held on the target for some seconds if the kinematic lead is to be computed. To obtain the gravity drop within 5 mils, one must know what is essentially horizontal range (see Figure 6) to within about 300 yd. The computed value of this variable may or may not be instantaneous. The computation of attack angle will probably be done as quickly as that of gravity drop.

Thus it is likely that the pilot's procedure will include holding a sight line on the target over a period of time, say several seconds. We now raise the question: What is the relation of this period of time to the moment of firing the rocket? If a firing course is to be followed, as in (b) of the last section, then the moment of firing lies anywhere during the tracking time, after the first few seconds. This suggests that the computation of lead should be taking place at the same time that this lead is being employed to bring the plane into the correct position. Hence, if the motion of the plane is itself being employed in the computation of lead, there is a circular process: computation of lead affects motion of the plane which affects computation of lead. This may lead to tracking instability, or at least a slowing down of the computation (see the next section).

In the design of a computer, such slowing down must be avoided as far as possible. The problem may be avoided by not letting the computation process affect the sight line, for example, by computing the proper lead while a fixed sight is held on the target. In this case either (a) one must arrive at the correct firing position at some moment during the process, at which moment the pilot must be told to fire or firing is automatic, or else (b) the pilot must bring the plane into firing position after the computation of lead is accomplished.

We give some examples of how (a) or (b) above may be accomplished. Suppose, at the start of tracking, the fixed sight has a lead which is somewhat less than that required for firing at the moment. As the pilot approaches the target, the correct lead diminishes; the computer is working out this lead; at the moment it decides that the desired lead equals that which the sight has, it fires the round automatically.

There are clearly some disadvantages in this procedure. The starting conditions must be arranged in some fashion so that the fixed lead will come about where the pilot expects; either he must be able to arrange to have the firing position (and hence the amount of the fixed lead) about where he desires it, or he must be able to know he is starting beyond his firing position. Having once started, he may not know how long he will have to wait, and he may begin to fear that he started too close, unless the computer tells him otherwise in some fashion.

These difficulties might be somewhat alleviated by letting the lead be variable, for example, letting it start at zero or a negative quantity, and gradually increase. Then the pilot will be sure that the proper lead will be reached before long. Moreover, it will take longer, and thus the computer will have more time to function properly, at the longer ranges (where more lead is required), which is desirable. (See Section 9.6.1.)

In case (b), one would like the procedure of bringing the plane into position and firing to be automatic. This can be accomplished by *tossing*,[18,20] which has the added advantage of combining the maneuver with the process of getting away. The procedure in brief is as follows. When the pilot has tracked for some seconds, he presses a button, which stops the computation of lead and starts the toss mechanism functioning. The pilot now pulls the nose of the plane up. The angle $\Delta\phi$, through which the flight path must turn, having been determined, the instrument now measures the normal acceleration $v\dot{\phi}$ (or perhaps $g\cos\delta - v\dot{\phi}$), integrates this to obtain $v\Delta\phi$, and releases the rocket when $\Delta\phi$ has reached the desired value.[21–23,25–27]

9.5.6 Operating Stability

The above discussion should be sufficient to show that *the manner of computing lead is tied up with the pilot's operating procedure*, and *the theory of the computer must go along with the operating procedure.*

Thus when the design of the mechanism has been determined, it is necessary to make a careful analysis of the way the mechanism will perform under actual conditions. In particular, any unevenness in the inputs, due to a variety of causes such as bumpy air, must be considered.

In this section, we shall study only the likely effect of imperfect tracking on the computation of lead and the resulting tendency to instability. Thus we consider here only effects resulting from false angular rates; if the mechanism operates independently of angular rates, there will be no tracking instability. (Another example of likely instability is that due to the use of an accelerometer.[18] If the instrument is very sensitive to accelerations and acts on the sight line gently, the accelerations due to bumpy air might cause the sight line to waver so that it cannot be held on the target.)

Consider the computation of kinematic lead, presumably based on the angular rate of the plane, or at least of the sight line. To see the effect of the false angular rate produced by getting on the target, let us take a numerical example. Suppose, in order to watch the target area, the pilot starts by diving 5° too steeply. Let us say he takes 1.5 sec to pull up the 5° to get on target. The average angular rate is then (5°/57.3°)/1.5 radians per sec = 58 mils per sec. If the range is 2,500 yd, this gives a hypothetical target velocity

$$v_t \sin\sigma = r\dot{\lambda}_k = 145 \text{ yd per sec}$$

(see Section 9.5.1), for which the proper kinematic lead would be in the neighborhood of

$$\lambda_k = \frac{v_t \sin\sigma}{500} = 290 \text{ mils},$$

which is far beyond reality. If this lead were actually fed into the sight line, the pilot would have to pull up 290 mils = 16.5°. If he did so, and stopped there, the computer would now decide that no lead was required, and the pilot would start pulling the nose down again, putting in a false angular rate in the opposite direction; clearly the situation is unstable.

If the pilot were nearly on the target, and started pulling up the little bit necessary to get on, and if the computer worked rapidly, it would at once decide that considerable lead was required and make him pull up a lot more. The same instability is present.

The usual way around this difficulty is to have the computer use a lagged value of the angular rate to compute kinematic lead; see Section 9.6.3. By proper choice of the constants, the process is rendered perfectly stable; the primary remaining difficulty is the slowness of the computation. A limit on the speed of computation was derived in Section 9.5.4; making the tracking process stable may slow the computation further.

The above discussion illustrates some possible causes of instability; the whole problem has to be studied for any given computer.

9.5.7 General Summary: The Search for a Satisfactory Computer

The designer of an aircraft rocket sight is confronted with enormous difficulties. In brief:

1. The corrections that are to be made for the various quantities are difficult to determine, particularly for a fighter plane with no operator other than the pilot.

2. Assuming certain corrections are to be made, it is very hard for the pilot or a mechanism to know whether they are being made properly, primarily since the direction in which a fin stabilized rocket starts out is approximately the flight direction relative to the surrounding air mass, an elusive variable.

3. Solving problems of such difficulty is likely to involve complicated mechanisms and complicated procedures; but it is essential that at least the operating procedure be kept as simple as possible.

How should one attack the problem of designing a sight? One may attempt something fairly standard in principle, or one may try something which is essentially new. An inventor who brings to the problem a deep insight or a really novel approach cannot be greatly helped by advice. If he is a pilot, he may best make himself thoroughly familiar with the problem by firing rockets himself, before he formulates his ideas. But when ideas for a solution are once formulated, then a study of the most important considerations is in order. It is at this and later stages of the problem that the work of AMP may prove of future service. It should help find omissions in the theory, help relate the theory to known theories, and help remind the designer what things to watch for.

The fundamental question that must be kept in mind about a gadget is, *how will it work when actually put to use?* This must be studied thoroughly back in the design stage, particularly in the present problem, where a pilot, with his life in danger, has to think of a myriad of things in a few seconds if he is to accomplish his mission and come back alive.

9.6 THE DESIGN OF A SIGHTING MECHANISM

The quantities that appear in the lead formula (13) are determined by the values of certain other quantities, as is implied by the formulas that we have written. The following table gives an indication of this dependence.

We have not listed angle of attack α as an input. While it is true that it may become possible to measure the *aircraft's* angle of attack directly (by differential pressure methods, for example), such a measurement would appear to offer no advantage over a computation of the *rocket's* angle of attack from formula (5), with $\dot{\phi}$ taken as zero. Observe that it is the *rocket's* angle of attack that is pertinent. Properly calibrating a measuring device appears as difficult as determining the correct constants for formula (5).

A discussion has been given [11] of the potentialities of certain devices for measuring altitude, indicated airspeed, time intervals, radar range, attitude of launching aircraft (from a gyro), angular rates (from gyros), angles of attack and skid, linear accelerations,

Function	Variables on which it depends critically	Variables on which it depends less critically
λ_g, gravity drop.	R, range from launching point to position of target at impact.	γ, angle of depression of effective launcher line. v, own speed. T, propellant temperature.
R	r, range from launching point to position of target at firing instant.	σ, angle of depression of sight line. v_t/V, ratio of enemy speed to average rocket speed.
f, launching factor.		v_0, indicated own speed T
v_0	v	h, altitude.
v	v_0	h
α, angle of attack.	v_0 δ, angle of depression of datum line.	v $\dot{\phi}$, rate of turn of flight path in vertical plane.
$v_t \sin \sigma$, target velocity across line of sight in the vertical plane.		
V average rocket speed.	v	

This table refers only to one aspect of the aiming problem: the component of lead in the plane of symmetry of the launching aircraft, assuming no skid or bank. (If bank β were present, it would be necessary to replace g by $g \cos \beta$ in the formula for α and to replace λ_g by $\lambda_g \cos \beta$.) The lead in the wing plane of the aircraft should be considered; it depends critically on the skid and bank.

We see from the table that a computer could produce the lead λ in the plane of symmetry if it had estimates of

$$r,\ v_t \sin \sigma,\ \delta,\ v_0;$$
$$T,\ h \text{ or } v,\ \sigma,\ \dot{\phi}.$$

(In determining λ_g, the angle γ may be replaced by δ.) The fact that these variables are sufficient does not imply that a machine must use them explicitly.

angular accelerations, weight of aircraft, and propellant temperature. There is a discussion also, for example, of the determination of range from altitude difference and dive angle (δ, γ, ϕ, or σ), dive angle from normal acceleration, the correction of an accelerometer for attack angle, dive angle from a forward accelerometer, dive angle from the rate of decrease of altitude, range from angular rates, attack angle from airspeed and dive angle.

9.6.1 Estimates of Situation by Pilot — Firing Projectiles for Information

The pilot, while flying, may make various estimates of variables affecting the sighting problem; he

may put his estimates into a computer, or he may try to fly so that these variables take on prescribed values. Examples of such variables are: dive angle, range to the target, expected airspeed, magnitude and direction of wind and target motion. Contrivances may be used to help him in his estimates. For example, if the target is of known dimensions, he may span the target with a reticle of variable size; the reticle size then determines the range. If the pilot flies level and at right angles to the direction to the target, and tips his wing over so that it lies just below the target, he may take his angle of bank as the dive angle to the target (if he goes at once into the dive), and read this angle from the gyro horizon. Or again, if he flies level without banking until the target is visible just ahead of some point of the wing, this point determines the dive angle to the target.

Firing cannon or machine guns at the target, or along a line running up to the target, furnishes the pilot with information which can be helpful in aiming rockets. Or again, watching where the first rockets land, the pilot may attempt to correct his aim with later rockets. In this regard, it should be noted that the plane will be nearly half way to the target before rockets with a corrected aim may be fired, and the conditions (range, wind) may then be considerably different from what they were before; moreover, the pilot will not know what the cause of the miss was, and the required correction depends on the cause of the miss.

By a proper approach the pilot may be able to make the azimuth lead small or zero.

If the target is fixed relative to the earth and if the wind velocity is known, the target velocity is then known.

9.6.2 Rate of Rotation $\dot{\sigma}$ of Sight Line

We continue to study the problem in a vertical plane. The angular rate of rotation

$$\omega = \dot{\sigma} = \frac{d\sigma}{dt}$$

of the sight line is very likely to be one of the quantities involved in future sights. It is clear from Figure 9 that

$$\omega = \dot{\sigma} = \frac{v \sin(\sigma - \phi) - v_t \sin \sigma}{r},$$

the first term being the contribution of own speed, the second, that of target motion. Now

$$\sigma - \phi = \lambda - \alpha,$$

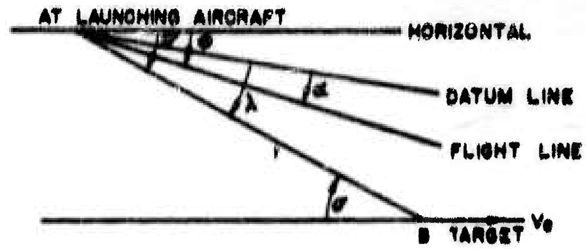

FIGURE 9. Figure for computing lead in terms of angular rate of rotation of sight line.

(see Figure 9). This is a small angle; we replace its sine by its radian measure. Thus

$$r\dot{\sigma} = v(\lambda - \alpha) - v_t \sin \sigma. \tag{14}$$

This relation may be used with equation (13) to eliminate $v_t \sin \sigma$, r or α. Eliminating $v_t \sin \sigma$ gives:

$$\lambda = \frac{fV - v}{V - v}\alpha + \frac{V}{V - v}\lambda_0 - \frac{r}{V - v}\dot{\sigma}. \tag{15}$$

We put

$$h = \frac{fV - v}{V - v}. \tag{16}$$

This quantity is near unity, since

$$1 - h = 1 - \frac{fV - v}{V - v} = \frac{1 - f}{1 - \dfrac{v}{V}},$$

which is near zero. We write

$$V_R = V - v; \tag{17}$$

thus V_R is the average speed of the rocket relative to its launching aircraft.

Thus equation (15) becomes

$$\lambda = h\alpha + \frac{V}{V_R}\lambda_0 - \frac{r}{V_R}\dot{\sigma}. \tag{18}$$

The coefficient h is given by equation (16) and is close to unity.

9.6.3 Computing Sight which Predicts Full Lead in Vertical Plane

The mechanism to be described is of the same sort as the Army Draper-Davis sight and the Navy PUSS,[4] when either is used for rockets.

We shall suppose that range r will be measured by a radar mechanism and will be available for the computation. With estimates or approximate measurements of dive angle, propellant temperature, and

own speed, the knowledge of range r permits a satisfactory mechanization to produce the gravity drop λ_g.

The angle of attack α can be computed mechanically from a measurement or estimate of indicated own speed and dive angle. The speeds V and V_R are determined by own speed v in a linear relationship. Thus, in principle, all the quantities in equation (18), except $\dot{\sigma}$, can be considered accessible. However, a mechanism for computing all the quantities that we have here discussed according to the equations given would be overcomplicated. In practice, average values would be used instead of certain variables, and engineering compromises would be appropriate.[26]

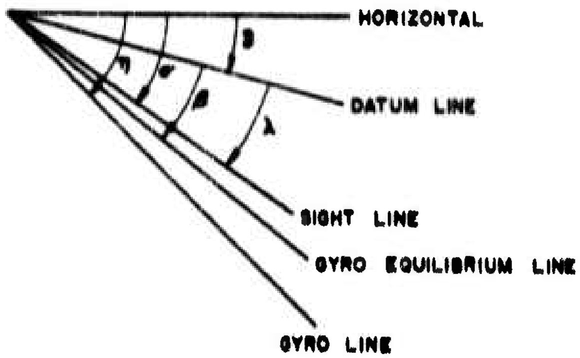

FIGURE 10. Position of gyro.

To measure $\dot{\sigma}$, a gyro could be mounted with axis (always in the plane of symmetry of the aircraft) at an angle η below the horizontal. As in the usual rate sight, the gyro, sight line, and datum line are coupled so that

$$\eta = \sigma + e\lambda = (1 + e)\sigma - e\delta, \qquad (19)$$

where e is a positive constant (often called $-a$). The gyro is made to precess toward an equilibrium position which is at an angle β below the datum line, at a rate proportional to the angular displacement:

$$\dot{\eta} = k(\delta + \beta - \eta) = k(\delta + \beta - \sigma - e\lambda) = k(\beta - (e + 1)\lambda), \qquad (20)$$

where k is a positive constant. Now direct substitution shows that equation (20) reduces to equation (18), if k and β have the following values:

$$k = \frac{V_R}{(e + 1)r}, \quad \beta = (e + 1)\left[\frac{V}{V_R}\lambda_g + h\alpha + e\frac{r}{V_R}\dot{\lambda}\right]. \qquad (21)$$

All the quantities used in defining k and β may be considered known, except $\dot{\lambda}$. One may use an anticipated average value of $\dot{\lambda}$. If one puts $\dot{\lambda} = 0$ in the expression for β, the gyro equation (20) reduces to

$$ea\dot{\lambda} + \lambda = h\alpha + \frac{V}{V_R}\lambda_g - \frac{r}{V_R}\dot{\sigma}, \qquad (22)$$

where

$$a = \frac{r}{V_R} = \frac{1}{k(e + 1)} \qquad (23)$$

is the *sensitivity* of the gyro system.

The differential equation (22) implies that the mechanization has the following properties:

1. $\dot{\lambda}$ at each instant is such as to make λ change in the direction which brings λ closer to the right hand member (which is the true lead), the rate of change increasing with the discrepancy. This can be seen by solving equation (22) for $\dot{\lambda}$ and observing that ea is positive. Thus the solution will be a lagged value of the correct lead (equation (23)).
2. The transient is eliminated after about $2ea$ sec. In the Draper-Davis sight, $e = 0.2$; in PUSS, $e = 0.25$.
3. If the sight line is subject to disturbances, the resulting disturbance in λ will not be unduly large compared to the disturbance in σ even though the disturbance in $\dot{\sigma}$ be large (as it may, indeed, be).[30]

The optimum choice of the functions a and β has been discussed [26] as well as the errors which can be expected from such choices of a and β, under ideal conditions.

9.6.4 Sights Whose Prediction Neglects Kinematic Lead in Whole or in Part

In this section we suppose that the target motion relative to the air mass is neglected. In the case of targets fixed relative to the earth, this implies that wind is neglected.

When v_t vanishes the equation (14) for sight rate $\dot{\sigma}$ becomes

$$\dot{\sigma} = \frac{v}{r}(\lambda - \alpha). \qquad (24)$$

The equation (13) becomes

$$\lambda = f\alpha + \lambda_g. \qquad (25)$$

If the launching factor f were unity, these relations would imply that $\lambda - \alpha = \lambda_g$ and

$$\dot{\sigma} = \frac{v}{r}\lambda_g. \qquad (26)$$

Now equation (2) implies that

$$\lambda_g = (A_0 + A_1 \bar{r}) \cos \phi,$$

since $f = 1$ implies that $\phi = \gamma$. If we take for example $A_0 = 0.013$, $A_1 = 0.000016$, and use $\bar{r} = r/1000$, this gives

$$\dot{\alpha} = \left(0.016 + \frac{0.013}{\bar{r}}\right) v \cos \phi \quad \text{mils per sec}, \quad (27)$$

where v is in yards per second. If $v = 160$ yd per sec = 300 knots, $\phi = 40°$, $r = 2{,}000$ yd, then $\dot{\alpha} = 2.9$ mils per sec. *At medium or long ranges, and at a given dive angle*, this shows that *$\dot{\alpha}$ is nearly constant.*

This fact may be used in the following way. Use a gyro to turn a sight line at the rate shown above. Then if the sight line is held on the target, $\dot{\alpha}$ will have the above value, and hence the lead has the correct value. Note that (assuming $f = 1$) the attack angle need not be known, since it does not occur in the formula. Thus this method uses "target coordinates," and is independent of what the air mass may be doing.

This method is useful if bombs are being dropped, in which case $f = 1$ and $A_0 = 0$; but in the case of rockets, the method results in a large overcorrection of kinematic lead, since $\dot{\alpha}$ will be appreciably incorrect when target motion is actually present. This overcorrection could be taken care of as follows. First, fly a pursuit course, holding a fixed sight on the target; measure the angular rate; then fly the course described above, but making a correction dependent on the angular rate that was found.

As a modification of the above method, one could use a coupling of the gyro to axes in the plane; instead of the equation (27)

$$\dot{\alpha} = \omega,$$

use

$$\dot{\alpha} = \omega + k\dot{\delta}. \quad (28)$$

This may be mechanized by a slight alteration of the usual rate sight. The same essential features are present as in the former case, but tracking will be easier.

One may make a sort of compromise between the above method and that of Section 9.6.3 by using the rate sight of Section 9.6.3 with a fixed sensitivity (to take care of kinematic lead at an average range), and with a fixed position of equilibrium of the gyro (to give an average gravity drop). This actually is much more successful than would appear at first sight, for the following reason. At long ranges, the lead is too small; at some intermediate range, the correct lead will have decreased to the lead actually used. As the range decreases further, the angular rate due to the velocity of the plane, given by equation (26) if $v_t = 0$, $f = 1$, increases since r is decreasing. The gyro lags behind, making the sight lag also (relative to the plane), thus decreasing the lead actually obtained. Thus the correct lead and the lead obtained are both decreasing. The actual curve of lead obtained may be made to fit the curve of desired gravity drop without very large errors, say if the range is less than 2,500 yd. At short ranges, the mechanism is apt to overcompute greatly. The dive angle must be taken care of separately.

The British method of using the Mark 23 sight is precisely of the sort just described. It has been studied in detail.[28] Optimum settings and errors under ideal conditions are discussed.

9.6.5 Peanut as a Rocket Sight

The sights considered thus far have been designed for firing courses, courses during which the rocket could be released at any instant, after the computer had settled down. The sight *Peanut* is of a different type: it automatically releases the rocket when a fixed lead, previously set into the sight, is the correct lead, in the absence of target motion. This sight has been discussed in detail.[29] Peanut is based on the equation

$$\dot{\alpha} = \frac{H_1 v(\lambda - \alpha)}{\lambda - f\alpha - H_0} \quad (29)$$

obtained by writing

$$\lambda_g = H_0 + H_1 r$$

and eliminating λ_g and r from equations (24) and (25). The quantities H_0 and H_1 are proportional to $\cos \gamma$. [See equation (2).] A fixed λ is put into the mechanism, which then releases the rocket when a smoothed value of $\dot{\alpha}$ assumes the value given by equation (29).

In the case of bombs, $f = 1$ and $H_0 = 0$, so that equation (29) reduces to

$$\dot{\alpha} = H_1 v. \quad (30)$$

A sight like Peanut is open to the objection that the pilot cannot fire when he feels it appropriate to fire. He must wait for the release and he may be unable to judge when the release will come. In a strong head wind the release may be delayed dangerously.

PART III

ANTIAIRCRAFT ANALYSIS

Chapter 10

STUDIES OF ANTIAIRCRAFT EQUIPMENT

10.1 INTRODUCTORY REMARKS

Division 7 of the NDRC, in charge of research and development in the whole field of fire control, was fortunate in having, both on its own technical staff and as employees of its contractors, a considerable number of individuals who were highly expert and experienced in the analytical aspects of fire control. Thus it is natural and proper that the well-balanced and broad program on antiaircraft equipment, including the more mathematical aspects, was carried out by Division 7, and will be found reported in their technical papers.

The Applied Mathematics Panel was, nevertheless, asked from time to time to do various jobs of analysis referring to antiaircraft equipment. The present chapter reports the AMP activities in this field. And since the AMP activities were of a rather assorted and catch-as-catch-can nature, it has seemed desirable to preface the chapter with some brief general comments on antiaircraft problems and equipment, in the hope that these general remarks will furnish a background against which the separate — and necessarily somewhat disconnected — sections of this chapter will be more understandable.

As far as the words themselves are concerned, the phrase "antiaircraft fire" should include all fire directed against aircraft. Military usage, however, seems to restrict this phrase to fire directed against aircraft from guns located on ground or ship, the phrase "plane-to-plane fire" being more common for fire directed from one aircraft against another.

The present chapter follows accepted usage, and thus refers to ground-to-plane or ship-to-plane antiaircraft fire. The phrase "antiaircraft fire control" is often interpreted as including various weapons, such as rockets and guided missiles, in addition to the orthodox guns of various calibers. The present chapter, however, refers only to antiaircraft *guns*. It is customary to divide antiaircraft guns into two main classes. *Automatic weapons* include machine guns of various calibers which utilize belted ammunition, and certain larger guns such as the 20-mm, and the 40-mm Bofors which automatically fire clips of shells. Automatic weapons normally fire solid slugs, or projectiles with contact fuzes, but they do not employ projectiles with time fuzes, nor does it seem likely that their dimensions would ever permit the use of proximity fuzes. *Major caliber antiaircraft* guns (sometimes called *heavy antiaircraft*) include the various larger sized guns which load single shells of calibers 3 in., 5 in., 90 mm, etc., these projectiles normally being time-fuzed.

An antiaircraft fire control system involves the following

1. Target data gathering.
2. Transmission of these data to predicting mechanism.[a]
3. Prediction; or estimation or calculation of gun orders (i.e. position of gun and fuze setting if this is involved).
4. Transmission of these gun orders to gun.
5. Orientation of gun (and setting of fuze, if involved) in accordance with these gun orders, and firing of gun.

In the case of a strafed infantry man, firing at a plane with his rifle, the target data are collected by his eyes, transmitted by his nervous system to his brain, which estimates the proper position for his gun. This estimate is again transmitted by the nervous system to muscles which orient the gun and press the trigger. The elements of a complete fire control

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[a] In the older terminology of the Army, the predicting mechanism, usually containing the mechanisms for obtaining angular target data but not target range, was known as a *director*. In the Navy, the word *director* usually refers to the target data mechanism, and the predictor is called a *computer*. In more recent Army equipment, the target data device is called a *tracker* and the predictor a *computer* (as in the Navy). To avoid this confusion of terms, we will use the general, and hardly misinterpretable, term *predictor* for the device which accepts target data and predicts the future position of the target.

system are all present, but the system is unmechanized and wholly personal in character.

In the intermediate case of a 40-mm gun, the target data are obtained by two men who, one working in elevation and one in azimuth, "track" the target by keeping the cross hairs of telescopes aligned on it. These tracking telescopes are an integral part of the predicting mechanism, so that transmission of target data presents no problem. A mechanical (and electrical) device accepts these target data, and continuously computes what position the gun should have in order that projectiles fired from it meet the target in space. These gun orders are transmitted by an electrical data transmission (selsyn) system to the hydraulic or electric servos which, obeying the signals they receive, automatically keep the gun properly oriented. The tracking and predicting mechanism is normally some distance from the gun, to avoid shock, smoke, etc., so that transmission of gun orders from prediction to gun constitutes a real (although satisfactorily solved) problem.

In a still more completely mechanized system, the target data might be obtained from a radar set which follows the target automatically, once its beam is placed on the target, and which makes continuously available the angular azimuth and elevation of the target and its slant range in yards. These data are then transmitted by a selsyn or similar system to the predicting mechanism which computes the gun orders. The gun orders are transmitted electrically to the servos, hydraulic or electric, which position the gun and which set the fuzes. Such a system is almost wholly mechanized and automatic, the human element normally entering only at the initial moment of choosing the target, and at the final moment of firing the gun. In the most completely developed systems, even the loading of the gun is largely or wholly mechanized.

The testing or evaluation of an antiaircraft fire control system involves a large number of considerations, some of which prove to be involved and difficult. The principal points appear to be as follows:

I. The determination of the errors in components of the system, or in the total system, *when the inputs are without error.*

The component in question might be a single amplifying circuit in the radar range equipment, or a slide multiplier in the predictor, or the fuze timing mechanism; or a larger component such as the whole predictor, or the gun servo.

An important part of this error (namely that part which would remain if all mechanization were perfect), can often be analyzed and computed mathematically, whether or not the physical equipment exists. Many studies of this sort were carried out by Division 7, and some by AMP.

But actual test of physical equipment is also essential, for even with perfect inputs, real equipment fails to perform exactly in accordance with the calculated theory, there being certain inevitable errors due to imperfection of parts, friction, spring of parts, wear, uncompensated temperature effects, etc.

Experiment is essential in the determination of these errors for another reason — namely that certain phenomena are not amenable (at least at this stage of affairs) to reliable predictive analysis. Thus the fragmentation characteristics of a certain shell must be determined experimentally. (See Chapter 11.)

II. The determination of *input errors*, and the determination of the *effect of these input errors on the output performance* of components of a system, or of the whole system.

At this point is introduced the vitally important matter of the alertness, accuracy, and stamina of the human operating personnel, as these factors are influenced by selection and training, and by operating conditions. The design of the system should take intimate account of the ability and limitation of the personnel. For example, the smoothing of data which accompanies the prediction process must be based directly on a knowledge of the fluctuations due to tracking errors, and an increase in theoretical precision of a device may be more than cancelled out by increased difficulty of operation. This is a field in which Division 7 has done much pioneer and fundamental work.

It is clearly important that all such tests be carried out under as realistic conditions as possible, that the testing be quantitative and objective, and that the tests be adequately planned and reliably interpreted by use of statistical techniques. Here again Division 7 has set new standards, notably through the development of dynamic testers for prediction, the development of great testing engines for aircraft fire control equipment, and the development of extensive and specialized computing equipment. The AMP's contributions have been minor and incidental.

III. The determination of the overall effectiveness of the fire control system.

The ultimate test of an antiaircraft defense is its ability to shoot down enemy planes. Thus it is natural

to attempt to evaluate, for any given system, some overall index such as "probable rounds per bird," assuming a certain type of target and a certain course of target relative to gun.

Such calculations involve, first of all, a knowledge of the pattern of error of the actual burst points in the sky relative to the desired burst location. This knowledge comes primarily from the studies and tests outlined above under I and II. It is describable in terms of the error in the mean point of impact of many bursts, this error in the mean point of impact normally changing its value as the target moves along its course. In addition to this slowly changing bias in the location of burst points, there is a random error of the individual bursts with respect to the mean point of impact, and under certain circumstances a correlation of greater or lesser importance between the position of individual succeeding shots. In the case of automatic weapons firing in bursts, it is sometimes convenient to speak of bias errors which are steady during the burst, but whose value would fluctuate from burst to burst, and might change systematically from point to point on the target course.

These studies of effectiveness must also deal with the whole complicated business of terminal ballistics — the size and weight distributions of shell fragments, the angular distribution of fragments, the velocity distribution of fragments, and the fall-off law for the fragment velocities as this depends on altitude. The vulnerability of the target plane must also be carefully considered, studying the probable effect of single or multiple hits of fragments of various characteristics on all the aircraft components. The orientation of the target is involved, as well as the relative velocity of target and shell.

It is also important to question how the effectiveness of the antiaircraft fire control system is affected if the target plane follows a course (curved flight or evasive action) which was not contemplated in the design of the predicting mechanism.

And "trial fire" procedures, intended to furnish optimum empirical values for certain correction parameters (such as ballistic wind, muzzle velocity, air density), have an important influence on overall effectiveness.

Relative to the items which come under this heading, AMP carried out a considerable amount of work which will be briefly summarized or referred to in this chapter and in Chapter 11. In this chapter, Section 10.1 is introductory. Sections 10.2 and 10.3 refer to dry run errors in tests of fire control systems without shooting. Section 10.4 is a theoretical comparison of a linear and a quadratic predictor. Section 10.5 gives estimates of the contributing errors of a particular fire control system. Section 10.6 discusses two nonlinear variants of a conventional linear prediction circuit. Section 10.7 discusses trial fire procedures in the case of a projectile with preset fuze. Sections 10.8, 10.9, and 10.10 are brief discussions of special topics involving antiaircraft equipment with which AMP was concerned. In Chapter 11, AMP's analytic studies of fragmentation and damage are reported, together with certain pioneering British and Section T papers.

10.1.1 Fire Control Systems

Consider a target aircraft in motion along its course. Suppose that a shot is to be fired at the instant at which the target is at a point T_1 of its course, called the *present position*. If the motion of the target in the instants following the firing instant is considered as known, there is then determined the *future position* T_2 on the course, namely, that point on the course at which the projectile should be aimed in order that the shot be most likely to hit the target. Thus the pointing of the gun (and the fuze setting, in the case of a preset fuze) should be such as would produce a hit at the future position T_2, according to the ballistic tables.

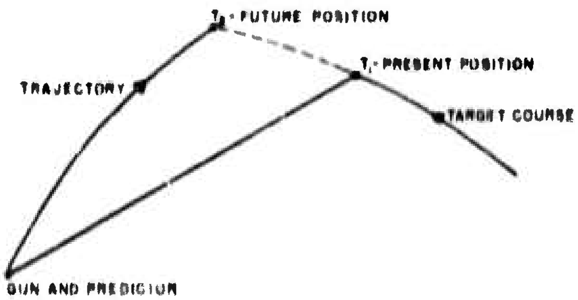


FIGURE 1. Position of gun and target.

The *predictor* is that part of the fire control system which determines the gun settings which are appropriate for each shot. It is clear that the motion of the target in an interval preceding the firing instant must influence the solution produced by the predictor, for such prior motion of the target is the only available source of information about the motion of the target during the time of flight of the projectile.

It is also clear that the solution produced by the predictor cannot be correct in all cases, since the target may move in different ways during the time of flight of the projectile, and, in certain cases, will be able to avoid damage by the projectile directed at a future position T_2 appropriate for a course different from the one actually flown after T_1.

Thus the operation of the predictor includes an extrapolation of the past motion of the target into the future.

The way in which the motion of the target prior to the instant of prediction is fed into the predictor is as follows. Range to the target at each instant is measured by radar or optically. The angular direction to the target at each instant is established by moving a line (the axis of a telescope) in the predictor so as to point at the target continuously. This process is known as *tracking*. Thus, except for the ranging and tracking errors, the present position of the target (relative to the predictor platform) is an input to the predictor at each instant.

In addition to taking account of the motion of the target, the predictor must make use of ballistic information of the sort contained in the firing tables for the gun and projectile. In particular, the appropriate pointing of the gun will not usually be toward the future position; it will be such as to achieve a hit on the future position.

In the case of heavy antiaircraft, the input to the predictor will include the ballistic wind, the ballistic temperature, the ballistic air density and an estimate of the muzzle velocity, and perhaps other settings.

The mechanism which transmits the angular gun orders (that is, the desired gun settings which refer to the pointing of the gun barrel) from the predictor to the gun operates so rapidly that one may consider the transmission as effectively instantaneous. The calculation of the angular gun orders by the predictor is therefore based on the presumption that the gun is fired at the very instant of prediction. In the case of a preset fuse, however, the fuse must be set before the shell is placed in the breech of the gun and fired. The interval of time between the instant at which the predictor computes the fuse setting and the instant at which the projectile is fired is the *fuse dead time*. It must be known in advance and be fed into the predictor.

Considering the entire fire control system, one may say that its input consists of the motions of the target and the gun and predictor platform, the tracking input, the ranging input, a number of settings related to wind, air density, temperature, muzzle velocity and the like, and, in the case of a preset fuse, the fuse dead time.

The output of the fire control system is the sequence of gun settings effected by the gun and the resultant trajectories. The gun settings are the angular gun settings which determine the direction of the rifle of the gun, and in the case of a preset fuze, the fuze setting.

The behavior of a particular projectile is not determined exclusively by the gun settings at its firing; there is superposed on these gun settings a dispersion or random variation: The bursts or trajectories, obtained from a series of shots at precisely the same gun settings, will be distributed about their average position (which is given in the ballistic table). In the case of proximity and contact fuzes, the detonation of the projectile is determined in part by the target itself.

A general (although incomplete) discussion of the character of antiaircraft weapons, heavy antiaircraft predictors, fire control equipment for light antiaircraft, and antiaircraft radar equipment as well as the reported operating characteristics of a number of specific weapons is given in a memorandum.[6]

10.1.2 Time Base of Predictor

Because the prediction is in part an extrapolation of the prior motion of the target as observed, the effect of errors of observation such as ranging and tracking errors would be serious if the observations were not averaged over a sufficiently long time interval.

To illustrate this point, consider a predictor which extrapolates in the following linear fashion: The course of the target during the time of flight is presumed to be along the straight line joining the present position T_1 of the target with the position T_0 of the target C seconds before the present instant, where C is some constant known as the *time base* of the predictor. Suppose the target is indeed flying at a constant velocity (that is, in a straight line at a constant speed), but suppose that the input to the computing mechanism of the predictor corresponds to a wavy motion which deviates from being straight because of input and machine errors. Then the extrapolation may be grievously wrong, if the time base is small compared to the time of flight, as is indicated in Figure 2. This observation is related to the fact that although an error may be small its rate of change

may be large; and the rate of change of the observed target position affects the extrapolation to the future position.

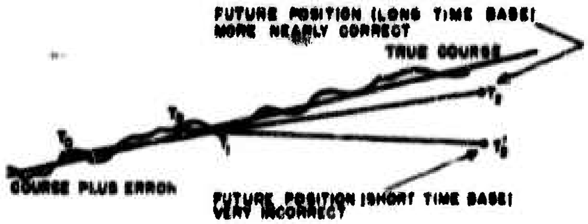


FIGURE 2. Long vs short time base.

Thus, other things being equal, a long time base improves prediction. Antiaircraft engagements are of limited duration and it is desirable to achieve effective shooting as early in the engagement as is possible. For this reason, a short time base would be more advantageous than a long one, if there were not compensating disadvantages.

10.1.3 Assessments of Fire Control Systems

Practically all the work of AMP on antiaircraft had to do with the *assessment* of fire control systems in the following sense: The AMP work was an appraisal of the effectiveness of a fire control system or of one of its parts, either by observation of performance in the field or in a laboratory, or by the study of its design.

An empirical method of assessing a fire control system is to set it up against the enemy and to count the number of planes shot down and the number of rounds fired or the number of engagements. Assessments of this sort apply specifically to the inputs which occurred in the cases observed. Accordingly, a comparison of two fire control systems on the basis of such observations should be tempered by an awareness of the extent, if any, to which the actual input tended to favor or disfavor one system against the other. For example, one system may be superior to another against targets taking evasive action. Such a superiority will not be in evidence if the enemy flies only straight courses in the observed engagements.

An assessment of a fire control system, of the sort just described, in the field against the enemy should be the last of a series of assessments carried out all during the interval from the initial design of the fire control system to its production and use in quantity. For, if these earlier assessments are not carried out, adjustments and improvements of the fire control system would be either delayed or lost altogether.

One may consider a series of types of assessment, of which the one just discussed is at the extreme, since it is an observation of the fire control system in action in which the inputs are subject to a minimum of control on the part of the observer. At the other extreme from the assessment just described is one which consists entirely of paper work. The component parts of the fire control system are described mathematically; the output is then determined by calculation, and appraised. Fire control systems are so complicated that a certain amount of idealization has been necessary in assessments of this type. Accordingly, the validity of the results depends upon the extent to which the idealizations approximate the essential elements in the situation. Experimental checks of the entire assessment or of parts of the assessment can establish the soundness of a theoretical assessment and permit one to have confidence in its conclusions.

One advantage of an assessment which is entirely theoretical is this: Initial conditions can be varied readily and the whole range of appropriate inputs can be studied. Thus the scope of the analysis is substantially broader than can be achieved by observing the fire control system in action. For example, if one wishes to consider the behavior of the fire control system under n_1 types of tracking, n_2 types of ranging, n_3 types of target motion, n_4 types of predictor, $\cdots$, one must consider in all $n_1n_2n_3n_4\cdots$ situations. The magnitude of the number of cases soon becomes prohibitive. Nonetheless, the theoretical analysis can be carried out, and a moderate number of experiments can be judiciously chosen and used as checks of the theory. Judicious and properly planned experimental investigations combined with theoretical analyses provide a powerful method of research.

Any experimental assessment should be as realistic as possible. This implies that it should involve shooting, provided the attainable motion of the target and other conditions are representative of actual conditions. As the maneuverability of tow targets is definitely limited, there will be situations in which a camera test against fast moving real airplanes, possibly flying evasive courses, will be a more realistic simulation of combat conditions than is attainable with shooting. In a camera test, everything but the

shooting may be simulated experimentally. The effect of shooting is then superposed on the experimental observations.

For the most part, the theoretical analyses of antiaircraft equipment conducted by the AMP applied to parts of the fire control system, rather than the whole.

10.1.4 Calculation of Probabilities

In any assessment, experimental or theoretical, it is desirable to calculate the probability of destroying the target during the engagement or the probability of hitting the target with one shot, or some similar probability. (The first probability is an estimate of the proportion of engagements in which the target would be destroyed. The second is an estimate of the proportion of shots which would hit the target.) These probabilities are a better measure of the performance of the fire control system than is a description of the amounts by which the hypothetical projectiles miss the target. For the probabilities are a measure of precisely that effectiveness for which the fire control system is intended. It would be proper to stop with a description of the misses only if such a description were, in effect, a description of the above probabilities. Now the process of converting the misses into probabilities involves the angular target size (which varies throughout an engagement) and the gun dispersion; the probability of hitting the target or of destroying the target is not at all proportional to the amount of the miss. The vulnerability of the target is sometimes complicated.

Thus the step from the amounts by which the hypothetical projectiles miss the target to the probabilities is essentially complex and often cannot be accomplished by an intuition based on a brief description of the amounts of the misses alone. It would therefore seem worth while to compute the probabilities whenever the gun errors are sufficiently reliable. There will be cases in which a rough computation of the probabilities will be sufficient. This matter is discussed further in Section 10.2.

10.2 ANALYSIS OF DRY RUN ERRORS

Suppose that a series of trials is carried out (in reality or perhaps only conceptually) in which all the elements of the fire control system are involved with one exception: There is no shooting; instead a record of the gun settings is taken, photographically or otherwise. Each trial is a simulation of an engagement of the fire control system against an enemy plane. Each trial will be called a *dry run*.

The *dry run error* at any firing instant may be defined as the displacement from the target to the average position of a projectile fired at the firing instant with the gun settings that were in effect in the dry run, the displacement being measured at the instant at which a correctly aimed projectile with zero dispersion would hit the target. In the case of a proximity or contact fuze the displacement is measured perpendicular to the average trajectory (relative to the target); in the case of a preset fuze it is measured as a vector in 3-dimensional space. An indication of a method of calculating dry run errors, together with explicit definitions, is given in Section 10.3.

The present section will consider the analysis of records of dry run errors. The concepts and methods discussed are pertinent to antiaircraft problems (particularly those which are concerned with automatic weapons) as well as to aerial gunnery and rocketry problems. The work of AMP along the line indicated was principally (but not entirely) in connection with air-to-air gunnery.

We conclude this introduction with a remark about the planning of the dry runs. The performance of a fire control system in an actual situation is the result of a large number of component causes, some of which are:

1. The type of engagement (that is, the particular motions of the centers of mass of the target and the gun platform).
2. The yaw, pitch, and roll of the gun platform, especially important in naval engagements and in air-to-air gunnery.
3. The tracking errors.
4. The ranging errors.
5. Atmospheric conditions, accuracy of adjustment, degree of wear, the particular harmonization, and the like, of the system.

Consider an investigation whose object is to determine the average or expected behavior of the fire control system when some or all of the conditions mentioned above (1 to 5) vary. In certain investigations one may wish to restrict as many of the conditions as possible, and study the effect of the variations of the others. For example, in an experimental investigation, one could specify that the target fly a particular course relative to the gun, and that there be no yaw, pitch, and roll of the gun. In all investi-

gations, there should be an adequate representation of the different combinations of the conditions (1 to 5) which are to be varied, and, in so far as it is possible, the weights given to the different conditions that are considered should correspond to the frequency with which the conditions are expected to occur in reality.

10.2.1 Superposition of Firing Effects

The effects of firing are adjoined to the dry run errors by means of the theory of probabilities. The dry run error refers to the behavior of an average projectile; any particular projectile, if it is fired during the dry run, would deviate from the average of all projectiles that might be fired at the same instant, in accordance with the probability distribution function which describes the behavior of the set of all projectiles. The variation from the average to the individual will be referred to as the *dispersion*; this dispersion is to comprehend those effects (and only those effects) which would be present with actual shooting but which are absent in the dry runs. The dispersion is an elusive quantity to determine, but there is no reason to suppose that the approximations that are made are unsatisfactory.

Formulas will be given in Section 10.2.6 which will permit the calculation of the probability p that a particular shot at a particular instant of a dry run would destroy the hypothetical target. The formulas themselves are less important than the way in which the quantities p are used once they have been determined.

The meaning of the probability p is as follows: It is the proportion of shots which, fired under the conditions which prevailed in the dry run at the firing instant against a target which moved before and after the firing instant in precisely the way that the target did move, would have destroyed the target, supposing that many such shots were fired.

The probability p will depend on the dry run error at the instant in question, the vulnerability of the particular target to the ammunition, and the dispersion.

10.2.2 Unconditional Vulnerability

A target is said to be unconditionally vulnerable if the probability that any shot will destroy it is independent of any damage (other than complete destruction) that the target may previously have sustained. Thus, an unconditionally vulnerable target is destroyed, if it is destroyed at all, by the effects of one shot. Although actual targets are probably not unconditionally vulnerable in the strict sense with respect to any particular kind of ammunition, nevertheless, the idealization that they are unconditionally vulnerable is useful, and at least in many cases, gives results which are essentially correct.

One reason for this is as follows: The probability that two or more projectiles will produce hits on mutually supplementary parts and thereby destroy the target, when each projectile by itself would fail to destroy the target, is small compared to the probability that one projectile will destroy. The presented area of the component parts which are multiply but not singly vulnerable is small, and the probability per unit area of obtaining precisely the required set of hits is small.

Moving pictures of a target being downed by a stream of caliber 0.50 bullets or a mass of fragments of a shell may give the impression that the target is downed by the combined effect of many hits. But this is not an inescapable conclusion. There is usually a multiplicity of hits with each kill of an unconditionally vulnerable target for several reasons: (1) The effect of the lethal hit may be delayed. Consequently all hits occurring after the lethal hit but before the destruction appear to contribute to the destruction. (2) The probability that a fragment is lethal is less than the probability that a fragment will hit the target. Consequently, many non-lethal hits occur for each lethal one, on the average.

The reader is referred to Appendix I of a report [21] for a discussion of one aspect of this subject, that is, the vulnerability of a fighter aircraft to machine gun fire.

A conditionally vulnerable target is one for which there is an appreciable probability that two or more projectiles will together destroy the target, each one of itself failing to destroy. A target which cannot be destroyed but which can be damaged is the opposite of an unconditionally vulnerable target. An extreme example of the former is the City of London and of the latter a compact ammunition dump.

10.2.3 Survival Probability

Henceforth it is assumed that the analysis is directed toward a determination of the effectiveness of the fire control system against an unconditionally vulnerable target.

Suppose that shots are imagined during a dry run and spaced so as to conform to the rate of fire of the fire control system, and that $p_1, p_2, \cdots, p_n$ are the probabilities defined in Section 10.2.1, that the respective shots would, of themselves, destroy the target. Then $1 - p_1, 1 - p_2, \cdots, 1 - p_n$ are the respective probabilities that the shots will fail to destroy the target. Accordingly, the *survival probability* for the n shots is

$$s = (1 - p_1)(1 - p_2) \cdots (1 - p_n), \qquad (1)$$

since the target is unconditionally vulnerable. The *destruction probability* is

$$d = 1 - s.$$

The probabilities s and d are the proportions of engagements in which the target would survive or be destroyed, respectively, it being supposed that the entire dry run is repeated exactly many times with shooting each time. Thus it is presumed that the pilot of the target in combat would not take any special evasive action (other than that present during the dry runs) as the result of the effects of the shots early in the engagement.

The probability s as given by equation (1) is too large, if the target is conditionally vulnerable.

10.2.4 Expected Values — Measures of Performance of Fire Control System

The probabilities d, s, and p apply to a particular run or a particular instant of a particular run. For this reason they are not measures of the entire performance of the fire control system. Such measures are rather the average or expected values of d, s, and p over the set of all real runs. *We shall use capital letters to denote estimates of the expected values of the corresponding small letters.* Thus, D, S, and P are estimates of the expected values of d, s, and p, respectively.

The quantity D is an estimate of the proportion of engagements in which the target would be destroyed. It is, therefore, a sound measure of the performance of the fire control system. It takes account of the dispersion of the projectile, the vulnerability and size of the target, and the serial correlation of the dry run errors along each course. (The serial correlation is discussed in the next section.)

The quantity P is an estimate of the proportion of shots which would destroy the target. It may be a sound measure of the performance of the fire control system, even though it does not take account of the serial correlation.

Let us denote by E an estimate of the expected value of the radial dry run errors. The quantity E is a measure of the performance of the fire control system which does not take account of projectile dispersion, target vulnerability, target size, or serial correlation. It may, nonetheless, be adequate in certain cases.

Suppose that each gun of a fire control system is replaced by two guns and that this replacement does not change the dry run errors. Then the survival probability s will be replaced by its square s^2. This follows directly from formula (1). It has been suggested [30] that

$$c = -\log s$$

be a measure, the *comparison index*, of the performance of a fire control system. The comparison index c has the following properties. (1) The greater c is, the greater the probability of destroying the target. (2) Replacing each gun of the system by two guns (subject to the same dry run error) has the effect of doubling c. Properties (1) and (2) hold also for C, the expected value of c over all runs.

To compare the meaning of C with that of D, one might note that knocking down twice as many enemy planes in the same number of engagements has the effect of doubling D.

10.2.5 Serial Correlation of Dry Run Errors

The dry run errors at the firing instants of a run are *uncorrelated* (*statistically independent*) if they have the following property: The error at an instant is statistically independent of the errors at earlier instants so that a knowledge of the errors at the earlier instants would not help a person to predict the error at the later instant.

It is clear that the dry run errors are correlated since the causes of the errors at different instants are, in part, the same or related. However, if it were true that the errors were uncorrelated, the calculation of S, an estimate of the expected survival probability, would be considerably simplified. This is because the expected value of a product of statistically independent factors is the product of the expected values. Accordingly, equation (1) would imply that the expected value of s is

$$(1 - P_1) \cdots (1 - P_n),$$

when $P_1, \cdots, P_n$ are respectively the expected values of $p_1, \cdots, p_n$. The effect of serial correlation will be important to the extent to which the above product fails to approximate the expected value of s. Among other things, rate of fire will certainly affect the importance of the serial correlation.

One may illustrate the effect of the correlation of dry run errors by the following examples:

Illustration 1. Consider two fire control systems A and B. Suppose that system A is half on and half off the target during each run, and that system B is entirely on the target during half the runs and entirely off during the other half. Assume that bullet dispersion is zero and that one hit on the target would destroy it. Assume (as we do throughout) that the fire control system itself is invulnerable. Then system A will then destroy the target in all engagements, and system B will destroy the target in only half of the engagements. Nevertheless, the probability that one shot will destroy the target is the same for both systems. For these two systems,

$$P_A = P_B = \tfrac{1}{2};\quad D_A = 1;\quad D_B = \tfrac{1}{2}.$$

It has been conjectured that the contrast in Illustration 1 is partly due to the absence of dispersion, and that a comparison of systems on the basis of the quantities D and P would give essentially the same results when the dispersion is large compared to the target.

The reader may get some insight into this question by considering the following modification of Illustration 1.

Illustration 2. Consider two fire control systems A and B. Let θ be any positive number less than unity.

Suppose that system A is, on the average, squarely on the target during a θth part and badly off the target during a $(1-\theta)$th part of each run, all dry run errors being statistically independent.

Suppose that system B is either squarely on or badly off the target during the entire run, and suppose that the first possibility occurs, on the average, for a θth part of all the runs, and the second, for a $(1-\theta)$th part.

Taking into account target size, vulnerability, and dispersion, let s be the probability that a single shot will destroy the target when the error is zero. Suppose that n shots are fired, and that the angular target size is taken as constant for all n shots.

Then the two systems have equal expected single shot probabilities,

$$P = \theta s.$$

However, the expected destruction probabilities for the two systems are (omitting details),

$$D_A = 1 - (1 - \theta s)^n,$$
$$D_B = \theta[1 - (1 - s)^n].$$

Accordingly,

$$\frac{D_A}{D_B} = 1 + \left(\frac{n-1}{2}\right) s(1-\theta) + \text{higher powers of } s.$$

Thus, if the quantity $(n - 1/2)s(1 - \theta)$ is sufficiently small, the D criterion will be equivalent to the P criterion. If the quantity is large, the criteria will be essentially different.

Illustration 2 is an extreme case in the sense that system A contains no serial correlation at all, and system B contains the ultimate of positive serial correlation: if one error is zero, all others are zero; and if one error is different from zero, all are very large. It is possible that real fire control systems exist which contain negative correlation, that is, in which there will be a tendency for the system to get on the target when it is off, or vice versa. It might be conjectured, however, that the P criterion will be adequate whenever it is adequate in the Illustration 2, for n, θ, and s that are appropriate.

It has been pointed out [28] that there are cases in which serial correlation will increase the probability, contrary to its effect in Illustrations 1 and 2.

10.2.6 Calculation of p, Probability That a Shot with Given Dry Run Error Will Destroy Target

Let us consider the case of a contact or proximity fuze. Let (x,y) be coordinates with origin at the center of vulnerability of the target in a plane perpendicular to the trajectory of the average perfectly aimed projectile (see Figure 3). Strictly speaking, the trajectory is to be taken relative to the target, that is, the trajectory is to be the motion of the projectile as viewed from the target. The reason for this is that the effect of the hits on the target is due in part to the velocity of the hitting particles, relative to the target.

Consider an instant at which the dry run error is (a,b). It is generally assumed that the actual errors (x,y) of trajectories of real projectiles are distributed *normally* about (a,b), that is, that there are quantities μ_{11}, μ_{22}, μ_{12} such that $g(x,y)\,dx\,dy$ is the probability that the shooting error will lie in a rectangle of sides dx, dy parallel to the axes about (x,y), where

$$g(x,y) = \frac{1}{2\pi\sqrt{\Delta}}\, e^{-Q/2},$$

$$Q = \frac{1}{\Delta}\left[\mu_{22}(x-a)^2 - 2\mu_{12}(x-a)(y-b) + \mu_{11}(y-b)^2\right],$$

$$\Delta = \mu_{11}\mu_{22} - \mu_{12}^2.$$

Here μ_{11} and μ_{22} are called the *variances* of x and y, respectively, and μ_{12}, the *covariance* of x and y. These

quantities are presumed known. In many cases, the pattern is taken as circularly symmetric, that is, $\mu_{11} = \mu_{22} = \sigma^2$ and $\mu_{12} = 0$, where σ, the *standard deviation*, is a known quantity. Ballistic tables should give values of the μ's, at least in the case of a fixed target and fixed gun. When the target motion is important, tables describing the behavior of a proximity fuze should give appropriate values of the μ's. In the case of a contact fuze, the target motion is probably unimportant. The variances and covariances are discussed further in Section 10.3.2.

At a particular instant of a run, the dry run error will not be known exactly. Rather (a,b) will be the dry run error contaminated by a *measurement error*, if actual observations are involved in the determination of (a,b). The measurement errors can be studied. It will be assumed that the measurement errors are unbiased (have mean $(0,0)$) and have known variances and covariance $\mu_{11,m}$, $\mu_{22,m}$, and $\mu_{12,m}$ respectively, then it can be shown that instead of the normal function $g(x,y)$ given above, one should use the normal function $g^*(x,y)$, obtained by replacing the variances and covariance of $g(x,y)$ by

$$\mu_{11} - \mu_{11,m}; \quad \mu_{22} - \mu_{22,m}; \quad \mu_{12} - \mu_{12,m}$$

respectively, provided the latter differences are nonnegative.

The above subtraction of the measurement variances and covariance is discussed in two reports.[118,93] That the subtraction is reasonable can be seen as follows. Consider a hypothetical case in which the measurement variances and covariance precisely equal the bullet variances and covariance, respectively. In such a case, the measurement effects which are present in the recorded dry run errors are statistically precisely those which would be present in a shooting test. In the latter test, the bullet errors would replace the measurement errors. Under our hypothesis, the two are equivalent. Accordingly, no further dispersion should be added to that already present due to measurement errors; the variances and covariance should be zero.

Let $\delta(x,y)$ be the probability that a projectile with error (x,y) will destroy the target. Since the origin is the center of vulnerability of the target, $\delta(x,y)$ will vanish when (x,y) is sufficiently far from the origin.

The probability that a shot at an instant at which the recorded dry run error is (a,b) will destroy the target should be taken as

$$p = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g^*(x,y)\delta(x,y)dxdy \,. \tag{2}$$

If the function δ is, except for a constant factor, itself normal, that is, of the form of the function g, the above integration can be carried out explicitly. In any case, the function δ can be approximated by a constant times a normal function. In this way it can be shown that *the probability of destroying the target with a single shot is approximately equal to a constant multiple of the value, at the center of vulnerability of the target, of the normal probability density with mean at the center of the bullet pattern and with variances and covariance equal respectively to those of the vulnerability distribution plus those of the bullet pattern less those of the measurement errors.* The variances and covariance of the vulnerability distribution referred to are defined as follows.

Suppose that

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(x,y)dxdy = c \,.$$

Since the origin of coordinates is at the center of vulnerability of the target,

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x\delta(x,y)dxdy = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y\delta(x,y)dxdy = 0.$$

The variances and covariance of the vulnerability distribution are

$$\mu_{11,v} = \frac{1}{c}\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^2\delta(x,y)dxdy,$$

$$\mu_{12,v} = \frac{1}{c}\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy\delta(x,y)dxdy,$$

$$\mu_{22,v} = \frac{1}{c}\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y^2\delta(x,y)dxdy.$$

The italicized assertion above is equivalent to the following: The probability of destroying the target is approximately c times the value of g at $x = 0$, $y = 0$, with μ_{11}, μ_{22}, μ_{12} replaced by $\mu_{11} + \mu_{11,v} - \mu_{11,m}$; $\mu_{22} + \mu_{22,v} - \mu_{22,m}$; $\mu_{12} + \mu_{12,v} - \mu_{12,m}$; respectively. For a proof of the assertion, the reader is referred to an AMP report.[186]

The Applied Physics Laboratory of Johns Hopkins University has found that the probability that a shot will destroy the target is approximately

$$\frac{kd^2}{d^2 + 2s^2}e^{-h^2/(d^2+2s^2)} = (k\pi d^2) \cdot \frac{1}{2\pi\left(s^2 + \frac{d^2}{2}\right)}e^{-h^2/[2(s^2+d^2/2)]}$$

where k and d are constants (operability and radius of lethal area) appropriate to the particular fuze and target, s^2 is a variance, and h is the radial dry run error.[108] This formula is of the type that we have just

discussed, except that the measurement errors are excluded, probably as being of small influence.

10.2.7 Procedures for Estimating Expected Survival Probability

What may be termed the direct procedure for estimating the expected survival probability has been studied.[29,30] The procedure is to calculate the survival probability s for each dry run according to the formula (1) and to take as S, the estimate of the expected survival probability, the average of the s's for the individual runs. If desired, the runs can be given different weights in the averaging. The estimate D of the expected destruction probability is then $1 - S$.

The direct procedure has the advantage that no special hypothesis is made as to the nature of the dry run errors; they are taken as observed. It has the advantage, also, that the calculation is fairly simple. It has the disadvantage that the calculation appears to possess great intrinsic variability. If the dry run errors constitute a normal universe (of $2n$ dimensions), the procedure is statistically inefficient, that is, extracts from the data a less reliable conclusion than one to be described below.

It should perhaps be noted that statistical efficiency at the expense of experimental or mathematical simplicity is not necessarily a good thing. Overall efficiency and soundness are desired for the investigation. It may be easier to fire four times as many projectiles or photograph four times as many dry runs and then execute relatively simple calculations than it would be to carry out a complex investigation of the results of fewer runs. It has been pointed out [30] that a procedure which is statistically efficient for a normal universe may be definitely inefficient for a universe which, in some sense, is close to being normal.

The direct procedure has been carried out in a number of investigations. Numerical results are given in two reports.[16, 17]

Another report [30] also presents two alternative methods for estimating the expected survival probability. These probably possess less intrinsic variability than is present in the direct procedure. The methods are based on what may be called a hypothesis of *quasi-steady* error: that the dry run error is the sum of two components, the first of which is the same at the same instant of all runs considered and is called the error in mean point of impact or the quasi-steady error; and the second of which is a fluctuating error which possesses no serial correlation from instant to instant along a run. The quasi-steady error might be assumed constant throughout straight runs, or it might be obtained by a smoothing of recorded dry run errors.

The hypothesis of quasi-steady error imposes on the dry run errors a particular type of correlation. Shots at different instants are related in that each has a quasi-steady contribution. These contributions are functionally related; in other respects the shots are independent. It is not known whether the hypothesis of quasi-steady error is a useful idealization of the actual behavior of dry run errors or not.

The calculation of the expected survival probability is described [31] in the case in which the functional and statistical characteristics of the dry run errors are known and in which the statistical characteristics are those of a normal universe of $2n$ dimensional vectors, where n is the number of shots. When the statistical characteristics are indeed of this sort and are known, the calculation described is the most desirable from the point of view of statistical efficiency. A procedure which analyzes the dry run errors, estimates the variance-covariance matrix of their presumed normal universe of $2n$ dimensions, and then estimates the survival probability by the analysis given in the report [31] would presumably be the most efficient procedure statistically, provided the universe were indeed of the type assumed. Such a calculation would be quite involved, however; among other things it would require the evaluation of determinants of order $2n$ or $2n + 1$.

Certain assumptions about the behavior of successive dry run errors which would make the computation of the determinants referred to somewhat simpler are suggested.[31] The complexity of the method described [31] depends very much on the number of shots.

10.2.8 Report of Analysis of Dry Run Errors Obtained on Dynamic Tester [b]

Several variants of the Army M-9 and T-15 directors used with preset fuzes are considered in a report.[18] These were studied in the *dynamic tester*, a device which makes it possible to feed the same air-

[b] In this and succeeding sections we shall use the term *director* rather than *predictor* since all bibliographical references adopt this usage.

craft course into several directors and to record the dry run errors for each director. Only the radial dry run errors are used; the directions of the point of aim from the true point of aim are not used. The radial errors are termed "vector errors" in the report.[13]

The radial errors were recorded at 5 or 1 sec intervals. Several of the courses fed into the tester were based on flights of enemy aircraft over England.

The method of comparing the directors was to compare the averages of the single shot probabilities p of destroying the target at all the firing instants of each run. This method of scoring is sound, if the effect of serial correlation is unimportant.

The observed differences in directors found[13] were for the most part small. Among the findings were the following: The T-15 with curved flight prediction had a significantly higher mean score than the same director without curvature, although the number of courses on which each was better was the same. The M9 director, with curvature calculated from the second derivative of rectangular coordinates and with 10 sec smoothing time for the first derivative and 20 sec smoothing time for the second derivative, had a significantly higher mean score than the T-15 director with curved flight prediction.

10.3 CALCULATION OF DRY RUN ERRORS

It may be of interest to discuss aspects of the calculation of the dry run errors, since the entire discussion of Section 10.2 supposed that the dry run errors were available to start with.

10.3.1 Backing-up Process

Suppose that for each dry run there is available a record of the target and gun platform motions and of the gun settings throughout the entire engagement. One can calculate the dry run errors by backing up from impact instants to firing instants in the following way. On the basis of the recorded motions, one can calculate coordinates (q_1, q_2, q_3) of the target in a coordinate system which is stable during the time of flight, that is, which does not roll, pitch, or yaw, and which moves, if at all, with a constant known translational velocity. It is presumed that the center of mass of the gun platform moves with a constant known velocity. If the gun platform moves with a known but varying velocity, the calculation of dry run errors is more complicated. Stabilization is discussed in Section 10.3.3. The coordinates (q_1, q_2, q_3) are functions of the clock time t; accordingly,

$$q_1 = q_1(t), \quad q_2 = q_2(t), \quad q_3 = q_3(t).$$

To compute the dry run error, start with an arbitrary instant $t = b$ which is taken to be an instant at which a perfectly aimed average shell would hit the target. The target would then have coordinates $q_1(b)$, $q_2(b)$, $q_3(b)$. In terms of these coordinates and the motion of a stable coordinate system, the ballistic tables give the time of flight t_f of the perfectly aimed projectile to the impact point. Then the firing instant for the perfectly aimed projectile must have been $t = b - t_f = a$. Next look at the record of gun settings and see what the actual gun settings were at $t = a$. The ballistic tables give the position after t_f seconds of an average shell fired at $t = a$ with the observed gun settings. The displacement to this position from the point $[q_1(b), q_2(b), q_3(b)]$ is the dry run error for the projectile fired at $t = a$. This displacement is a vector which shall be denoted by $\mathbf{e}$. (See Figure 3.)

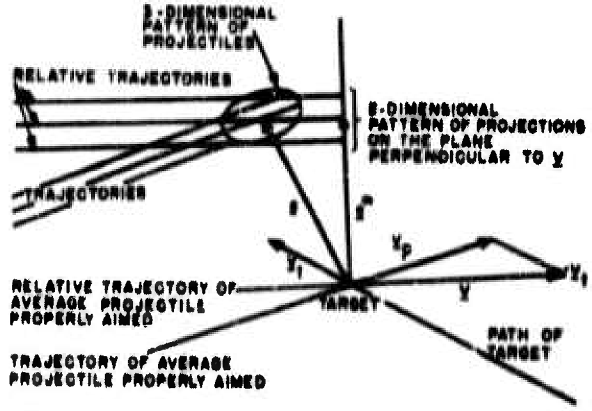


FIGURE 3. Trajectories as varied from the air mass and from the target: $\mathbf{V}_p$, remaining velocity of projectile; $\mathbf{V}_t$, target velocity; $\mathbf{e}$, dry run error; (these vectors are not necessarily coplanar). $\mathbf{V} = \mathbf{V}_p - \mathbf{V}_t$, velocity of projectile relative to target; $\mathbf{e}^*$, the projection of $\mathbf{e}$ on the plane perpendicular to $\mathbf{V}$.

10.3.2 Dry Run Error Perpendicular to Relative Trajectory

In the case of a proximity or contact fuze, the pertinent dry run error is the projection of the full dry run error $\mathbf{e}$ (discussed in Section 10.3.1) on the plane perpendicular to the relative trajectory. Let the projection of $\mathbf{e}$ on the plane perpendicular to the relative trajectory be the vector $\mathbf{e}^*$. Let $\mathbf{V}$ be the

velocity relative to the target of a perfectly aimed average projectile at the impact instant $t = b$; $\mathbf{V}$ is the remaining vector velocity of the projectile less the vector target velocity. Then, since $\mathbf{e} \cdot \mathbf{V}$ is the projection of $\mathbf{e}$ on $\mathbf{V}$ multiplied by the length of $\mathbf{V}$,

$$\mathbf{e}^* = \mathbf{e} - \frac{\mathbf{e} \cdot \mathbf{V}}{\mathbf{V} \cdot \mathbf{V}} \mathbf{V},$$

as can be seen from Figure 3. This relation permits one to obtain the error $\mathbf{e}^*$ pertinent to a proximity or contact fuze from the full error $\mathbf{e}$ and the relative remaining velocity of the projectile. In many calculations only the length of the vector $\mathbf{e}^*$ will be needed. This length is

$$\left| \mathbf{e} \cdot \mathbf{e} - \frac{(\mathbf{e} \cdot \mathbf{V})^2}{V \cdot V} \right|^{\frac{1}{2}}.$$

Since the time of flight of the properly aimed and actually aimed average projectiles are the same, the vector $\mathbf{e}$ will be approximately perpendicular to the line from the gun to the target.

At the impact instant $t = b$, the set of all projectiles that might have been fired at $t = a$ precisely as aimed in the dry run constitutes a three-dimensional pattern about their average position. Their velocities relative to the target will differ from $\mathbf{V}$ by negligible amounts as far as estimating the error is concerned. Accordingly, the three-dimensional pattern may be projected onto the plane through the target perpendicular to $\mathbf{V}$; this gives the two-dimensional pattern that was considered in Section 10.2.7. (This question in the case of caliber 0.50 bullets is discussed in a report.[27])

The variances and covariance μ_{11}, μ_{22}, and μ_{12} of the two-dimensional pattern will, strictly speaking, depend on $\mathbf{e}$ and $\mathbf{V}$. Future studies of vulnerability and kinematics would seem appropriate to determine average or suitable values of these variances and covariance.

10.3.3 Stabilization of Observations Made from Rotating Gun Platform

The calculation of the stabilized coordinates from coordinates relative to a rotating gun platform and measurements of the rotation of the gun platform is a problem which was considered by AMP in the case of airborne fire control systems. The methods apply to the antiaircraft case and are as follows.

An *absolute stabilization* is a determination of target coordinates in a system which is unrotated throughout the entire experiment. A *local stabilization*, on the other hand, is a determination of target coordinates in a system which is unrotated throughout the time of flight of the projectile, but which is not necessarily the same for all projectiles. If the object of the calculation is to determine the radial dry run error, a local stabilization is often sufficient. The reason for this is that the radial error is a quantity which, itself, is independent of coordinate systems. Whether a local stabilization is sufficient in a particular case will depend, in part, on the deviation of the "up" direction of the particular stabilized coordinate system from the true direction of gravity.

When a local stabilization is permissible, the advantages gained from its use are twofold: (1) For the two instants, impact time and firing time, only one stabilization is necessary. (2) The amount of roll, pitch, and yaw during a time of flight is small; hence, one may use certain correction formulas which are valid for small rotations, but not otherwise. These formulas are first order corrections. The process is analogous to estimating the increment of a function by the increment along its tangent line. Local stabilization is discussed in three reports.[18, 22, 23]

When an absolute stabilization is required, there are different procedures of calculation. One may use the exact but cumbersome equations of rotation. A second method is the use of the gnomonic charts.[18a, 19] A third method involves the following idea:[24] For each direction in the celestial sphere, the changes in zenith distance and bearing for one degree of roll are calculated. The changes for R degrees of roll are then approximately R times the changes for one degree. This approximation is very satisfactory for rolls up to 7°, provided the target elevation is under about 30°. Actually, the tabular correction for one degree of roll is not that correction which would be exactly accurate for such a roll, but rather that correction which, used in the way described, gives the best results for rolls between 0° and 7°.

Essentially the same table is used to correct for pitch. After coordinates have been corrected for roll and pitch, the correction for yaw is trivial. This presumes the convention that the change from stabilized to unstabilized coordinates is accomplished in the order: yaw, pitch, roll. If other conventions are followed, the procedure of correction is changed accordingly.

The above method is analogous to approximating the increment of a function by measuring the increment along a judiciously chosen chord of the graph.

This method may be extended to permit one to correct for rotations up to 15°. The extension involves the use of other tables, which would apply to the range 7° to 15°. For each zenith and bearing, one such table would give a constant value of roll and the correction to zenith per degree of roll in excess of the constant amount given. A similar table would refer to bearing. The approximation would again be a linear one (a chord of the graph), but the chord would not pass through the origin.

10.4 CURVED FLIGHT DIRECTOR

This section gives the results of a theoretical comparison of a particular linear and a particular quadratic director. It is pertinent to the problem of the design of new directors for several reasons: (1) The methods used may be suggestive to other investigators. (2) The conclusion that the quadratic director would be of marginal value compared to the linear director may be of interest. (3) Though particular directors are considered, there is no reason to doubt that their performance is indicative of the performance of a wide class of directors.

The work of AMP in this problem [11, 12] was aided by the cordial and effective collaboration of the Bell Telephone Laboratories.

10.4.1 Types of Directors

Consider a director which operates on the basis of rectangular coordinates (x,y,z) of the target: The director contains a unit which converts the range and the angular coordinates of the target (obtained from the ranging and tracking) into rectangular coordinates referred to axes fixed in the director platform. (The advantage of such conversion stems from the fact that the components $\dot{x}$, $\dot{y}$, $\dot{z}$ of target velocity are constant, whereas the rates of change of range or angles are not normally constant when the target moves at a constant velocity.

The coordinates x,y,z are functions of clock time, t:

$$x = x(t), \quad y = y(t), \quad z = z(t).$$

At a particular instant $t = t_1$ these coordinates have been fed into the computing unit for all instants up to t_1. In order to produce a solution at the instant t_1 the director must have at that instant explicit functions $X(t)$, $Y(t)$, $Z(t)$ in which values of t later than t_1 can be substituted. Thus the extrapolation that has been referred to in Section 10.1.2 can be performed by means of the functions $X(t)$, $Y(t)$, $Z(t)$.

A large class of directors can be defined as follows. (Only the x coordinate will be considered since the y and z coordinates are treated similarly.) Let a time base c be chosen and a weight function $w = w(u)$ defined for u between 0 and c. Let $X(t)$ be that linear function of t which minimizes the integral

$$\int_{t_1-c}^{t_1} [X(t) - x(t)]^2 w(t_1 - t)\,dt \qquad (3)$$

of the square of the deviation from the observed x to the explicit X over the interval c seconds before the present instant t_1, the squared deviation being weighted according to the function w as indicated in equation (3). It can be shown that there is precisely one linear function X which minimizes the integral (equation 3).[11] A director which uses this function X will be called the *linear least square director with time base c and weight function w*. The *quadratic* least square director with time base c and weight function w is defined in exactly the same way except that the function $X(t)$ is that *quadratic* function in t which minimizes the integral (equation 3).

Suppose there were neither errors in the input to these directors nor machine errors in the directors. Then the prediction of the *future position* would be correct, provided the target motion is such that the coordinates x,y,z of the target can be expressed as quadratic or linear functions of clock time t, respectively, in the case of the quadratic or linear directors.

As a matter of fact, by varying the time base c and the weight function w one may obtain a great variety of types of extrapolation: The class of directors that has been defined is very wide indeed, and it is the conjecture of the present author that the class comprehends approximations of all types of extrapolation that have been used in directors.

In the following subsections of Section 10.4, the special case of constant weights

$$w = 1$$

is considered. Any other particular case can be investigated by the same methods.

10.4.2 Prediction Errors, Supposing Perfect Input, Perfect Operation, and Perfect Determination of Time of Flight

The errors in prediction by the linear and quadratic directors for various types of target motion are

The Function L

Time of flight $T = 10$ sec

For a helical course, the error in future position is RL.
For a sinusoidal course, the error in future position is $R(L/\sqrt{2})$.
L is given in the body of the table.
c is the time base of the director.
$2\pi q$ is the period of the helical or sinusoidal motion of the target.
R is the radius of the helix or amplitude of the sinusoid.

Linear Director

Period $2\pi q$ seconds	q seconds \ c seconds	0	5	10	15	20
12.6	2	6.0	5.35	2.8	0.70	0.07
18.8	3	4.05	4.05	4.7	3.4	1.6
25.1	4	2.6	3.6	4.1	4.1	3.5
37.7	6	1.3	1.6	2.4	2.0	3.15
56.5	9	0.59	0.90	1.2	1.5	1.8
75.4	12	0.34	0.52	0.71	0.92	1.1
94.2	15	0.22	0.335	0.47	0.605	0.75

Quadratic Director

Period $2\pi q$ seconds	q seconds \ c seconds	0	5	10	15	20
12.6	2	13.2	18.0	17.0	11.0	2.0
18.8	3	5.0	8.2	11.0	11.0	9.6
25.1	4	2.3	4.1	5.8	7.25	8.0
37.7	6	0.73	1.3	2.1	2.9	3.7
56.5	9	0.22	0.42	0.675	0.98	1.3
75.4	12	0.095	0.18	0.29	0.43	0.6
94.2	15	0.049	0.093	0.15	0.23	0.32

considered.[12] An analysis is accomplished in the following way: It is first supposed that the time of flight T is accurately known. The average (root mean square) error in future position is then determined for certain specified courses. The average is taken over the set of all possible positions of the gun for which the time of flight against the target will be precisely T seconds. The average error for more general courses is then compounded from the errors for the particular courses.

Thus suppose that the target is moving in a helix of radius R yards, that it makes a complete revolution in $2\pi q$ seconds, and that the time base of the director is c seconds. The component of target velocity parallel to the axis of the helix and the angular velocity about the axis are supposed constant. Then if the inputs are entirely correct and if the computing mechanism is mechanically perfect, the error in future position will be RL, where L depends on q, c, T, and n:

$$L = L_n(q,c,T),$$

the index n being equal to 1 for the linear director and 2 for the quadratic director.

The formulas for the function L and the results of certain computations based on these formulas are given.[12] Certain of the computations are cited below.

The error function L applies also to *sinusoidal* target motions. Suppose that the target moves along a sine curve of amplitude R yards, period $2\pi q$ seconds, at a constant velocity parallel to the axis and a simple harmonic velocity normal to the axis, in the plane of the sine curve. Then the root mean square prediction error (subject to the same qualifications as in the helical case) is $RL/\sqrt{2}$.

For time of flight 10 sec and for different values of the time base c and the period of oscillation, the following tables compare the performance of the linear and quadratic directors.

The case $q = 6$ sec and $R = 1{,}000$ ft or more may be considered an example of an evasive action. For the same time base, the errors cited above for the quadratic director are less than those for the linear

director in the case of low frequencies (large q) and are greater than those of the linear director for high frequencies. As will be seen later, the effects of other errors are such that the time base for the quadratic director should be two to four times the time base for the linear director. For this reason it would appear that the quadratic director is not significantly superior to a linear director against the courses considered above. Tables, of the sort given above, for $T = 5$, 10, 15, and 20 sec are given.[19a]

The numerical results and the formulas for L have an import in the case of courses which are far more general than the simple helix or the simple sinusoid. In effect, courses of very general types can be approximated or built up from simple sinusoids. Suppose that the target motion is such that it can be represented in the following way. Each rectangular coordinate is a sum of sines and cosines in t plus a linear or quadratic function of t, where t is clock time. Then the prediction error for the linear or quadratic director, respectively, is obtained by combining in quadrature the errors of the sort cited above for the separate sine and cosine terms: The square of the error on the compound course is the sum of the squares of the errors on the component sinusoidal courses. Since very general functions can be represented as sums of the sort described above[a] the prediction error that we have been considering can be computed for very general courses by the above methods.

As an illustration, consider the target course

$$x = 250 + 150t + 25 \sin \frac{t}{2} + 400 \cos \frac{t}{12},$$

$$y = -100 + 10t + 50 \sin \frac{t}{6},$$

$$z = \text{constant},$$

x, y, and z being measured in yards, t in seconds, and angles in radians. Consider a time base of 10 sec and a time of flight of 10 sec. Let e be the root mean square prediction error in yards under the conditions of perfect input and no machine error. Then, by the above tables,

$$e^2 = \frac{25^2(2.8)^2}{2} + \frac{400^2(0.71)^2}{2} + \frac{50^2(2.4)^2}{2} = 224^2$$

for the linear director and

$$e^2 = \frac{25^2(17)^2}{2} + \frac{400^2(0.20)^2}{2} + \frac{50^2(2.1)^2}{2} = 319^2$$

for the quadratic director.

[a] The process of approximating functions by certain sums of sines and cosines is considered in works on Fourier series.

The errors referred to above are the prediction errors, supposing that the calculation unit of the director has determined the time of flight correctly. Actually the time of flight itself is based on the prediction. Accordingly, there will be another contribution to the prediction error due to the fact that the time of flight will not be known exactly.[19]

The error function L has the following convenient property:

$$L_n(q,e,T) = L_n(kq,ke,kT), \tag{4}$$

where k is any constant different from zero. Approximate formulas for the function L, together with an indication of the range of values in which the formulas are valid, are as follows:

$$L_1(q,e,T) = \frac{1}{2q^2}\left(\frac{e^2}{6} + eT + T^2\right), \quad q \geq 15,\ e \leq 20,\ T \leq 20;$$

$$L_2(q,e,T) = \frac{1}{6q^3}\left(\frac{e^3}{20} + \frac{3e^2T}{5} + \frac{3eT^2}{2} + T^3\right), \quad q \geq 15,\ e \leq 15,\ T \leq 15.$$

Exact equations for the function L are available.[19b]

10.4.3 Effects of Other Errors

To be able to make a comparison of the linear and quadratic directors, it is proper to consider not only the theoretical errors in points of aim that have been discussed, but all other errors and random effects which enter into the determination of what each shell does to the target. An approximate analysis is carried out in the following way.[19] There is superposed on the calculated theoretical error in point of aim a spherically symmetric normal distribution of the displacement from the theoretical point of aim to the actual point of burst, the standard deviation of one component of this distribution being σ^*.

The principal elements which contribute to the value of σ^* are the gun and fuse performance errors, the machine errors in the mechanism, and the effect of the tracking errors. The elements other than the effect of the tracking errors are assumed to be the same as are applicable to the Army M-9 director and 90-mm gun. It is assumed that the actual tracking errors that would be committed with either director are the same as those which have been observed with the M-9 director. The effect of the tracking errors in the computing mechanism is then estimated by replacing the least square directors that we have been considering by directors whose extrapolation is of the

following type: The linear director is one which extrapolates along a straight line determined by the present position of the target and its position c seconds previously. The quadratic director is one which extrapolates along the parabola determined by the present position of the target, its position $c/2$ seconds previously, and its position c seconds previously. Under these assumptions, the following estimates of the standard deviation σ^* are obtained. The σ^* of the quadratic director with time base 20 sec is comparable to that of the linear director with time base 5 sec.

Estimated standard deviation σ^* (in yards) of the spherically symmetric normal distribution that is to be superposed on the theoretical point of aim.

Time of flight $T = 10$ sec†

c	5 sec	10	15	20
Linear Director	161 yd	106	80	82
Quadratic Director	1,280	451	205	103

† Values of σ^* also for $T = 5$, 15, and 20 sec are given.[10]

For purposes of comparison it is assumed[12] that the probability of damage is

$$p^* = \frac{1000}{\sigma^{*3}} e^{-\frac{1}{2}(r^2/\sigma^{*2})}$$

where r is the distance of the target from the presumed future position as discussed earlier. The probabilities calculated by this formula are not absolute probabilities; but the relative sizes of p^* for the linear and quadratic directors are presumably an indication of the effectiveness of these two directors. If p^* were correct, it would give the proportion of shots which would damage the target.

The probabilities for the case $T = 10$ sec are cited below. The cases $T = 5$, 10, 15, and 20 sec have been considered.[10] Probabilities were calculated only in those cases in which the theoretical prediction error was less than the radius of the helix or the amplitude of the sinusoid. The probabilities refer to a target flying either a helical course with a radius of 300 yd or a sinusoidal course with an amplitude of $424 = \sqrt{2} \times 300$ yd.

10^7 times the hypothetical probability of damaging the target. (The upper entry applies to the linear, the lower to the quadratic, director.)

$T = 10$ sec

q seconds \ c seconds	5	10	15	20
0	500	30	0.02	
	5	100	200	
12	1,500	1,100	120	10
	5	110	480	600
15	1,500	3,500	1,800	420
	5	110	520	1,200

In order to allow for various uncertain effects, including the effect of the error in time of flight, and also in order to test the sensitivity of the results to changes in the estimated σ^*, the probabilities were also computed with σ^*'s which were, except for rounding, 1.5 times those cited above. The results, in the case $T = 10$ sec, are as follows:

10^7 times the hypothetical probability of damaging the target, based on standard deviations 1.5 times those cited above. (The upper entry applies to the linear, the lower to the quadratic, director.)

$T = 10$ sec

q seconds \ c seconds	5	10	15	20
0	300	170	33	
	1.4	28	120	
12	520	1,000	500	43
	1.4	20	150	310
15	500	1,800	1,400	1,000
	1.5	20	150	350

The probabilities are quite sensitive to the standard deviation. Nonetheless they indicate that the linear director with a comparatively short time base is the appropriate fire control instrument to cope with the type of maneuver considered. The performances of the quadratic director with time base 20 sec is comparable to that of the linear director with time base 5 sec.

10.4.4 Theoretical Performance of Directors against Recorded Course

The analysis that has been described has been devoted to helical and sinusoidal courses and courses by implication which might be built up therefrom. It seemed desirable to study the behavior of linear and quadratic directors on a standard course which might be deemed typical of a course flown by an actual enemy aircraft. Such a course was supplied by Lt. Col. A. H. Musson of the Inspection Board, United Kingdom and Canada. It is a two-minute course of an enemy aircraft over England. The course

is far from straight, but it perhaps does not represent deliberate evasive action. If one introduces rectangular coordinates in a certain way, the z axis being vertical, the course can be approximated by the following equations:

$$x = -0.00001287t^4 + 0.003405t^3 + 0.02044t^2 - 116.0t - 137.0$$

$$y = -0.00003448t^4 + 0.02367t^3 - 0.02200t^2 - 67.83t - 4{,}183$$

$$z = -0.00003103t^4 - 0.001870t^3 + 0.16095t^2 - 5.235t + 1333$$

These equations apply to the interval $-60 \leq t \leq 60$ sec; x, y, and z are measured in yards. The x and z coordinates have approximately linear trends. The y coordinate consists approximately of an oscillation of period of the order of 120 sec and of amplitude of the order of 1,500 yd. (This can be seen from the graphs following page 82 of reference 12.)

Theoretical errors and probabilities for this recorded course, for T = 5, 10, and 15 sec are presented.[12e] The results for T = 10 sec are as follows.

Errors and probabilities in the case in which the target flies the recorded course (described in the text)

T = 10 sec

Root mean square error in prediction in yards. (Subject to the conditions of perfect input, perfect mechanization and perfect determination of time of flight.)

c	5 sec	10	15	20
Linear director	385 yd	540	740	963
Quadratic director	47.3	70	121	174

10^7 times the hypothetical probability of damaging the target. (The upper entries apply to the linear, the lower to the quadratic director.)

c	5 sec	10	15	20
Standard deviation σ*	140	0.02	0	0
as cited above	5	110	480	980
Standard deviation 1.5	240	3.5	0	0
times those cited above	14	20	150	310

For the test course, the quadratic director with time base 20 sec is the best of those considered. That the results obtained for the test course are consistent with those earlier obtained may be seen as follows: The test course has the general character of a sine curve of period $2\pi 18$ sec, that is, with q = 18 sec, and amplitude 1,500 yd. According to the earlier theory then, the root mean square prediction error is $1{,}500L/\sqrt{2}$, where $L = L_n(q,c,T)$ is evaluated at $q = 18$, $n = 1$ or 2 for the linear or quadratic director, respectively, c = 20 sec, say, and T = 10 sec. Now by the property of the function L given in equation (4)

$$L_n(18,20,10) = L_n(9,10,5);$$

the latter quantity has been tabulated.[12e] Its value is 0.54 for the linear case, and 0.19 for the quadratic. This gives errors of 574 and 202 yd, respectively, for the linear and quadratic directors; these are of the same order of magnitude as the 963 and 174 yd cited above for c = 20 sec.

10.5 ESTIMATES OF CONTRIBUTING ERRORS[a]

The estimates to be given apply primarily to the Army M9 director and 90-mm gun with preset fuze, as of 1945. The estimates are of uneven reliability.

1. *Errors in Static Alignment of Director and Gun.* It is probable that these errors are very small.

2. *Tracking Errors in Angles.* The probable error of each component of optical tracking is estimated at 0.3 mil for a stable platform; it is also stated that this error is roughly proportional to the angular velocity of the target. The probable error of radar tracking is 2 mils for the best manually controlled equipment.

3. *Tracking in Range.* The best radar range has a probable error of about 20 yd. The probable error of optical range is approximately

$$\frac{60r^2}{206{,}265bm}\ \text{yd},$$

where b is the base length in yards, m is the magnification, and r is the range in yards.

4. *Errors in Selsyns Transmitting Data to Directors.* These are of the order of 0.1 mil and are negligible.

5. *Predicted Position.* The errors in predicted position depend not only on the errors 1, 2, 3, 4, already discussed, but also on the frequencies present in the input data (the power spectrum or the input) and on the time of flight. For the Army M9 director, radar SCR-584, and 90-mm gun, estimates are as follows (assuming no evasive action):

Errors in Predicted Position
M9 Director; SCR-584 Radar

Time of flight	5	10	15	20 (sec)
Probable error in angles (optical tracking)	3	4	5	6 (mils)
Probable error in angles (radar tracking)	4	5	6	8 (mils)
Probable error along the trajectory (radar ranging)	40	75	110	150 (yd)

6. *Superelevation*, that is, the increment to be added to the elevation of the predicted target position to compensate for the gravity drop of the projectile. Errors in superelevation are considered negligible.

7. *Muzzle Velocity. Case I. Preset Fuze.* Aspects of this question are discussed at length.[8] The muzzle velocity varies with the age of the gun, with different lots of ammunition, and with different rounds of the same lot. Accordingly, the actual muzzle velocity for any shot will differ from the input muzzle velocity. In some cases, the input muzzle velocity is set in accordance with a rule depending in part on the age of the gun. A better estimate of the muzzle velocity can be attained by the use of a chronograph which measures the muzzle velocity of each shot. When the chronograph is used, the average muzzle velocity of 8 or 12 preceding shots can be used as an estimate of the muzzle velocity of the next following shot. It is concluded that the probable error in one's estimate of muzzle velocity without a chronograph (using the formula $MV = 2{,}730 - 0.087R$ fps, where R is the total number of rounds fired by the gun) is 17 fps, and with a chronograph is 7 fps in the case of the 90-mm gun. These errors in muzzle velocity result in an error in the burst point of the shell as follows:

Probable Error in Yards Along the Trajectory, Due to Incorrect Muzzle Velocity. (Preset Fuze)

Time of flight	4	15	25 (sec)
Error, without chronograph	17	42	60 (yd)
Error, with chronograph	7	17	20 (yd)

Case II. Proximity Fuze. In this case, the effect of the error in muzzle velocity should be comprehended in the gun dispersion (item 13 below; see also Section 10.3.2). A numerical appraisal has not been given.

8. *Dead Time (Preset Fuze Only).* For hand-loading, the probable error in dead time, that is, the time between the prediction of the fuze setting and the firing of the gun, is about 0.02 sec. This corresponds to a probable error along the trajectory of about $0.06v$ yd, in the case of the 90-mm gun, where v is the velocity of the target in miles per hour.[10] For a target speed of 300 mph, the probable error, due to the error in dead time, is thus about 18 yd along the trajectory. These figures refer to fuze prediction of the conventional type.

For automatic loading the probable error of dead time is of the order of 0.03 sec and has a negligible effect.

9. *Errors in Selsyns Transmitting Gun Orders.* The probable error in the angle transmitted is 0.1 mil. The probable error in fuze transmitted is 0.02 sec, corresponding to errors along the trajectory of approximately 13, 7, and 5 yd for times of flight 4, 15, and 25 sec, respectively.

10. *Gun Following Errors.* The probable error in each angle is about 0.4 mil, which is negligible, compared to the errors in the transmitted signal.

11. *Fuze Setting Errors (Preset Fuze).* The probable error in the case of hand setting is 0.04 sec, corresponding to range errors of approximately 26, 14, and 10 yd for times of flight 4, 15, and 25 sec, respectively. The probable error in the case of mechanical fuze setter is about half that for hand setting.

12. *Fuze Performance Errors (Preset Fuze).* For times of flight of the order of 20 sec, the probable error in fuze performance is from 0.1 to 0.2 sec. The resulting probable error along the trajectory is about 30 yd, more or less constant for different times of flight because of the fact that the time fuze error increases with increasing time while the residual velocity of the shell decreases.

13. *Gun Dispersion.* Two types of errors under this category have been listed:[9] (1) The errors due to faulty estimates in the values of the ballistic wind and the ballistic density which are used on the wind and density dials of the director. This error is very difficult to estimate. Its effect along the trajectory has been variously assumed as 10, 20, 40, and 60 yd for time of flight of 25 sec. (The latter errors correspond to faulty estimates of range wind, amounting to about 1, 2, 4, and 6 mph, or to errors in air density of 0.25, 0.5, 1, and 1.5 per cent. At 15 sec time of flight, these errors would be reduced by about 1/3.) The resultant probable error in azimuth appears to be of the order of one mil, and in elevation of the order of 2 mils.

(2) The error due to the deviation of the ballistic characteristics of gun, projectile, and atmosphere from those which are presumed in the ballistic tables, other than those characteristics discussed above (in items 7 or 13-1). The probable error along the trajectory from this source is estimated as 10 yd; in elevation, as 1 mil; and in azimuth, as negligible.

Errors in a quantity arising from independent sources should be combined in quadrature. Thus, the square of the probable error due to causes a and b, which are independent, is the sum of the squares of the probable errors due to a and to b separately.

The term *gun dispersion* has different meanings in different problems. For example, the gun dispersion

that was considered in Section 10.2 is not the gun dispersion here considered. Dispersion usually refers to a variable whose value depends on chance according to its probability distribution. In each application the variable in question should be precisely defined.

The reader is referred to a report[8] for a consideration of the variation of muzzle velocity from shot to shot, from batch to batch, and with the age of the barrel; for a study of errors along the trajectory (part of which is subsumed above); and for an actual composition of the errors from different sources.

10.6 PREDICTION CIRCUITS

The term *prediction circuit* is here used to denote a device which approximates at each instant the future value of an input function. The input to the prediction circuit is a quantity: h seconds (the prediction interval) and a quantity: $x(t)$ (the predicted function), which varies with clock time, t. The output of the prediction circuit at the instant t is a quantity $z(t)$, which is an approximation of the future value $x(t + h)$ of $x(t)$, h seconds later.

As one application, x may be the fuze setting appropriate at the instant of prediction, in the case of heavy antiaircraft with preset fuze, and h may be the fuze dead time. Then z will be the fuze setting appropriate at the instant of firing. As another application, x may be any coordinate of the target, and h may be the time of flight of the projectile. Then z will be the coordinate of the target at impact. In the latter case, the time of flight h itself must be predicted by another mechanism or another part of the same mechanism.

Basic requirements of a prediction circuit are that its output $z(t)$ approximates the desired future value $x(t + h)$ in a satisfactory fashion without undue delay and that this approximation is not unduly disturbed when the input function $x(t)$ is itself disturbed.

10.6.1 Particular Circuits

A number of prediction circuits have been studied.[9] Some of the results and methods may be of general interest, even though the problem of fuze prediction is itself of limited interest.[a]

The circuits considered are characterized by mathematical relations of the form:

$$z = x + y,$$

where y is variously defined.

In the *conventional* circuit (also called the tangential circuit with feedback), y satisfies the equation,

$$b\dot{y} + y = h\dot{x},$$

where b is a positive constant. Strictly speaking, the quantity y is not uniquely determined by the inputs x and h, but depends also on certain initial conditions. However, the effect of these initial conditions is not important after an initial settling time. For example, after $4b$ sec the difference between any two outputs will be less than 2 per cent of their initial difference. In effect then, the output of the circuit after settling depends only on x and h. The quantity b is known as the time constant of the circuit.

In the *Sperry A-circuit*, y is an empirical function of h and w, where w satisfies the equation

$$k_1k_2\ddot{w} + k_2\dot{w} + w = \dot{x},$$

k_1 and k_2 being positive constants.

In the *Tappert circuit*, y satisfies the equation

$$b\dot{y} + g(y,h) = h\dot{x}, \tag{5}$$

where g is a function of y and h chosen so as to give optimum results. For the particular application to the fuze dead time problem, g was determined as $y - c_1(y + c_2h)^2$, where c_1 and c_2 are constants. If c_1 is zero, this circuit reduces to the conventional circuit.

In the tangential circuit with follow-up motor, y is the amount of rotation of a motor which receives a signal to increase whenever y is definitely less than $h\dot{x}$ and to decrease whenever y is definitely greater than $h\dot{x}$.

Mechanizations of these circuits are indicated in two AMP reports.[9, 10]

Both the Tappert and the Sperry A-circuits are interesting refinements of the conventional circuit designed to give more effective prediction fairly simply. The Sperry A-circuit involves the use of a three-dimensional cam. The Tappert circuit involves a judiciously chosen and judiciously placed nonlinear gear or other nonlinear device.

The tangential circuit with follow-up motor does not seem to provide adequate smoothing of certain input disturbances.[10] This result may be of interest, as it might have been thought that the lagging effect of the follow-up motor would in itself provide smoothing of input disturbances.

[a] The work to be reported was aided by the cordial and effective collaboration of the Research Laboratories of the Sperry Gyroscope Company.

10.6.2 Method of Analysis

The method of studying the responses of these circuits to a perfect input in the case of dead time fuse correction circuits was as follows:[9] The functions x were calculated from the ballistic table for typical target courses in which the target flew at a constant velocity. Because the ballistic functions cannot conveniently be represented in explicit analytic form, the same was true of the functions x. In any particular case, one could construct a numerical table of x, but no useful explicit analytical expression for x in terms of the initial conditions was found. Now a numerical description of a family of functions is cumbersome at best. In this case, a numerical description was particularly awkward as the prediction circuits made use of $\dot{x}$ as well as x, and numerical determinations of $\dot{x}$ were obscured with errors due to rounding and errors in reading graphs, and so forth. By plotting the functions $\dot{x}$, as determined numerically, against t, it was found that each function $\dot{x}$ could be approximated (according to a least-square criterion) by a function of the form

$$a + \frac{b}{c - t},$$

where a, b, and c were constants depending on the course. Thus the family of ideal inputs $x(t)$ was approximated by a three-parameter family of explicit functions for which the difference between the output $z(t)$ and the desired output $x(t + h)$ could readily be worked out.

The representation of the functions $\dot{x}$ as a three-parameter family of explicit functions was particularly useful in the determination of the function g of the Tappert circuit which makes the circuit most effective. The method was as follows: For each course, explicit expressions for $\dot{x}$ and for $\dot{y}$ (supposing perfect prediction) were substituted in equation (5). Thus the ideal g for each course was determined as

$$g = h\dot{x} - b\dot{y}.$$

A function g of y and h was then chosen which was an average of the appropriate functions for the different courses.

10.7 TRIAL FIRE METHODS

This section applies to projectiles with preset fuses. A *trial fire procedure* is one which makes use of observations of a number of trial shots in order to obtain corrections, where necessary, to inputs to the director such as muzzle velocity, air density, wind velocity, or other factors. Several trial fire procedures are described briefly below. Which procedure is most satisfactory in a particular set of circumstances depends on those circumstances. It depends also on a knowledge of the relative magnitudes of the various sources of error to be expected in the firings. These points are discussed in detail in a report[1] for the Army 90-mm gun.

If the trial fire procedure is to be based on four shots all fired at the same gun settings of the 90-mm gun, then no trial fire corrections are warranted if the observed center of burst differs from the trial shot point, that is, its expected position, by less than 60 yd in slant range, 2 mils in elevation, and 1 mil in azimuth. Furthermore, the set up of the gun and its equipment should be checked for serious errors if the observed center of burst differs from the trial shot point by more than 275 yd in slant range, 7 mils in elevation or 10 mils in azimuth.

The trial fire procedures that are considered are the following. All involve four trial shots.

1. Four shots are fired at a single trial shot point. The observed deviations of the center of burst of the four shots from the trial shot point are corrected by *spot* corrections in elevation, azimuth and percentage altitude (or range) settings of the director, a spot correction being one which is the same for all trajectories.

2. Two shots each are fired at two trial shot points on the same trajectory. The observed deviations of the centers of burst from the trial shot points are attributed to muzzle velocity, wind and air density, and linear equations are set up to determine the corrections in the latter quantities.

3. Like item (2), except that the two trial shot points are chosen at the same altitude but at different horizontal ranges.

4. Four shots are fired at a single trial shot point and the deviation of the center of burst is attributed to wind and to a combination of muzzle velocity and air density.

5. Methods of the sort already described, except that radar is used to determine range to the points of burst.

6. Two shots each are fired at two trial shot points at exactly the same gun settings except that the azimuths differ by 90°, thus interchanging the range wind and the cross wind.

16.8 TRACER STEREOGRAPHS

The problem has been considered[8] of measuring the amount by which a tracer bullet misses a towed target by means of two sets of motion pictures of the tracer bullet taken from two different fixed points.

The *Stibitz* method consists essentially of finding the point of intersection of the apparent tracer paths, relative to the target, as given by the two sets of motion pictures. The method is an approximation. It fails if the tracer curves do not intersect at all, and is ambiguous if they intersect more than once. It is shown[8] that the method is entirely satisfactory with a vertical or nearly vertical base line of about 20 yd, and with targets at a slant range of from approximately 1,000 to 1,500 yd, at elevations of 40° or less, provided the residual velocity of the projectile is at least about 10 times the velocity of the target. In this case, the maximum error in the determination of the miss by the Stibitz method is less than about 10 per cent of the miss.

The *Radial Grid* method does not require a vertical or a short base line. It does require, however, either quite accurate timing of the different frames (to within about 1/40 sec) or nearly perfect synchronization of the two cameras. The operator is supplied with a set of radial grids. These are made of transparent material about a foot square and are provided with straight lines radiating from a central point, marked from 0° to 360°, but not uniformly. From a short calculation based on the known approximate estimates of the azimuth and elevation of the axis of the camera in question, the operator selects the suitable radial grid to be used with each of the apparent paths of the tracer, relative to the target. He superposes the grid on each apparent path and looks for a pair of corresponding readings, one from each grid, which contains simultaneous tracers. For accurate work interpolation is necessary.

16.9 KINEMATIC MODELS FOR TRAINING PURPOSES

There is given[7] a study of a device in which a model airplane is exhibited at various distances and at various orientations to simulate the appearance of an actual airplane in flight, the observer viewing, not the model airplane itself, but rather the image of the model in a mirror. The device was designed for military training purposes. The airplane model is mounted on a universal joint on a vertical axis, and its orientation is at the disposal of the operator through the universal joint. The apparent distance of the model from the observer is changed by moving the mirror in which the observer views the image of the model.

Formulas and tables are supplied in the report referred to[7] which solve the problem of determining the positions of the mirror and those angles of rotation of the elements of the universal joint which would cause the model to produce a simulation of an airplane in a specified position or sequence of positions.

16.10 NUMERICAL DIFFERENTIATION AND SMOOTHING

In many investigations it is desirable to determine the rate of change of a function that is given numerically at successive instants. As the given numerical values are subject to error (due to rounding and perhaps other causes) and as such errors can have a large effect in the calculated rate of change, some method of smoothing is appropriate. A method is proposed[32] which has been used at Brown University, the Frankford Arsenal, and elsewhere. The method is as follows: A quadratic function is fitted by least squares to each successive 9 values of the function to be differentiated, and the first and second derivatives of this quadratic function at the central point are taken as the first and second derivatives of the tabulated function. The fitting and differentiation is carried out, once and for all literally, and formulas are obtained for the estimates of the first and second derivatives of a tabulated function in terms of the 9 ordinates or their differences. These formulas provide a method for simultaneously differentiating and smoothing a tabulated function. Functions of degree other than the second and numbers of tabulated values different from 9 can be used.

Chapter 11

THE RISK TO AIRCRAFT FROM HIGH-EXPLOSIVE PROJECTILES

11.1 INTRODUCTION

THIS CHAPTER is mainly concerned with some mathematical problems arising in attempts to estimate the probability of damage to an aircraft or group of aircraft from one or many shots from heavy antiaircraft [AA] guns. Related problems arise in air-to-air bombing and in air-to-air or ground-to-air rocket fire, but the major part of the mathematical analysis so far performed has been devoted to problems of flak risk.

The following basic numerical problem underlies the whole discussion of flak risk:

Problem (A). Find the probability of damage to an aircraft from a single shot aimed at an arbitrary point of space.

The solution of this problem, once it is attained, is used to investigate two major problems of flak risk.

Problem (B). To estimate the effects of range, altitude, speed, arrangement, and spacing of aircraft in a given bombing unit on the probability of damage to aircraft of the unit from one or many shots.

Problem (C). To study the influence of altitude, size of bombing units, and spacing between units on the probability of damage to aircraft in a large bombing mission composed of many bombing units attacking the same gun-defended area.

The major difference between problems (B) and (C) is one of scale; in problem (B) all the aircraft under consideration are flying close together in one rigid formation; in problem (C) all the aircraft attacking a given target area on a single bombing mission are considered.

Most of the work on problem (A) has been devoted to the following special case:

Problem (D). Find the probability of damage to an aircraft from a single shot aimed at the aircraft itself.

This special case is of greatest interest in comparing the relative effectiveness of two different shells and is, therefore, the question of first interest to anyone planning AA equipment. A solution of problem (D) is also useful to an air force since it can be used to study risks of damage to a single aircraft in terms of slant range and altitude. In addition, an air force also needs information about problem (A) in order to work out reliable solutions of problems (B) and (C) for any formation with more than a single aircraft in it.

This chapter is divided into sections as follows:

Section 11.2 discusses the intricate process of shooting down a bomber by a fragmenting projectile.

Section 11.3 outlines the mathematical problem involved in the computation of risks from AA fire.

Section 11.4 is a summary of the basic reports in the field including not only work done for the AMP but also some earlier reports dealing with problems (A) and (D). The first serious work on problem (D) available to the writer, and the first considerations involving problem (A) are to be found in two British reports [14, 15] issued by the Exterior Ballistics Department in 1940 and 1941. The first of these [14] is mainly concerned with developing the method for comparison of different shell types; the second [15] is principally devoted to a rapid method of finding an approximate solution of problem (D) from parameters describing the distribution of shell bursts. In 1942, in response to a request from the Chairman of NDRC, a comparison was made [16] of the probable effectiveness of time and proximity fuzes in AA fire against high level bombers attacking a ship at sea. Soon thereafter work was begun on a related study of the dependence of effectiveness of time and proximity fuzes on the fragmentation characteristics of the shell. The report on this study [18] was distributed in 1945. The last study of this group [11] uses an approximate method of solution of problems (A) and (D) in order to have some information with which to attack problems (B) and (C).

A similar problem in which the projectile is an airborne rocket is described in Section 11.4.5. Many

smaller studies, mostly concerned with comparison of effectiveness of different guns and projectiles, are grouped together in Section 11.4.6.

Section 11.5 outlines some of the applications of these damage calculations to problems (B) and (C); these applications are discussed more fully in the flak analysis manual [8] and in an AMP report.[11]

Section 11.6 is a brief concluding remark.

It is the aim of the chapter to describe a method for treating problems of risk. Specific numerical conclusions are likely to become obsolete before further need for them arises, but the technique by which the results were obtained will be useful as long as weapons which destroy by means of flying fragments are in use.

It may be well to try to describe, roughly, what AMP has and has not contributed to this subject of risk from high-explosive [HE] projectiles. The AMP does not supply the original experimental information on which the computations are based; this comes from a variety of sources, mostly Army and Navy, OSRD, and British reports. Much of the fundamental mathematical theory is contained in any elementary statistics text. The theory outlined in Section 11.3 is essentially that given in the early British report.[3] The principal problems are those of obtaining accurate original information (for which the AMP could take only what existed), of developing computational procedures which could be carried through before the project became obsolete, and of applying these techniques to selected examples. The selection of pertinent examples, the development of computing techniques, and the carrying through of the resulting involved numerical work have been the principal contributions of the AMP to the problem of risk from flak. The greater part of the work done for AMP in the field was carried out by the Statistical Research Group of Columbia University; one major report [11] came from the Applied Mathematics Group of Brown University and another [13] was worked out directly by the AMP staff rather than by its contractors.

11.2 THE PHYSICAL PROCESS OF ANTIAIRCRAFT FIRE

The physical process whose mathematical probability is to be computed consists of the following steps:

1. The original projectile is launched, arrives at some point in space, and bursts.
2. Fragments fly off from the point of burst in many directions at high speed.
3. Some (or none) of these fragments strike the target.
4. Some (or none) of the fragments striking the target do some damage.

To compute the resulting probability of damage we must be able to follow all these steps and collect the total risk from all the possible occurrences. Let us begin with a discussion of step (1).

The projectile we are discussing does not arrive in space of its own volition but is sent there by some mechanical device. If the device were perfect, the projectile would always burst in contact with its target and the probability of damage would be unity. However, guns, bombsights, and other existing devices have inaccuracies in design, construction, and operation, so the projectile is not very likely to burst just where it would be most effective. We must, then, consider in each small volume of space the likelihood, large or small, that the projectile aimed at the given target may burst in that region.

A great deal of machinery must be used to get the projectile to the point of burst. There must be some sort of fire control device to estimate the location of the best place for the projectile to burst; the projectile must be fired or launched with the proper direction and speed to get it somewhere near the desired point, and there must be some sort of fuze to set off the explosive charge in the projectile.

Let us discuss the problem of the heavy AA gun in some detail to get some idea of the imperfections that plague the first step in the process of shooting down aircraft with ground weapons. The principal parts of the mechanism and the function of each part is given in the following list.

1. Tracking instruments, optical or radar, measure the present bearing, elevation, and range of the target and feed this information continuously to the fire control computer.
2. The fire control computer uses the ballistic functions of the gun, estimates of the present position and rates of the target, and some assumptions about the kind of path the target is following to estimate where the gun should be pointed in order that a shell fired at present time with the assumed ballistic properties will reach the target's path at the future time when the target is there. For time fuzes, the computer also estimates the proper fuze setting to make the shell explode at the target.
3. Automatic transmission systems carry this in-

formation to the gun and fuze setter; servomotors point the gun continuously in the direction ordered by the computer.

4. The shell is fired from the gun along a trajectory determined by the bearing and elevation set in by the servos and by the muzzle velocity with which the shell leaves the gun.

5. If the shell has a time fuze, this explodes the shell some time after it leaves the gun. If the shell has a proximity fuze (also called VT fuze), and if the fuze operates, it explodes the shell if it comes close enough to the target, the required proximity depending on direction and rate of closure.

6. If the gun happens to be mounted on shipboard, the roll and pitch of the ship under the gun and director must be compensated for in steps (1) to (4).

Errors can occur in all these stages of the process. Tracking errors are normally small (of the order of several mils). These tracking errors tend to increase with large angular accelerations. The fire control computer has such a complex problem to solve that it usually is built with a number of simplifying approximations so that considerable errors can arise. The largest of these (common to all predictors) come from two sources: (a) linear prediction for curved paths, and (b) the time lags caused by the necessity of smoothing the rough information supplied by the tracking mechanism and by the time of flight from gun to target. The transmission and servo systems have a time lag and also have internal oscillations or "jitter." The fire control computer is usually exceedingly complex so that there is a certain amount of lost motion in gear trains and servos inside the computer itself.

After the shell is finally fired, it starts out along a trajectory which can no longer be influenced by any action of the gun crew. The purpose of most of the complicated fire control machinery is to get it started along a trajectory which will bring the shell close to the target.

If the shell is equipped with a proximity fuze, this is all that can be done; the fuze explodes the shell if the shell passes near enough to the target under suitable conditions of closure. If the shell has a time fuze other sources of error are still present. The computer has estimated where the target will be when the shell can reach it, has tried to set gun bearing and elevation to start the shell trajectory through that point, and has tried to set the fuze to explode the shell when it passes through that point. In addition to inaccuracy of prediction, errors along the trajectory are increased by the variability of time fuzes.

Once the fuze operates and the shell bursts, the next step in the process begins. In this step the nature of the shell and its remaining velocity are the important factors. The bursting explosive charge shatters the shell case into a number of fragments of various weights which fly off with various initial velocities. The number, the weight distribution, and the pattern in which they fly off from the burst vary with such quantities as the thickness of a shell casing, the total weight of the shell, the explosive charge, and other characteristics of the shell itself. The initial velocity a fragment possesses is the vector sum of the velocity it would have from the burst of a shell at rest in space plus the remaining velocity of the shell at the time of burst. Air resistance slows the fragments as they fly from the burst until they are going too slowly to do any significant damage; this air resistance varies with the air density, that is, with the altitude. Also, the fragments spread farther apart as they fly from the burst; the number of deadly fragments passing through each square foot of the sphere of radius 100 ft about the burst would be only one-fourth of the number of deadly fragments passing through each square foot of the sphere of radius 50 ft about the burst even if there were no air resistance whatever.

If the burst occurs close enough to the target, some fragments may strike with more or less effect. If the target is an exceedingly complex device, like a heavy bomber, there is still the difficult problem of estimating the probable effect of those fragments which strike even after we have an estimate of the probable mass and velocity distributions of the shell fragments at various distances and directions from the shell burst. Parts of the target are more vulnerable than others, some parts are more vulnerable from one direction than from another, some parts shield others. Nothing precise is known about the kind of fragment strikes required to kill or injure personnel. It is not known what characteristic of the flying fragment makes it deadly. Is it the kinetic energy, the momentum, or some other combination of mass and velocity? This information is needed in deciding which fragments to consider at any distance from the shell.

If at this point in the discussion it appears to the reader quite improbable that any aircraft is ever brought down by AA fire, he will have the correct impression as far as single shots at high altitude bombers are concerned. Records of American, British,

and German ground-based 90-mm to 5-in. guns collected at various times during World War II show that 1,500 to 2,500 shells were fired for each aircraft knocked down. This means that single AA guns are not much more than a nuisance to attacking bombers; *the Germans made their AA defenses extremely dangerous but only by massed firepower with good overall command.*

11.3 THE MATHEMATICAL ANALOGUE OF THE FIRE CONTROL PROBLEM

Because of the extreme complexity of the fire control problem, which can be solved only approximately by any mechanical device, it is not possible to decide whether or not a shot fired at a given instant will bring down the target; even a complete knowledge of the path of the target up to the time the shot was fired would not suffice for perfect prediction since the target has some considerable amount of time to get off its original course after the shot has been fired. It would, perhaps, be natural to try to formulate the problem of risk from a single shot by asking where the shell will burst and whether a burst there will destroy the target. Unfortunately these questions cannot be answered; we replace them by other questions phrased in terms of probabilities.

If a region of space is given, what is the probability that a shell will burst in that region?

If the shell bursts at a given point in space, what is the probability that the target will be destroyed?

How can the total probability that the target will be destroyed be computed from the answers to the preceding questions?

These first two questions do not have the same answers under all possible tactical situations, for example, the answer to the first question certainly depends, to some extent, on the range, altitude, speed, and direction of the target, and the answer to the second depends greatly on the structure of shell and target. However, once numerical expressions are obtained for the first two questions in a given situation, the third question can be answered by the computation of certain definite integrals.

In Sections 11.3.1 to 11.3.3 we discuss these three questions separately. Before going on to these separate questions we shall describe certain coordinate systems which will be used consistently through all that follows in order to help describe the tactical situation and the functions to be used in the formulation and solution of the problem. To see why such a diversity of coordinates appears in the problem, it is only necessary to note that the mere description of a tactical situation in which the probabilities are to be computed requires knowledge of the (future) range, elevation, bearing, direction and speed of motion of the target, and the type of gun, shell, and fuze.

For each shot from a heavy AA gun in a given tactical situation, the shell is intended to burst at some point O in space at a time when the target is at or near that point. We shall set up a rectangular coordinate system for locating O from the gun as follows (see Figure 1). The origin of the system is at the gun, the H axis is vertical, the X axis is horizontal and points in the direction of flight of the target aircraft; the Y axis is also horizontal and at 90° counterclockwise from the x axis, when viewed from above. Then the intended point of burst, O, can be described by its coordinates (X,Y,H) in this system. Various other coordinates are also used to locate O. Among these are the slant range $D = \sqrt{X^2 + Y^2 + H^2}$, the ground range $R = \sqrt{X^2 + Y^2}$, the angle θ between the X axis and horizontal projection of the line from the gun to O (this angle is the azimuth or bearing of O from the gun relative to the X direction), and E, the elevation angle of the line from the gun to O.

Parallel to the X,Y,H coordinate system take an $X_0Y_0H_0$ system with origin at O. This coordinate system is useful in describing the position relative to O of target planes at the time of burst.

We shall ignore the drift of the projectile and assume that the vertical plane through O and the gun contains the entire trajectory of the shell. We define an x,y,z coordinate system with its origin at the point of aim O, with the z axis pointing away from the gun along the trajectory, with the x axis pointing upward, perpendicular to the z axis in the vertical plane through the trajectory, and with the y axis chosen to make x,y,z a right-handed rectangular coordinate system, that is, at 90° counterclockwise from the x axis when viewed from the z direction. We shall also have occasion to use an a,b,c coordinate system superposed on the x,y,z system when discussing formations of aircraft in Section 11.5.

We shall assume, in the neighborhood of O to be considered, that the curvatures of the shell trajectories are small enough to neglect, and that these trajectories can be considered as straight lines parallel to the z axis. We shall also neglect any yawing of the shell and assume that the shell at the time of burst is spinning about an axis parallel to the z axis.

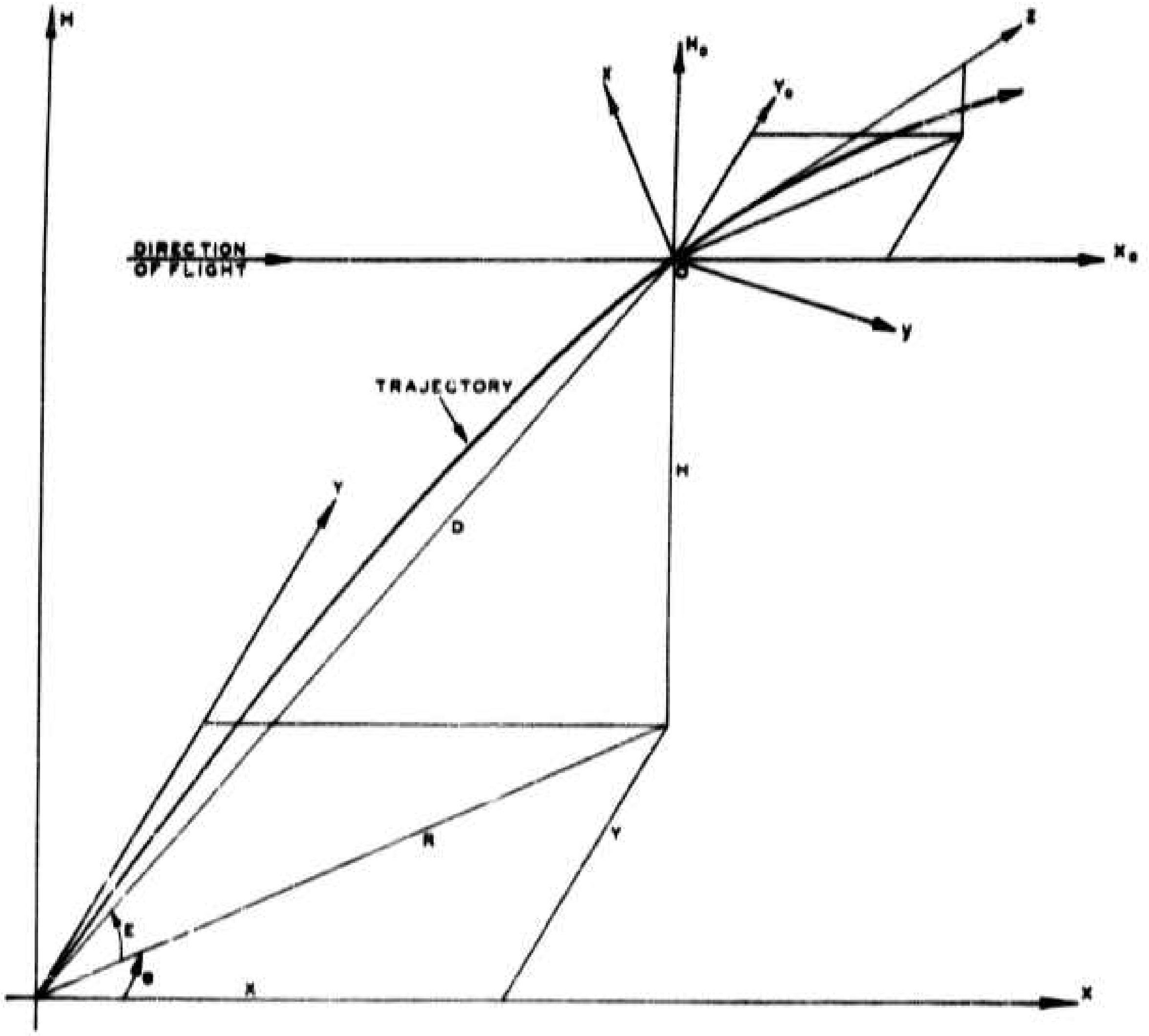


FIGURE 1. Coordinate systems at gun and point of aim.

In discussing the scattering of fragments from the point of burst it will be most convenient to use spherical coordinates (r,ϕ,ψ) where (see Figure 2) r is the distance from the shell burst, ψ is the latitude (so that $\psi = 90°$ defines the direction of motion of the shell and $\psi = 0°$ defines the equatorial plane of the shell), and ϕ is the remaining angular variable measured from the x direction toward the y direction in the equatorial plane of the shell.

Sections 11.3.1 to 11.3.3 discuss the separate questions raised at the beginning of this section. Together they give the mathematical background for solution of the problem of risk from a single shot and point out some of the difficulties in both theory and application. The mathematical technique is essentially the same throughout most of the major reports to be discussed in Section 11.4; it is that technique which is to be presented in this part of the chapter. The principal differences in method come in the use of special devices and methods of computation rather than in the basic theory.

In one sense it may be said that the present theory has yielded about all the general information which can be derived from it. To improve the results greatly will require new hypotheses in closer relation to the physical process and more accurate values of the numerical information which must be determined experimentally. At present, the effect of the speed of

the target is neglected in most parts of the computation and is included only in the study of the number of shots which can be fired while the target is

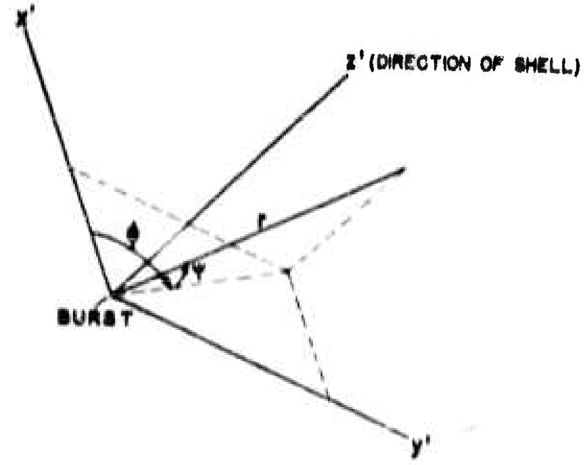


FIGURE 2. Coordinates of actual point of burst (parallel to the x,y,z system of Figure 1 at intended point of burst).

within range (see Section 11.5.1). The magnitude of the errors resulting from this neglect of the influence of speed on accuracy of fire and on the deadliness of the individual fragments is not known.

11.3.1 The Distribution of Shell Bursts

If in a given tactical situation (as nearly as it could be reproduced) a large number of shots were to be fired, they would not all burst in the same place but would be scattered through a large volume of space; the better the fire control mechanism, the nearer the center of this scattering of bursts will be to the point of aim.

We assume for each tactical situation that there exists a function $F(x,y,z)$ which we shall call the *probable density of bursts*, such that if a large number N of shots were fired in the given tactical situation, then the number of shots expected to burst in a cubic foot of space around (x,y,z) would be $N \cdot F(x,y,z)$. Regarded in another way, $F(x,y,z)$ is the probability that if a single shot is fired in the given tactical situation, the burst will occur in a cubic foot of space around (x,y,z). By the usual process of setting up a definite triple integral (see any good calculus book) it can then be seen that if U is any region in space, the probability that a single shot fired in the given tactical situation will burst somewhere in the region U is given by $\iiint F(x,y,z)dxdydz$ where the triple integral is taken over the region U.

The actual nature of this function $F(x,y,z)$, which expresses the probable density of shell bursts throughout space, is not very well known. Since the integral

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(x,y,z)dxdydz$$

expresses the probability that the shell will burst somewhere in space, this means that if all fuzes operate so that every shell fired bursts somewhere, then

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(x,y,z)dxdydz = 1. \tag{1}$$

If k per cent of the fuzes fail to operate, the triple integral would have the value $1 - k/100$. However, since this is a constant factor for a given fuze type it is easier to make the general computations under the assumption that all fuzes operate, and then correct the last stage of the calculations of risk by multiplying by this operability factor.

Recalling that the z axis points along the trajectory through the point of aim, we see that the distribution of bursts normal to the z axis is determined by errors and uncertainties in the fire control process and by the ballistic dispersion of the gun; the errors along the trajectory depend on these and on the fuze type as well. Hence the rest of this section falls naturally into two parts according as time or proximity fuzes are used.

Time Fuzes

For time fuzes it is generally assumed that the errors of the x-, y-, and z-coordinate directions are independent of one another. Mathematically this assumption of independence is formulated by saying that $F(x,y,z)$ is a product of three functions each depending on only one of the variables, that is, $F(x,y,z) = f_1(x)f_2(y)f_3(z)$. Of course the accuracy of this assumption is open to question, for example, with a fire control predictor using a preset altitude reading there is almost certain to be correlation between errors in the y and z directions (among others).

Since no further information is available about the structure of $F(x,y,z)$, it is further assumed that these errors in the three coordinate directions are Gaussian distributions with mean zero, that is, it is assumed that the errors in the three coordinate directions fit the normal error curve for the proper choice

of the standard deviation. Such a function has the form

$$f(x) = \frac{e^{-x^2/2\sigma^2}}{\sqrt{2\pi}\sigma},$$

so, multiplying together three such functions with the appropriate variables, and using the rules for multiplying exponentials, we see that, under all these assumptions, the probable density of bursts can be expressed in the form

$$F(x,y,z) = \frac{1}{(\sqrt{2\pi})^3\sigma_1\sigma_2\sigma_3} e^{-\frac{1}{2}(x^2/\sigma_1^2 + y^2/\sigma_2^2 + z^2/\sigma_3^2)} \quad (2)$$

where e is the base of natural logarithms and σ_1, σ_2, and σ_3 are the standard deviations of the errors in the x, y, and z directions, respectively. The values of σ_1, σ_2, and σ_3 must be found by experiment or by computations depending on the nature of the whole fire control situation. Their dependence on the tactical situation is not known at the time this is written although various estimates have been made.

A distribution law of the form (2) is sometimes called an *ellipsoidal* Gaussian distribution. This name is motivated by the shape of the surfaces on which $F(x,y,z)$ has any given constant value. If

$$e^{-\frac{1}{2}(x^2/\sigma_1^2 + y^2/\sigma_2^2 + z^2/\sigma_3^2)} = \text{constant},$$

then the exponent is also constant, that is

$$\frac{x^2}{\sigma_1^2} + \frac{y^2}{\sigma_2^2} + \frac{z^2}{\sigma_3^2} = \text{constant}.$$

It is shown in any elementary analytic geometry book that for any positive values of the constant this equation defines an ellipsoid with its principal axes along the coordinate axes.

Most of the reports summarized in Section 11.4 assume that the probable density of bursts is given by an ellipsoidal Gaussian law of the form (2). Most of these reports make the further assumption that lateral and vertical standard deviations are equal, that is, that $\sigma_1 = \sigma_2$. Then the probable density of bursts is given by a *spheroidal* Gaussian distribution of the form

$$F(x,y,z) = \frac{1}{(\sqrt{2\pi})^3\sigma_1^2\sigma_3} e^{-\frac{1}{2}[(x^2+y^2)/\sigma_1^2 + (z^2/\sigma_3^2)]}. \quad (3)$$

Letting $\sigma_1/\sigma_3 = r$ this can also be written as

$$F(x,y,z) = \frac{r}{(\sqrt{2\pi})^3\sigma_1^3} e^{-\frac{1}{2}[(x^2+y^2+r^2z^2)/\sigma_1^2]}.$$

One report,[16] discussed in Section 11.4.1, uses a more general ellipsoidal Gaussian distribution than that of equation (2). The formula is

$$F(x,y,z) = Ke^{-\frac{1}{2}Q(x,y,z)} \quad (4)$$

where $Q(x,y,z)$ is a general quadratic function of x, y, and z which can be written in the form

$$\begin{aligned} Q(x,y,z) = {} & a(x-\alpha)^2 + b(y-\beta)^2 + c(z-\gamma)^2 \\ & + 2f(y-\beta)(z-\gamma) + 2g(z-\gamma)(x-\alpha) \\ & + 2h(x-\alpha)(y-\beta), \quad (5) \end{aligned}$$

and where K is so chosen that the integral of $F(x,y,z)$ over the whole space is equal to one. It is shown in any solid analytic geometry text that the surfaces $Q(x,y,z) = \text{constant}$ are ellipsoids centered at (α,β,γ) when the coefficients satisfy certain conditions. These ellipsoids need not, however, be set with their axes along the coordinate axes. This more general distribution of bursts is studied in the report.[16]

Another report,[18] which will not be discussed further in this chapter, adds in the time errors due to the fuze as another Gaussian independent variable in addition to the space errors due to prediction and then gets the probable density function $F(x,y,z)$ by integrating over the time variable. In this way some account is taken of the effect of the speed of the aircraft on the distribution of bursts. The numerical calculations have not been reported, so the magnitude of this effect is still unknown.

PROXIMITY FUZES

The errors in the x and y directions are again assumed to be independent.

A proximity fuze contains a miniature radio transmitter and receiver. The fuze, as it travels through air, sends out radio waves and receives reflected waves from obstacles in the path of the radiation. When the reflected waves satisfy certain conditions, the fuze operates to fire a detonator. The point of such operation depends, for a given combination of fuze and projectile, not only on the size and reflecting properties of the target but also on the relative orientations and on the relative motion, including rate of closure, of projectile and target.

In the analysis which follows, the target is taken to be specified, and its position is regarded as fixed in the (x,y,z) coordinate system. The projectile is assumed to travel along a straight trajectory parallel to the z axis and is assumed to have a fixed velocity. These conditions are approximately fulfilled, for a given tactical situation, in a small neighborhood of the intended point and time of burst. Along such

trajectory there is assumed to be a theoretical point of burst, and the totality of such points is taken to be a smooth surface, called the *burst surface*, defined by an equation of the form $z = f(x,y)$.

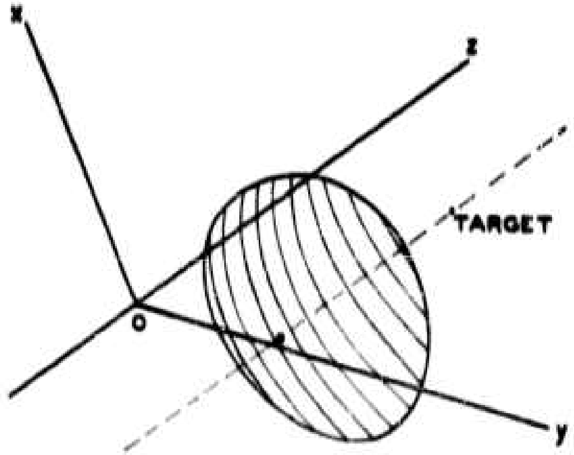


FIGURE 3. Sketch of a burst surface (projectile travels parallel to z axis).

For different types of fuze and projectile, the general form of the burst surface will vary. However, it will be assumed for the sake of mathematical analysis that this general form is fixed, but that its position and size, relative to the target, depend on a parameter S which can conveniently be thought of as the "sensitivity of the fuze." It is important to note, however, that this terminology is a mere convenience. It does not imply any particular principle of functioning. It merely lumps together certain more or less unpredictable sources of variation in the position of the burst surface B_S, whose equation, for any particular value of S, will be written in the form:

$$z = f_S(x,y). \tag{11}$$

Various assumptions will be made as to the dependence of $f_S(x,y)$ on the parameter S. These are merely working hypotheses to make a suitable mathematical analysis possible. For the sake of specialized practical applications, modifications might be required in the hypotheses and the analysis based on them. The principal assumptions and basic definitions follow.

1. Corresponding to each value of S, there is a region U_S in the (x,y) plane, which is the domain of definition of $f_S(x,y)$. If a trajectory passes through U_S, it meets the corresponding burst surface B_S and the fuze will function. If a trajectory misses U_S (and hence B_S) a fuze of sensitivity S on that trajectory will not function.

2. If the target is at the point (a,b,c), then, for each S, $f_S(x,y) < c$. In other words, the burst surface is closer to the gun than is the target, along any trajectory. Note that this would not hold for a VT fuze with a delay, intended to make it function slightly after passing the target.

3. As the sensitivity S increases, the domain U_S also increases and the burst surface B_S gets farther from the target along each trajectory. In other words, analytically speaking: If $S_1 < S_2$, then U_{S_2} contains U_{S_1} and $f_{S_2}(x,y) < f_{S_1}(x,y)$ wherever both functions are defined.

4. As S varies over its entire range, the burst surface B_S sweeps out a solid region V in space.

5. The function $f_S(x,y)$ has a continuous partial derivative with respect to S.

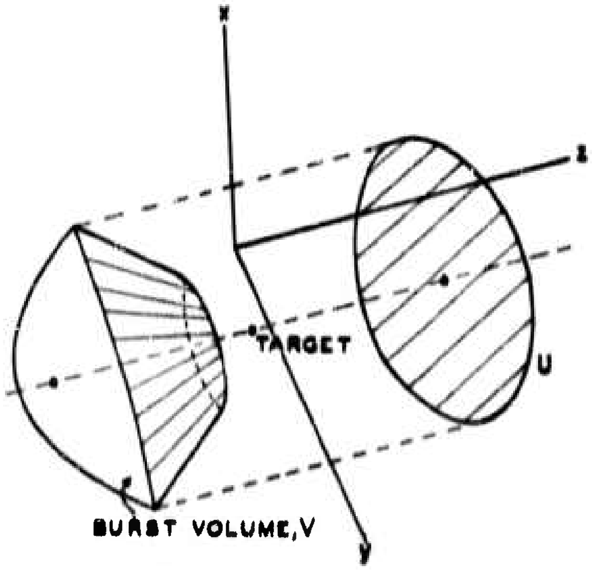


FIGURE 4. The burst volume, the part of space in front of the target in which proximity fuzes may operate.

To compute $F(x,y,z)$ it will also be necessary to use the distribution of sensitivities. Suppose that in a given tactical situation $f(x,y)$ is the probable density of shell trajectories, that is, the probability that the shell will pass through a small patch of area dA around (x,y) is given by $f(x,y)dA$. Hence the probability that the trajectory will lie in a region A of the x,y plane is given by the double integral

$$\iint_A f(x,y)\,dx\,dy.$$

Similarly, suppose that the probability that the sensitivity lies in the interval from S to $S + dS$ is $p(S)dS$. Then the probability that the sensitivity lies between S_1 and S_2 is given by

$$\int_{S_1}^{S_2} p(S)dS.$$

We also assume that fuze variations are independent of aiming errors.

Now to compute $F(x,y,z)$ from this we need to compute the probability of a burst in some small volume dV about (x,y,z). If (x,y,z) is not in the region V, then $F(x,y,z)$ is defined as zero. If (x,y,z) is in V, then dV will be chosen so small as to lie entirely in V, and will be taken as a block bounded by a pair of planes x = constant, a pair of planes y = constant, and a pair of surfaces S = constant. (See Figure 5.) The projection of this block on the

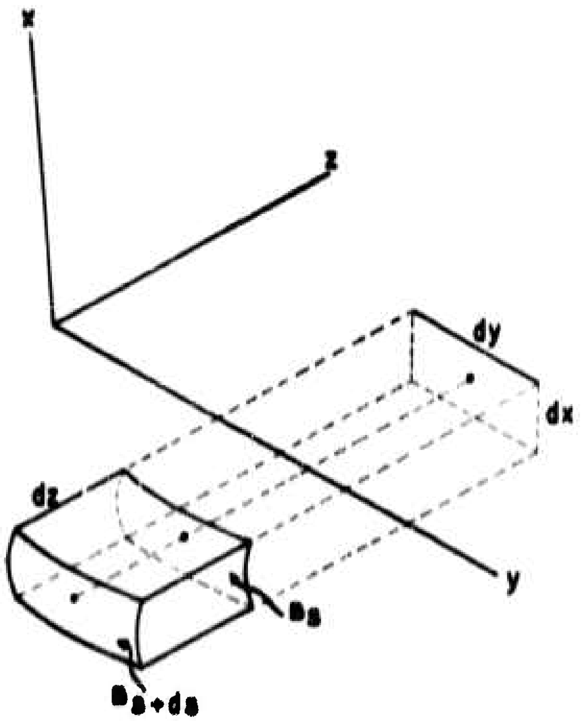


FIGURE 5. Typical volume element between two burst surfaces.

x,y plane is a rectangle with edges dx and dy. The curved faces of the block are cut off by the surfaces B_S and B_{S+dS}. We also assume that the fuze sensitivity is independent of the fire control errors. Then the probability that the trajectory passes through the rectangle is $f(x,y)dxdy$; the probability that the sensitivity lies between S and $S + dS$ is $p(S)dS$. By the assumption of independence of fuze and aiming errors, the probability that both these occur is just the product

$$p(S)dSf(x,y)dxdy. \tag{7}$$

Now the volume of the block, since its two curved faces are nearly parallel, is approximately $dxdydz$. Since $z = f_S(x,y)$, we can hold x and y fixed and find that

$$dz = \frac{\partial}{\partial S} f_S(x,y)dS.$$

Hence the volume of the block is

$$dV = \frac{\partial}{\partial S} f_S(x,y)dxdydS$$

and the probable density of bursts around (x,y,z), when $z = f_S(x,y)$ is given in terms of x,y, and S by

$$F(x,y,z) = \frac{p(S)dSf(x,y)dxdy}{dxdydz} = \frac{p(S)f(x,y)}{\dfrac{\partial}{\partial S} f_S(x,y)} \tag{8}$$

when $z = f_S(x,y)$. Since $z = f_S(x,y)$ we can define $F_0(x,y,S)$ to be $F[x,y,f_S(x,y)]$ and rewrite the equation as

$$F_0(x,y,S) = \frac{p(S)f(x,y)}{\dfrac{\partial}{\partial S} f_S(x,y)}$$

if (x,y) is in V_S;

$$F_0(x,y,S) = 0 \tag{9}$$

if not.

Inside V we can solve equation (0) for S in terms of x,y, and z. The solution is unique as a consequence of hypotheses (3) and (4), and we write

$$S = g(x,y,z)$$

if (x,y,z) is in V. Then

$$dS = \frac{\partial}{\partial z} g(x,y,z)dz$$

and

$$p(S) = p[g(x,y,z)].$$

Substituting in equation (7) and dividing through by $dxdydz$ gives

$$F(x,y,z) = f(x,y)p[g(x,y,z)]\frac{\partial}{\partial z} g(x,y,z)$$

if (x,y,z) is in V;

$$F(x,y,z) = 0 \tag{10}$$

if (x,y,z) is not in V.

If we assume (as is usual in fuze calculations) that the errors in the x and y direction have independent

Gaussian distributions with mean zero, we get specialized forms of equations (9) and (10).

$$F_0(x,y,S) = \frac{p(S)e^{-\frac{1}{2}(x^2/\sigma_1^2 + y^2/\sigma_2^2)}}{\frac{\partial}{\partial S} f_S(x,y)}$$

If (x,y) is in U_S;

$$F_0(x,y,S) = 0 \tag{11}$$

If (x,y) is not in U_S; and

$$F(x,y,z) = e^{-\frac{1}{2}(x^2/\sigma_1^2 + y^2/\sigma_2^2)} p[g(x,y,z)] \frac{\partial}{\partial z} g(x,y,z)$$

If (x,y,z) is in V;

$$F(x,y,z) = 0 \tag{12}$$

If (x,y,z) is not in V.

Most reports that have dealt with damage calculations for proximity fuzes have made various approximations instead of attempting to carry out this involved computation. The first approximation assumes that all fuzes have exactly the same sensitivity S so that all shells burst on a single surface B_S. As we shall show in Section 11.3.3, this assumption reduces the final computation of risk from evaluation of a triple integral to evaluation of a double integral.

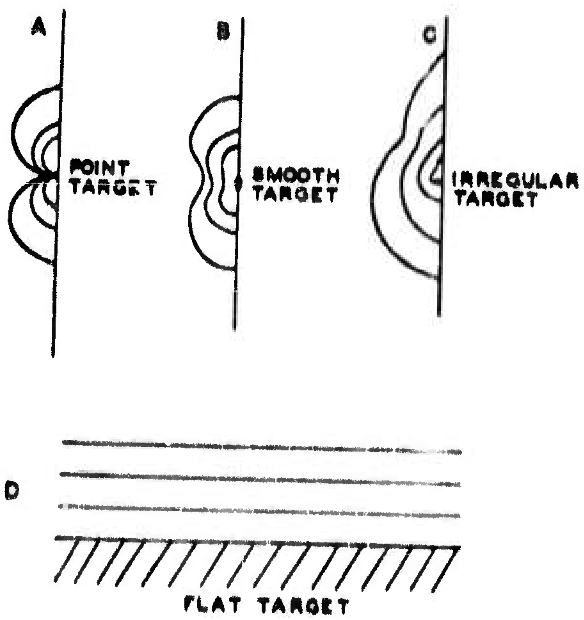


FIGURE 6. Cross sections through burst surfaces.

Further approximations are made by assuming special simple shapes for the burst surface. Two such assumptions used in two reference reports [9,10] are that the burst surface is a hemisphere centered at the target with axis of symmetry along the trajectory, or that the burst surface is a disk centered on the trajectory and lying in a plane at right angles to the trajectory.

The actual shape of the burst surface for a given sensitivity S will depend greatly on the size and shape of the target. If it is assumed that the antenna of the fuze acts like a radiating dipole, the burst surfaces for a point target should be symmetric about the trajectory through the target and should have cross sections resembling the half-circles in Figure 6A. If the target is still fairly small and reasonably smooth, the burst surfaces will be partly symmetrical with cross sections more like those of Figure 6B; if the target reflects with entirely different intensities from different directions, the burst surfaces will be much more lopsided as in Figure 6C. If the target is a large plane surface, such as a lake or ocean (targets commonly used in testing sensitivity), the burst surfaces for a given angle between the trajectory and plane surface will be planes parallel to the given plane surface. (See Figure 6D.)

The bulk of the computations that have been done with proximity fuzes was carried through before any proximity fuzes had been produced, so most computations carried out by the AMP are carried out for a single burst surface of simple type. The reports discussed in Section 11.4.5 are an exception to this; tests with proximity fuzes in airborne rockets were available to the men working on that project (AMP Study 21).

Contact Fuzes and Solid Projectiles

The problem of computing risk from solid projectiles, such as machine gun bullets, and from explosive projectiles with contact fuzes is a simplification of the problem for proximity fuzes. In this case the projectile must pass through some part of the target to be effective. The burst surface is then the surface of the target turned toward the gun, and the distribution of hits is determined by the probability density function $f(x,y)$ giving the distribution of trajectories cutting the (x,y) plane.

11.3.2 The Conditional Probability of Damage

The next problem to be dealt with is that of giving the risk of damage from a burst which occurs at any given point in the neighborhood of the target in the tactical situation under discussion. Here we do not

consider how the shell comes to burst at that point; the problem is: given a point of burst, what is expected to happen if the shell bursts there? All work with this problem so far has required rather strong assumptions to be made from a small amount of experimental information.

We can simplify our formulas slightly by assuming that the target is at the origin O. Later we can easily account for the trivial effect of moving the target to another point. We shall attempt to find a function $p(x,y,z)$ to describe the probability that a shell bursting at (x,y,z) will destroy the target at the origin. We shall call this function the *conditional probability of damage*.

The values of $p(x,y,z)$ depend on many characteristics of the shell and target. These factors are the weight and shape of the fragments from the bursting shell, the speeds and directions with which they are thrown off from the burst, the air density, the size, shape, and orientation of the target, the arrangement in the target of vulnerable parts such as pilots and fuel tanks, the vulnerability of these parts to fragments of the kind thrown off from the burst, the speed of the target, and its direction relative to the shell trajectory.

It should be noted here that many of these factors are ignored or suppressed in most calculations of risk. A common broad assumption is that the target can be represented by a sphere hanging in space at the point of aim; this assumption removes all effect of the speed of the target on the conditional probability and reduces the effects of the complex vulnerability characteristics of the target to the choice of a single variable, the size of the sphere.

For either an ideal or an actual target, the first step in computing $p(x,y,z)$ is to describe the fragmentation pattern of the shell used. This description is simplest in the spherical coordinate system r,ϕ,ψ shown in Figure 2 where the line $\psi = 90°$ points along the shell axis toward the nose of the shell. If we neglect yaw of the projectile and assume that the shell points along the trajectory, this makes the line $\psi = 90°$ parallel to the z axis.

Because of the spin of the shell and its axial symmetry we can assume that the number of fragments of a given mass and speed thrown out by the burst will be independent of the longitude angle ϕ. It is generally assumed that the effectiveness of a flying fragment must depend on its shape and mass, and on the relative speed of fragment and target at the instant the fragment strikes.

If the target speed is small enough, it makes no great difference to the effectiveness of a fragment if the target speed is neglected entirely and it is assumed that the target is stationary in space at the instant the shell bursts. For high speed targets the effectiveness of a fragment, particularly of a heavy fragment that can do damage at speeds of a few hundred feet per second, is largely dependent on whether it is going in the same or the opposite direction as the target.

Neglecting the target speed greatly simplifies the calculations of the conditional probability because it simplifies the problem of deciding which fragments to count. When target speed is zero, the relative speed of a fragment reduces to its speed through the air; as with mass and shape, there is no reason to believe that this speed varies with the longitude. Hence in case target speed is zero, we can choose fragments for consideration by criteria involving weight and speed and still see that the expected distribution of these selected fragments is independent of the longitude angle ϕ.

In most of the work done heretofore, the fragments to be counted have been selected on one of two bases: first, by putting up screens at various distances from a bursting shell and counting the number of fragments that penetrate the screens, and second, by assuming that a fragment must have a certain minimum kinetic energy to do damage and counting only those fragments with at least that much energy.

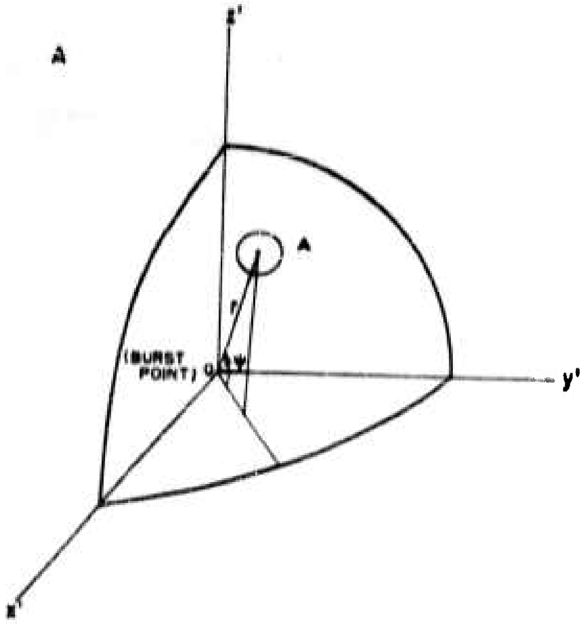
We have a choice of several similar methods of describing the pattern of fragments from a shell burst providing only that it is independent of longitude.

1. The angular density $\rho(r,\psi)$ of fragments at any point with coordinates (r,ϕ,ψ) is the number of fragments per solid unit angle[a] passing through the sphere of radius r about the burst in a small region about the direction denoted by (ϕ,ψ).

This angular density $\rho(r,\psi)$ is written without ϕ to show its dependence on r and ψ alone. To estimate it for a given position (r,ϕ,ψ), count the fragments going through a given small area A on the sphere of radius r, about any point at latitude ψ and divide this number of fragments by the solid angle A/r^2 subtended by the area A.

2. The area density of fragments is the number of deadly fragments per square foot of surface at dis-

[a] A unit solid angle is a cone reaching out from the origin which cuts off an area of 1 square unit on a sphere of radius 1 unit.



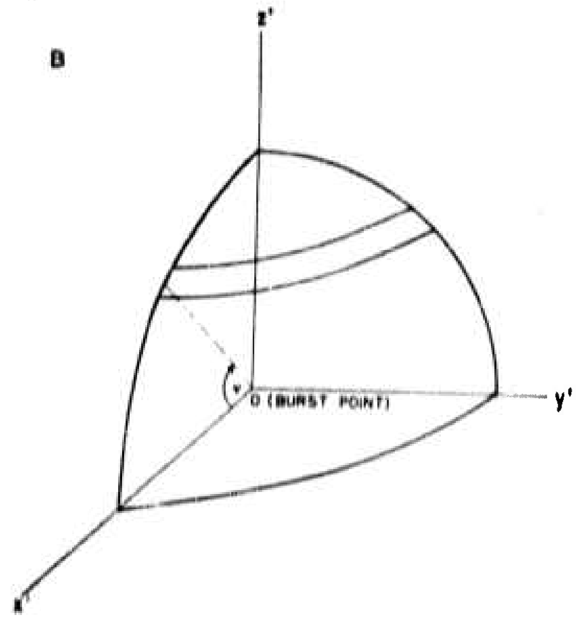


FIGURE 7A. Typical region for estimating angular fragment density, z' axis points toward shell nose. FIGURE 7B. Typical zone for calculating zone count.

tance r and latitude ψ. It is found from the formula $\rho(r,\psi)/r^2$.

3. The zone count $n(r,\psi)$ is obtained by dividing the sphere of radius r around the burst into a number of zones, say of width 2.5°, and counting the number of fragments passing through each zone. The term $n(r,\psi)$ is the number of fragments in the zone containing ψ on the sphere of radius r.

As the fragments fly further from the burst they are slowed by air resistance so that fewer of them are effective; hence $\rho(r,\psi)$ and $n(r,\psi)$ decrease with increasing r.[b] If the weight, speed, and ballistic shape distribution of fragments is independent of ψ we can break $\rho(r,\psi)$ into a product of two factors

$$\rho(r,\psi) = \rho(\psi)g(r) \tag{13}$$

where $\rho(\psi)$ defines the angular density of fragments which have energy enough to do damage very close to the burst, and $g(r)$ describes the decrease in the number of deadly fragments with distance r from the burst. Similarly we could write

$$n(r,\psi) = n(\psi)g(r) \tag{14}$$

with $n(\psi)$ the number of fragments in the zone about ψ with energy enough to do damage very near the burst, and $g(r)$ the same function as in equation (13). The term $g(r)$ is called the *fall-off law* for effectiveness of fragments.

Various types of experiments have been performed to find this *angular fragmentation pattern* and to get estimates for the fall-off law $g(r)$.

1. Wooden screens ruled off in square feet are placed at various distances from the bursting shell and the number of fragments penetrating each square foot area is counted. This gives values of $\rho(r,\psi)$ [or $n(r,\psi)$] for several particular values of r and some values of ψ. However, the values of ψ must be corrected in some way for the fact that the shell in the test is at rest while the shell in any tactical situation has some considerable remaining velocity which tends to throw the fragments in the moving burst pattern forward of the burst zone in which they appear in the static burst patterns.

2. Shells are burst in sand pits and in sandbag beehives; the fragments are sifted out of the sand and sorted by weight or by the fineness of the screen they sift through. These tests are to determine the distribution of weights of the fragments of the shell and give no information about the angular fragment pattern.

3. Various photographic and penetration tests have been devised to measure the speeds at which fragments are thrown off from the bursting shell. These are usually static tests and give values in the

[b] There are some indications that 50 or 100 ft from the burst, fragments may still be breaking apart. One experiment[14] shows no decrease in $n(r,\psi)$ as r went from 40 to 60 ft.

neighborhood of 2,500 fps for the initial fragment velocity.

4. Firing against actual aircraft on the ground under conditions where the location of the burst can be determined. This helps to determine the relation between angular density of fragments and the conditional probability of damage to aircraft.

In terms of the angular fragment pattern it is not difficult to estimate the expected number of hits on an aircraft from its size and distance from the burst. However, the aircraft is a complex target and fragment hits on parts of it are only a minor nuisance, not fatally damaging. To pass from the fragment density $\rho(r,\psi)$ to the number of *hits* on a target presenting the area A toward the burst at distance r away, we need only multiply the angular fragment density $\rho(r,\psi)$ by the solid angle A/r^2, subtended by the target. If we are interested in the expected number of *damaging hits* due to the fragments, we can use the same formula provided we use for A not the actual area presented by the target, wings, tail surfaces, and all, but only that area vulnerable to the fragments being used in the computations. This concept of the *vulnerable area* of the target is, of course, a fiction but an extremely useful one in making computations of the conditional probability of damage. Estimates of vulnerable area are usually arrived at by comparing experiments of the types (1) and (4) just referred to.

Data of the four types (1) to (4) have been collected by a number of organizations. A considerable body of data on fragmentation characteristics of U.S. Army projectiles is collected in a series of TDBS Reports.[a] A report of Section T of OSRD[17] contains the most exhaustive study extant of conditional probability and vulnerable area derived by test firing against aircraft on the ground. The British have also made a number of studies of fragmentation characteristics of their own shells.

Once the vulnerable area A is found, the expected number of damaging hits on the target at (r,ϕ,ψ) is given by the formula

$$m(r,\psi) = \frac{A\rho(r,\psi)}{r^2}.$$

The next problem is to pass from the expected number of damaging hits on the target to the probability of damage to the target. This depends on how nearly the actual scattering of fragments from a single burst conforms to the expected pattern for shells of its type. It is generally assumed that this scattering of fragments follows the so-called Poisson law (which has been observed in other problems of random scattering). This law implies that if k is a whole number and m is the expected number of hits on a given target, then the probability of getting *exactly* k hits is

$$e^{-m}\frac{m^k}{k!}.$$

That is, the probability of 0, 1, 2, 3, $\cdots$, hits is

$$e^{-m},\ me^{-m},\ \frac{m^2e^{-m}}{2!},\ \frac{m^3e^{-m}}{3!},\ \cdots.$$

Hence the probability of getting *at least* one hit is 1 minus the probability of getting no hits, that is

$$p(x,y,z) = 1 - e^{-m}, \tag{15}$$

where $m = m(r,\psi)$ and (r,ϕ,ψ) is the location of the origin O from the point (x,y,z) at which the burst occurs. Some experimental justification of this equation is given in a reference report.[18]

If we are interested only in the problem D of the introduction of finding the risk to a single aircraft at the point of aim, then $p(x,y,z)$ and $F(x,y,z)$ can be combined to obtain the probability of damage. To solve problem A, however, we must have the conditional probability that a shell bursting at (x,y,z) will destroy a target at some point (a,b,c). This new probability is easily computed from what we know already, for moving both burst and target by the vector $(-a,-b,-c)$ we see that the risk to a target at (a,b,c) from a shell burst at (x,y,z) is precisely the risk to a target at $(0,0,0)$ from a shell burst at $(x-a, y-b, z-c)$, that is, the risk is equal to $p(x-a, y-b, z-c)$.

It is to be remembered that in passing from number of hits to number of effective hits, we tacitly assumed that the vulnerable area A did not depend on θ; that includes the assumption that the speed of the target and the aspect from which it is viewed have no effect on the conditional probability. No careful study of this has ever been completed, for computational difficulties would be considerable. The effect of target speed has been considered,[19] but numerical results have never been published.

11.3.3 The Probability of Damage to a Single Aircraft

Once the probable density of bursts and the conditional probability of damage are given, the calcu-

[a] Technical Division, Ballistics Section, Office of the Chief of Ordnance.

lation of the risk from one shot at a single aircraft in a given tactical situation can be calculated by means of a triple integral. Let $P(a,b,c)$ be the probability of damage to a target at (a,b,c) from one shot fired at O in the given tactical situation. To set up the triple integral for $P(a,b,c)$, cut the space around O into small bricks by planes parallel to the coordinate planes, and consider any such brick with center at a point (x,y,z) and edges of length dx, dy, and dz. Then, from the definition of probable density of bursts, the probability that the shell will burst in

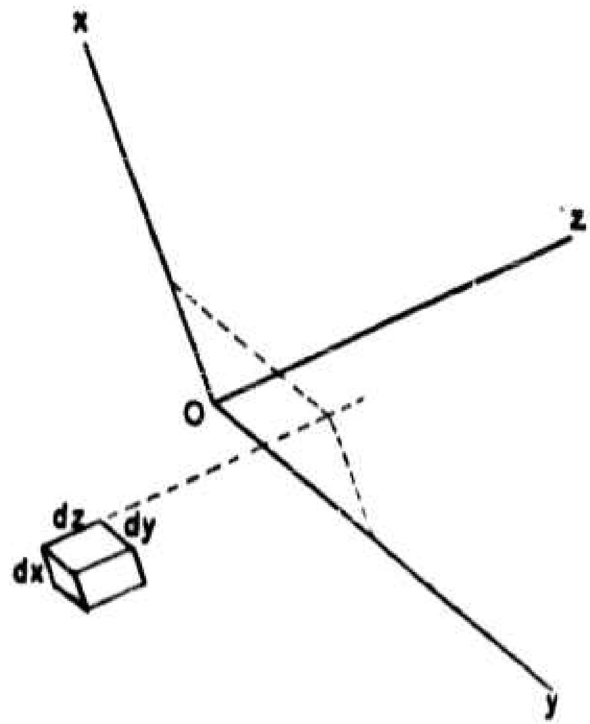


FIGURE 8. Volume element $dxdydz$.

this region is $F(x,y,z)dxdydz$. If the shell bursts there, the probability that it will damage the target at (a,b,c) is $p(x-a, y-b, z-c)$. Hence the probability that the shell will burst in this small region *and* damage the target at (a,b,c) is the product

$$p(x-a, y-b, z-c)F(x,y,z)dxdydz.$$

Summing up over all such small regions, we see that the risk $P(a,b,c)$ can be given by the triple integral

$$P(a,b,c) = \iiint_{-\infty}^{\infty} p(x-a, y-b, z-c)F(x,y,z)dxdydz. \quad (16)$$

In case the target aircraft is at the point O, we have $a = b = c = 0$ and the risk for such an aircraft is

$$P(0,0,0) = \iiint_{-\infty}^{\infty} p(x,y,z)F(x,y,z)dxdydz. \quad (17)$$

It is this last formula which has been used by most writers trying to solve problem (D) for time fuzes; the formula for $P(a,b,c)$ is needed in solving problem (A) and therefore has applications to problems (B) and (C). (Some discussion of this is contained in Section 11.5.)

For proximity fuzes and a single target at the origin, the integral has usually been replaced by a simpler double integral by making the assumption that all fuzes have the same sensitivity, that is, that all the shells burst on a single surface in front of the target. That surface has an equation of the form $z = f(x,y)$. Assuming independent Gaussian errors in x and y directions, the probability that the shell will pass through a small rectangle of sides dx and dy about (x,y) is

$$\frac{1}{2\pi\sigma_1\sigma_2} e^{-\frac{1}{2}(x^2/\sigma_1^2 + y^2/\sigma_2^2)}.$$

If the shell is on the trajectory through (x,y), then either it is too far away from the target to burst at all or it is in the region U where $f(x,y)$ is defined and the shell bursts at $[x,y,f(x,y)]$; hence the probability that a shell on the trajectory through (x,y) will damage the target at O is $p[x,y,f(x,y)]$ if (x,y) is in U, and is zero if not. Hence the risk from trajectories passing through the small rectangle is

$$p[x,y,f(x,y)]e^{-\frac{1}{2}(x^2/\sigma_1^2 + y^2/\sigma_2^2)}dxdy$$

and the total risk to a target at the origin is given by

$$P(0,0,0) = \iint_U p[x,y,f(x,y)]e^{-\frac{1}{2}(x^2/\sigma_1^2 + y^2/\sigma_2^2)}dxdy. \quad (18)$$

$P(a,b,c)$ could be computed by a similar process.

It is worth while to mention that risk from solid projectiles and contact-fuzed explosive projectiles can be computed from the formula (18) provided that proper interpretation of the various terms is made. For contact-fuzed projectiles the analogy is extremely close. The burst surface for contact-fuzed projectiles is the surface of the target turned toward the negative z axis. The term $p[x,y,f(x,y)]$ is a function $p'(x,y)$ determined by the risk from a burst of the given shell at that point on the surface of the aircraft; the risk is then given by equation (18), which can also be rewritten

$$P(0,0,0) = \iint_U p'(x,y)e^{-\frac{1}{2}(x^2/\sigma_1^2 + y^2/\sigma_2^2)}dxdy. \quad (19)$$

If $p'(x,y)$ is regarded as the probability that a given solid projectile entering the aircraft on the trajectory determined by (x,y) will damage the aircraft, then this same formula can be used for solid projectiles. This shows that there is a close analogy

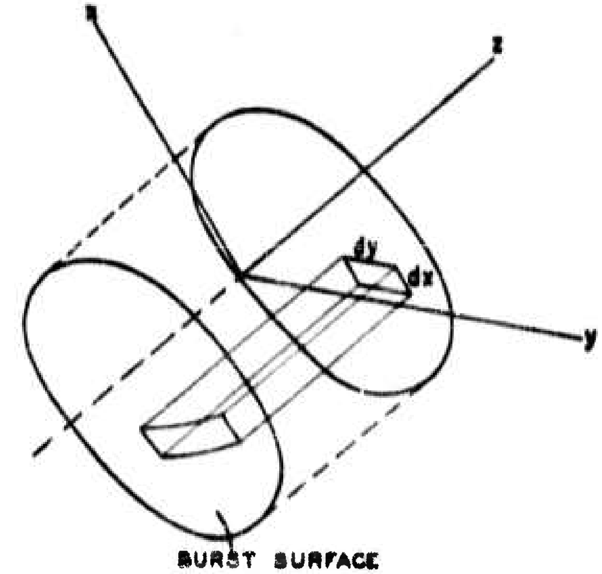


FIGURE 8. Area element and burst surface.

between the risks from contact-fuzed and proximity-fuzed projectiles; the difference is largely a difference of scale. The contact-fuzed shell may be effective if it passes through the target; the proximity-fuzed shell may be effective if it passes through a certain larger region near the target.

We may note that if a computation of $P(a,b,c)$ or $P(0,0,0)$ is made from one of these formulas on the assumption that all fuzes operate, then, to estimate the effect if k per cent of the fuzes fails to operate, it is only necessary to put a factor $(1 - k/100)$ in the probable density of bursts. Since this is a constant factor, it can be removed from under the integral sign in these equations for risk so the risk is easily corrected for operability of the fuze.

In applying the various formulas of this section it is soon discovered that the integrals can seldom be evaluated in any simple form but must be computed numerically. For any given special case, this evaluation is not too difficult; however, it becomes extremely tedious and expensive if computations must be made for a large number of examples.

The integrals for risk from time and proximity fuzes are usually evaluated by converting them to the (r,ϕ,ψ) coordinate system with origin at the shell burst.

11.4 HISTORICAL SUMMARY OF PRINCIPAL STUDIES OF RISK TO AN AIRCRAFT FROM A SINGLE SHOT

In this section we describe the specific problems studied in certain reports on risk of damage from high-explosive shells and rockets, outline the assumptions and the nature of the results, and give some ideas of the methods used.

Section 11.4.1 deals with two British reports;[14, 15] Section 11.4.2 discusses OSRD Report No. 738,[16] the first major study of this kind made in this country. Section 11.4.3 is devoted to another such study started soon after OSRD 738 but published years later as AMP Note No. 19.[18] Section 11.4.4 deals with the last large study of flak risk, AMP Report No. 185.1R.[11] Section 11.4.5 discusses some questions of risk to a bomber from airborne rocket fire.[3-5] Section 11.4.6 discusses some AMP reports of special aspects of the flak problem; for instance, one such report discusses the probability of shooting down a directly approaching aircraft.[8] Other reports in this section deal with comparison of different guns and projectiles, usually for directly approaching aircraft.

The attitudes with which these reports regard problem (D) vary widely because of the difference in the specific problems that required some study of risk. In the first British report,[14] the emphasis is on the comparison of the effectiveness of different types of AA shells and of different standards of accuracy in shooting. A large part of the report is also devoted to a full development of the theoretical methods used in the study of risk. The second British report[15] is vitally concerned with the effect of the distribution of shell bursts on the probability of damage and attempts to get a formula for quick approximate calculation of the risk when the probable density of bursts is given in a certain form. OSRD Report No. 738[16] is mainly concerned with comparison of the effectiveness of time and proximity fuzes for a given shell in half a dozen different tactical situations. AMP Note No. 19[18] also compares the risk for time and proximity fuzes; a tactical situation is fixed and the emphasis is placed on the effects of variations in the fragmentation characteristics of the shell. AMP

Report 185.1R [11] attempts to solve problems (A), (B), and (C) of the Introduction (Section 11.1) and therefore gives an approximate solution of problems (A) and (D) which can be computed for many different tactical situations; in particular, for many different ranges and altitudes of the target.

11.4.1 Two British Studies

In 1940, the British developed a method [14] for the comparison of the effectiveness of different AA shells under varying assumptions about the accuracy of the AA fire. The mathematical argument is, essentially, that outlined in Section 11.3 of this chapter for time-fuzed shells.

The probable density of bursts is assumed to be Gaussian with errors independent in the three coordinate directions and, moreover, it is assumed that the standard deviations σ_1 and σ_2 in the x and y directions (across the trajectory) are equal. Then the probable density of bursts takes the form

$$F(x,y,z) = \frac{1}{(\sqrt{2\pi})^3\sigma_1^2\sigma_3} e^{-\frac{1}{2}[(x^2+y^2)/\sigma_1^2 + z^2/\sigma_3^2]}.$$

The fragmentation data came from several sources:

1. Counts of the number of fragments penetrating wooden screens placed at different distances from the shell burst.

2. Counts of the number of disabling hits on a Blenheim bomber hung in the main fragment zone of the shell bursts.

3. Counts of number and weight distribution of fragments from shells exploded in sandbag beehives.

From the first source it was possible to estimate the fragment density $\rho(r,\psi)$ in terms of the distance and direction of the target from the shell. By comparing (1) and (2) a correlation between fragment density and damage to a Blenheim bomber was found for the type of shell used in this test. From (3) information as to the fall-off of the number of deadly fragments with distance from the shell is obtained for a number of different shells.

The actual complex target is replaced by a hypothetical target presenting a vulnerable area of 100 sq ft from any distance or direction. The fragmentation pattern has been described by dividing the hemisphere forward of the burst into zones of 2.5° width and giving the number of fragments in each zone which penetrate the 2-in. wooden screen of test (1) at the distance in question. The number of these fragments expected to strike a disk of 100 sq ft area was computed as follows: If the center of the disk is at (r,ψ), the disk itself overlaps some of the 2.5° zones. In each zone divide the area of the part of the disk in that zone by the area of the zone and multiply by the number of fragments in that zone to get the expected number of hits from that zone. Add the results to get the expected number of hits on the disk (at the range r and latitude ψ).

The comparison of tests (1) and (2) suggested that the Blenheim bomber received one damaging hit for each seven "throughs" (fragments penetrating a 100 sq ft area of wooden screen) at the same latitude and distance from the burst. Hence $m(r,\psi)$, the expected number of damaging hits on an aircraft at range r and latitude ψ, was found from the screen tests and fall-off data. Direct counts of the number of "throughs" were not available for values of r greater than 150 ft, so the dependence of $m(r,\psi)$ on r for larger r was estimated from the fall-off of velocity of the fragments flying through the air.

Once $m(r,\psi)$ is given, the probability of at least one damaging hit is, from the Poisson law, $1 - e^{-m}$, where $m = m(r,\psi)$. An appendix of this report [14] discusses in some detail the applicability of the Poisson law to the scattering of shell fragments; the evidence then available fitted very well.

The integration of the triple integral was done numerically in such a way that the portion of the risk due to certain regions around the shell could be found. It was found, for example, that under many circumstances more than half of the risk comes from bursts more than 100 ft from the target.

This early report is fundamental in the field; it contains a clear discussion of the aims and difficulties in attempting to find the risk from a single shot.

A second British report [15] describes a method of computing the probability of damage when the probable density of bursts does not have the simple form

$$\frac{1}{(\sqrt{2\pi})^3\sigma_1\sigma_2\sigma_3} e^{-\frac{1}{2}(x^2/\sigma_1^2 + y^2/\sigma_2^2 + z^2/\sigma_3^2)}$$

given in equation (2) but follows the more general ellipsoidal Gaussian law given in equations (4) and (5); that is

$$F(x,y,z) = \frac{K}{(\sqrt{2\pi})^3} e^{-\frac{1}{2}Q(x,y,z)}, \tag{4}$$

where

$$\begin{aligned} Q(x,y,z) = {} & a(x-\alpha)^2 + b(y-\beta)^2 + c(z-\gamma)^2 \\ & + 2f(y-\beta)(z-\gamma) + 2g(z-\gamma)(x-\alpha) \\ & + 2h(x-\alpha)(y-\beta). \end{aligned} \tag{5}$$

Q is any general quadratic form whose level surfaces are ellipsoidal, and K is chosen so that the integral of $F(x,y,z)$ over all space is one. Most of this paper deals with the case of no bias, that is, $\alpha = \beta = \gamma = 0$.

By using certain approximations in the numerical evaluation of the triple integral for probability of damage, a function $I_0(\lambda_0)$ is defined in terms of the type of shell used, the altitude of the burst, and the remaining velocity of the shell. This function is tabulated for two shells (British 3.7 in. and 4.5 in.) at altitude 10,000 ft and remaining velocity 1,350 fps. Two approximations to $P(0,0,0)$ are given in such a form that they can be computed quickly from the coefficients of the quadratic form $Q(x,y,z)$. The simplest approximation is

$$P(0,0,0) = KI_0(\lambda_0),$$

where

$$\lambda_0 = \frac{a + b + c}{3}$$

and a, b, and c are taken from the given polynomial Q. A somewhat more accurate approximation is

$$P(0,0,0) = KI_0(\lambda_0),$$

where

$$\lambda_0 = \left(\frac{a+b}{2}\right)\cos^2\psi_0 + \frac{c}{2}\sin^2\psi_0,$$

and ψ_0 is an angle depending on the direction of the main fragment zone of the bursting shell and, hence, depending on the type of shell and its remaining velocity at the time of burst. These approximations are reasonably good as long as the standard deviations are greater than 100 ft. Tables are given for correction of these approximations to other altitudes and remaining velocities, so a quick estimate of risk for either of the two shells considered can be given in any tactical situation in which the probable density of bursts can be described in the form of equations (4) and (5), with $\alpha = \beta = \gamma = 0$.

11.4.2 OSRD Report No. 738

This report [10] compares the effectiveness of time and proximity fuzes for Navy 5-in., 38-caliber shells. Since the report was written before any knowledge was available of actual burst surfaces of a proximity fuze, computations were carried through for two types of surfaces, hemisphere and disk, and for a wide variety of sensitivity ratios.

The probable density of bursts for time fuzes was assumed to follow a spheroidal Gaussian law (equation (3)) with standard deviations σ_1 and σ_3 across and along the trajectory. Dependence of the risk on σ_1 and σ_3 was studied along with the dependence on fuze characteristics. The computations for time fuzes are made from the formulas (3) and (17). The computations for proximity fuzes were made from the formula (18) with $\sigma_1 = \sigma_2$, with U a circle of radius b feet centered at O and with $f(x,y) = \sqrt{b^2 - x^2 - y^2}$ for the hemispherical burst surface of radius b, and $f(x,y) = -d$, a constant, for a disk burst surface of radius b centered d feet in front of the target.

Various tables give:

1. P_1, the risk from a time-fuzed shell, in terms of the standard deviations σ_1 and σ_3 across and along the trajectory.

2. P_2, the risk from a proximity-fuzed shell with hemispherical burst surface, in terms of the standard deviation σ_1 across the trajectory and of the radius of the hemisphere.

3. P_3, the risk from a proximity-fuzed shell with a disk burst surface, in terms of the standard deviation σ_1 across the trajectory, the radius b of the disk and the distance d from the center of the disk to the target.

4. The advantage ratios P_2/P_1 and P_3/P_1 in terms of these parameters.

It may be noted that for the range of values of σ_1 used in this report, the risks from proximity fuzes vary approximately inversely with the square of σ_1, the standard deviation across the trajectory. That is to be expected, for the proximity fuze risk behaves much like the risk from a solid projectile. The difference is in the area through which a proximity-fuzed shell may pass and still be effective. In both cases there is a certain area of the x,y plane such that a shell on a trajectory passing through a given point (x,y) of that area has a certain probability of doing damage. For solid or contact-fuzed projectiles, this area is the presented area of the target; for proximity-fuzed projectiles, this area is the presented area of the burst surface. For large values of σ_1, the function

$$\frac{e^{-\frac{1}{2}[(x^2+y^2)/\sigma_1^2]}}{2\pi\sigma_1^2}$$

is approximately equal to $1/(2\pi\sigma_1^2)$ over the burst surface. Substituting in equation (18) and taking the constant factor $1/(2\pi\sigma_1^2)$ from under the integral sign leaves an integral not dependent on σ_1; hence the risk is approximately proportional to $1/\sigma_1^2$.

Some study is also made of the effect of altitude

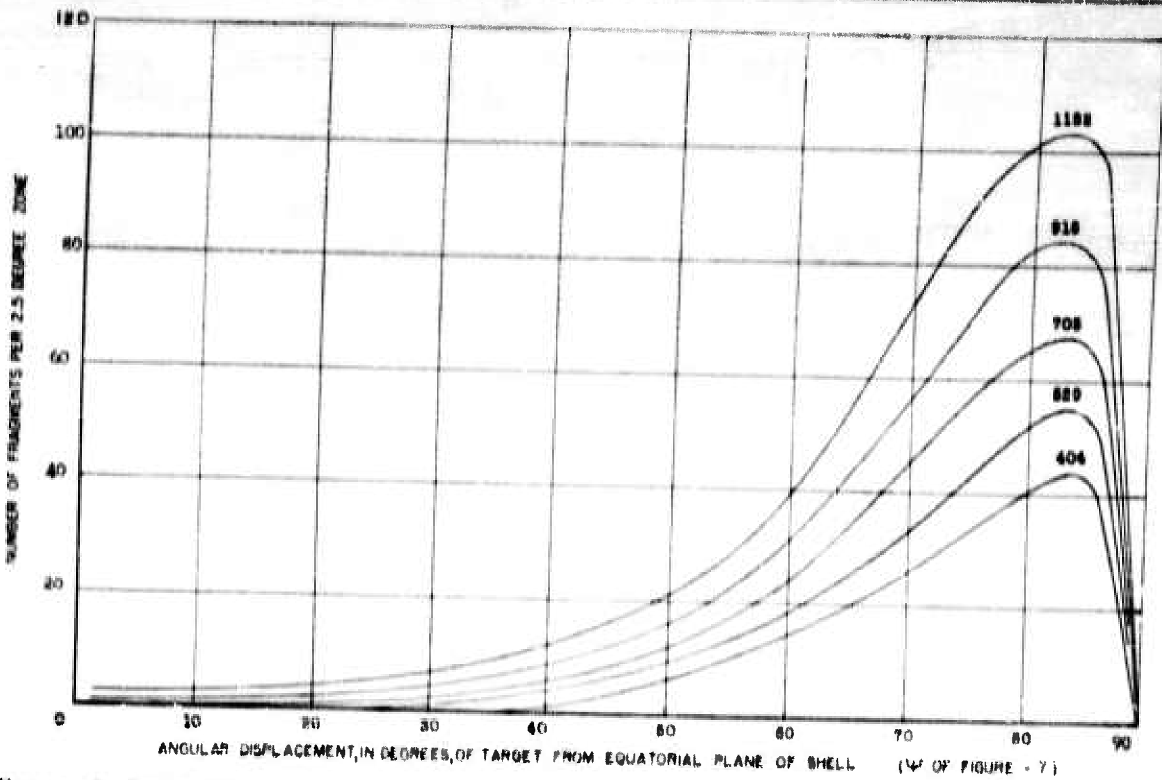


FIGURE 10. Optimum fragmentation patterns for various total numbers of fragments (time fuze shell; 5"/38 fall-off law).

on these risks. This information is used to find the advantage ratio of proximity over time fuzes in half a dozen different tactical situations specified by altitude, slant range, and fire control errors.

The results of this study indicate that in the typical tactical situations considered proximity fuzes would give from $2\frac{1}{2}$ to 5 times the probability of damage from time fuzes. The range of variation in relative effectiveness is due to the variations in aiming errors, altitude, and shape of burst surface. In spite of the uncertainties in the computation, this report indicated that if proximity fuzes could be made to operate even half or three-fourths of the time, then proximity fuzes would be measurably more effective than time fuzes. This prediction has been borne out by combat records. The results of the next study [18] also emphasize the same conclusion although the formal final report was not issued until long after proximity fuzes had been in wide use.

One section of the report compares the variation of effectiveness of shells of different weights. Another section discusses the dependence of the effectiveness on the weight distribution of fragments from the burst. This has its effect on the rate at which the fragments lose speed as they fly from the burst and on the speed a fragment must retain to have enough energy to do damage when it strikes the target.

In both this and the preceding report the target is assumed to be divided into three parts of different vulnerabilities (these parts are intended to correspond to pilot and bombardier, fuel system, and motor). The term $m(r,\psi)$, the number of fragments effective against this vulnerable area in direction ψ and distance r, is computed from two sets of data; first, the angular fragmentation pattern of the shell under discussion which gives the number of fragments in each zone at the burst itself, and, second, sand pit tests giving the number of fragments of given weight thrown off by the burst. Assuming that the weight pattern of fragments is the same in all zones, the number of fragments still effective at distance r is computed from the law for retardation of fragments by air friction and from the energy needed to be effective when the target is struck.

11.4.3 AMP Note No. 19

This report,[19] like OSRD 738, does not take into consideration the functioning characteristics of

proximity fuzes. Both reports compare time and proximity fuzes, but, in the later one, a particular tactical situation is specified and a study made of the dependence of damage probabilities on the following fragmentation characteristics: (1) angular distribution of fragment pattern, (2) total number of fragments, (3) the fall-off law for effectiveness, and (4) the weight distribution of fragments. The investigation compares time fuzes, proximity fuzes with hemispherical burst surfaces, and proximity fuzes with disk burst surfaces. Such burst surfaces are recognized as hypothetical, but the methods developed are of wide applicability.

For each fuze type, individual angular zone contributions to the total probability of at least one damaging fragment hit are analyzed. The zones are measured in terms of latitude relative to the equatorial plane of the shell. One of the principal results is the development of optimum fragmentation patterns for each type of fuze and for various given total numbers of fragments. In the case of a time fuze, for example, the optimum pattern concentrates most of the fragments in forward zones, the heaviest concentration being about 5° to 15° aft of the shell nose. Such patterns are graphically portrayed in Figure 10 (identical with Figure 8 of AMP Note 19). For the hypothetical burst surfaces considered, optimum fragmentation patterns give heavier concentrations of fragments further back from the nose, in a manner dependent on the total number of fragments and (especially for disks) on the diameter of the burst surface.

For a time-fuzed shell, effectiveness is revealed to be more sensitive to increases in the total weight of the shell (more fragments in each weight class) than for proximity-fuzed shells. On the other hand, a more considerable gain is indicated for proximity fuzes, rather than time fuzes, from increases in the fineness of fragmentation, when the total weight of the shell is held fixed.

11.4.4 AMP Report 185.1R

This report [11] is primarily concerned with the larger scale problems (problems (B) and (C) of the introduction) of flak risk and therefore requires solutions for problems (A) and (D) for a large number of values of the slant range and elevation. This is the first report so far mentioned in which large biases in the dispersion of shots had to be considered. In finding the risk to a large group of aircraft from a single shot it is obviously impossible to assume that every aircraft is at the point of aim; it is necessary to be able to estimate the risk to each aircraft of the formation when the burst distribution is centered at some given point (say the lead aircraft of the formation).

Since this report is concerned with problems (A), (B), and (C) as well as problem (D), some other simplifications had to be made to make it possible to take account of the effects of all the new variables such as slant range, altitude, and the location of the target relative to the point of aim. All computations are for time fuzes. The target (as in reference 14) is a sphere. A fictitious shell fragment pattern is used in which all the fragments are assumed to fly off in one nude fragment zone of width 20°; it is assumed that in that zone the fragment density is constant. Hence for any values of the dispersion parameters studied in the preceding reports, the solution of problem (D) given these for actual shell patterns will be more reliable than the solution given in AMP 185.1R. On the other hand, none of the other reports give any information about problem (A). The method of computation for this report assumes that the probable density of bursts is spheroidal Gaussian (equation (3)), but the magnitudes of the standard deviations σ_1 and σ_2 across and along the trajectory are taken much larger than in any of the preceding studies and are assumed to be determined by the (future) slant range from the gun to the target. Some account of remaining velocity of the shell is also taken as is the influence of altitude on air resistance, but the computations do not assume any dependence of $F(x,y,z)$ on the height or speed of the target nor any dependence of $p(x,y,z)$ on the motion of the target.

Since this chapter is being prepared by the author of AMP 185.1R, it need only be mentioned that the whole attitude of this chapter has been influenced by the work done and information gathered during the preparation of that report. It will be referred to again in connection with the discussion (Section 11.5) of applications of problems (A) and (D) to the general problems (B) and (C) which are of more obvious immediate concern to an air force. It may only be noted that all the computations carried out for this report on problem (A) indicate that for time fuzes and for large values of σ_1 and σ_2, $P(a,b,c)$ can be approximated by an expression very similar to that defining $F(x,y,z)$ except that the center is shifted forward and the lateral dispersion increased. The

forward shift is due, of course, to the forward spray of the fragments from the burst; the increase in lateral dispersion is caused by the wide side spray. That is to say, if $F(x,y,z)$ follows the spheroidal Gaussian law

$$F(x,y,z) = Ke^{-\frac{1}{2}[(x^2+y^2)/\sigma_1^2 + z^2/\sigma_2^2]},$$

then $P(a,b,c)$ can be approximated by a formula of the form

$$P(a,b,c) = e^{-\frac{1}{2}[(a^2+b^2)/(\sigma_1^2+r^2) + (c-c_0)^2/\sigma_2^2]}$$

where c_0 and r are determined by the shell and its remaining velocity as well as being influenced by σ_1 and σ_2.

Table 1 shows the variations of accuracy of fire with slant range assumed in AMP 185.1R using

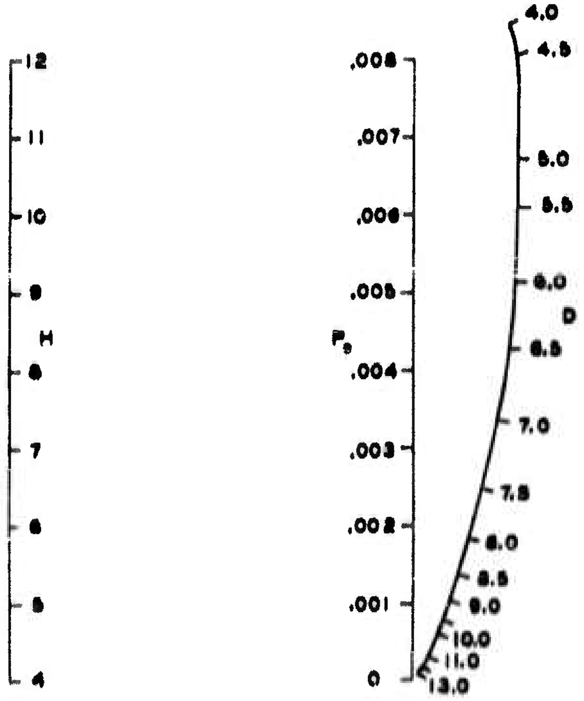


FIGURE 11. Nomograph showing the relationship under service conditions between P_0, the risk to a single aircraft at the point of aim; D, the slant range from gun to aircraft; and H, the altitude of the aircraft. Nomograph valid for $D \geq H$. D and H are measured in thousands of yards. To use the nomograph lay a straight-edge from H on the left-hand scale to D on the right-hand scale; read P_0 on the central scale.

these assumptions. Figure 11 shows a nomograph (reproduced from Figure 3s of that report) giving the relationship between P_0 the risk to a single aircraft at the point of aim, H the altitude of the point of aim, and D the slant range to the point of aim.

TABLE 1. Standard deviations σ_2 along the trajectory and $\sigma_1 = \sigma_3$ across the trajectory as functions of slant range D under "Service Conditions" assumed in AMP Report 185.1R.

D (1,000 yd)	σ_1 (feet)	σ_2 (feet)
4	98	1,200
5	104	1,130
6	158	1,084
7	225	1,070
8	330	1,120
9	450	1,190
10	580	1,300
11	750	1,420
12	940	1,600
13	1,130	1,800

11.4.5 Airborne Rocket Fire

In this study of the risk to a bomber from an airborne rocket fired from astern it was expected that fire control errors would be small enough that most of the risk would come from close bursts, bursts so close that the size and shape of the target would be important. Hence the conditional probability was computed for an actual aircraft (German Ju-88) rather than for a spherical target.

The rocket is assumed to have a proximity fuze; the whole study is largely motivated by questions of fuze design. This study is more fortunate than OSRD 738 and AMP Note 19, in that some firing test data were available for the proximity fuze T5 of the 4½-in. airborne rocket.

The technique of calculation of the conditional probability function is given in the first [8] of a series of reports on the study. The bomber is divided into sections and for each section the vulnerable area is estimated. For a given point of burst, computations from the fragment pattern of the rocket give the expected number of hits on each of the vulnerable parts. Taking account of the shielding of one part by another (from the given point of burst) the total expected number m of deadly hits can be found. The conditional probability is taken to be $1 - e^{-m}$ in the usual way. This report also calculates the errors in the conditional probability that would arise if the actual target with its scattered vulnerable parts were replaced by a sphere of the same presented area at the center of the actual target.

Table 2, reproduced from Table 1 of this report,[8] shows this conditional probability.

TABLE 2. Conditional probability that a 4½-in. proximity-fuzed rocket fired from astern will destroy a twin-engine bomber.
Ju-88 equipped with Jumo 211 (liquid cooled) engines and assumed unable to return to base on one engine.

Distance, (x) in feet, from tail of airplane measured toward nose	Probability that burst at indicated point will prevent return to base if hopset parameter, (p) in feet, is				
	20	40	60	100	140
Aspect angle (θ) = 0° (bursts directly above airplane)					
−12	0.130	0.027	0.100	0.075	0.020
−2	0.227	0.104	0.244	0.065	0.020
8	0.253	0.444	0.232	0.073	0.032
18	0.754	0.458	0.221	0.118	0.080
28	0.905	0.835	0.591	0.294	0.101
38	0.980	0.820	0.586	0.272	0.134
48	0.490	0.349	0.262	0.137	0.070
58	0.017	0.024	0.077	0.063	0.042
Aspect angle (θ) = 90° (bursts directly to one side of airplane)					
−12	0.101	0.091	0.053	0.038	0.013
−2	0.205	0.065	0.094	0.041	0.011
8	0.394	0.193	0.150	0.043	0.018
18	0.382	0.312	0.147	0.047	0.020
28	0.998	0.503	0.284	0.113	0.058
38	0.997	0.050	0.380	0.147	0.098
48	0.637	0.519	0.185	0.070	0.085
58	0.200	0.011	0.051	0.035	0.019

The conditional probability is used in a second report [4] to find the location of that surface which would give the greatest probability of damage if it were possible to design a fuze with that burst surface.

From this material and from records of actual fuze performance, in a third report [5] the risk to the bomber from a rocket fired from astern was computed in terms of the accuracy of aim of the rocket.

11.4.6 Comparison of Different Projectiles

AMP Study 27 compares the effectiveness of 5-in. high-explosive shell, 5-in. shrapnel (both with time fuzes), 40-mm and 20-mm HE (both with contact fuzes). The computations with time fuzes are made in more or less the way described in Section 11.3, but the other types of fuzes use modified procedures appropriate to their shell types.[6,7]

A shrapnel shell acts somewhat like a flying shotgun; when the fuze detonates the shell, a number of spherical balls are pushed out of the shell case and travel forward in the path the shell was following. Most of their speed comes from the remaining velocity of the shell but there is some scattering. Since the balls are a good ballistic shape and comparatively heavy, they retain their effectiveness at much greater distances from the burst than do most of the jagged fragments from the burst of a high-explosive shell. The conditional probability function for a shrapnel shell is zero except in a narrow cone (of aperture 5° or so) about the nose of the shell. This makes the computation for a shrapnel shell simpler than for a high-explosive shell where most of the fragments come out in a side spray somewhat forward of the shell equator.

The computations for contact-fuzed projectiles are simplified from those of proximity fuzes as we discussed near the end of Section 11.3.3. Those areas of the target vulnerable to the particular ammunition being used are projected on the x,y plane, giving an area U. Then for a hit in each vulnerable region, the probability of damage from a hit in that region is estimated and the final risk computed from a double integral

$$P = \iint_U F(x,y)\,p(x,y)\,dx\,dy$$

where $F(x,y)$ describes the probable density of trajectories in the x,y plane and $p(x,y)$ is the risk from a shell on the trajectory passing through x,y. The actual computation is done by dividing the target area U into sections on which p and F are practically constant, computing the risk for each section separately, and summing.

Comparison of the two 5-in. projectiles is com-

pleted with the calculation of the probability of damage with a single shot of each kind. To compare effectiveness of the other types of projectiles in stopping an attack, other factors, such as the rate of fire and the range at which effective fire can be begun, must also be considered.

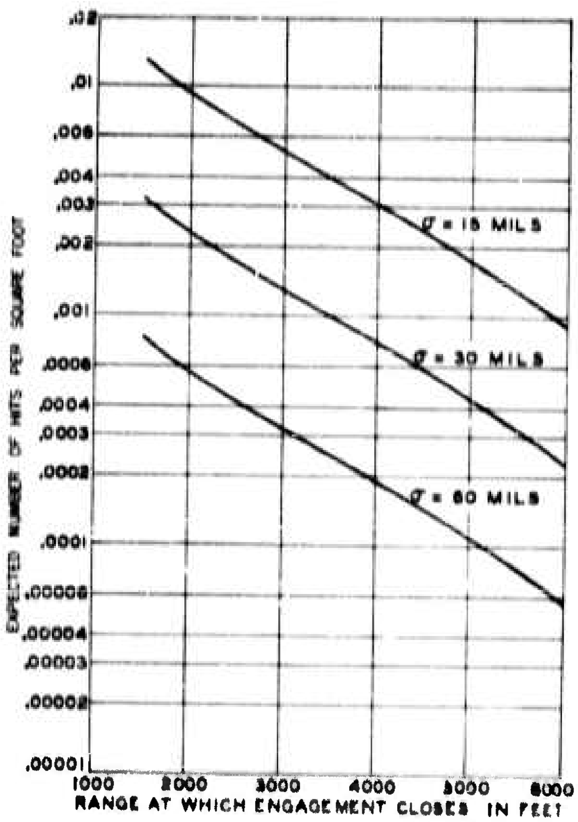


FIGURE 12. Expected number of hits on each square foot of target area presented to the gun as a function of the range at which the engagement closes. 7.7-mm gun firing 400 shots per minute; opening range is 7,500 ft, target velocity 250 fps; σ is the standard deviation of the bullet pattern.

The earliest AMP study of risk deals with a directly approaching aircraft and automatic weapons. It is assumed that the shots have a normal distribution about their mean point of impact and that the mean point of impact has a normal distribution about the target center. The report [1] gives the probable number of hits and quick means of estimating upper and lower bounds for the number of hits in terms of the presented shape of the target and the standard deviations of these Gaussian distributions.

Another study [2] is concerned with the increase in risk to a high-altitude bomber due to increase in the length of its bombing run. For various lengths of bombing runs the risk from both fighter aircraft and AA guns is considered. This study was motivated by the suggestion that aircraft dropping guided missiles might need to continue in steady straight-line flight until the missile struck, instead of beginning evasive action immediately on releasing bombs.

Another study compares the expected number of hits per square foot of target for four different automatic weapons: 7.7 mm, 13.2 mm, 20 mm, and 40 mm. This expected hit density is given on a number of charts in the report [8] in terms of the rate of fire, the range at which fire is begun and stopped, the target speed, and the accuracy of fire (assuming a constant angular dispersion at all ranges). Such a chart is reproduced here as Figure 12.

A later report [9] compares the effectiveness of the 20-mm gun, one existing machine gun (caliber 0.50), and one projected new machine gun (caliber 0.60) for installation in antiaircraft turrets. The comparison is made in terms of the accuracy of the guns, the rates of fire, and the relative effectiveness of the individual projectiles. Comparison of the caliber 0.50 and the 20 mm on various bases is reasonably reliable, but the comparison is quite sensitive to the accuracy of fire involved, so conclusions involving the caliber 0.60 (with its unknown accuracy) are somewhat more tentative.

A recent study [10] estimates the vulnerability of different parts of a very heavy bomber (B-29) from different directions and different ammunition types (armor-piercing, incendiary, high-explosive, and mixed). The numbers given in the tables cannot be used directly for other types of projectiles or for fragments from a heavy AA shell, but are indicative of the relative vulnerability of the different parts of the B-29. Table 3, reproduced from this report,[10] shows some of these vulnerability estimates.

The latest study [12] bearing on conditional probability compares three different types of bombs. For each bomb the surfaces are given on which the probability is 0.9 or 0.5 of destroying a bomber (Ju-88) by a burst of a given bomb on that surface. Characteristics of the bombs were taken from various TDBS reports (issued by the Chief of Ordnance); vulnerability characteristics of the target were those used in AMP Study 21 (described in Section 11.4.5).

TABLE 8. Vulnerability of B-29 to fighter attack, by components attack from directly ahead.

Component	Type of damage	Equivalent lethal area (sq ft) 7.7 mm AP	7.7 mm I	12.7 mm AP	12.7 mm I	20 mm API	20 mm HEI	Cooperative damage assumed required to prevent plane from returning to base
	Unconditionally lethal							
Gas tanks	Fire	0	6.0	6.0	21.4	25.8	51.4	
Fuselage	Serious or lethal wound to *both* pilot and co-pilot	0	0	0	0	0	6.6	
	Conditionally lethal							
Each inboard engine	Engine stoppage	6.1	5.5	12.7	11.4	16.8	15.1	Two out of four engines must be stopped; lethal hit on engine or associated fuel tanks sufficient to stop engine
Associated gas tanks	Leakage sufficient to stop engine; no fire	0	0	3.0	0	5.3	4.5	
Engine plus gas tanks	Engine stoppage; no fire	6.1	5.5	10.0	11.4	22.1	19.0	
Each outboard engine	Engine stoppage	6.1	5.5	12.7	11.4	16.8	15.1	
Associated gas tanks	Leakage sufficient to stop engine; no fire	0	0	3.5	0	4.0	4.0	
Engine plus gas tanks	Engine stoppage; no fire	6.1	5.5	10.2	11.4	21.4	19.1	
Fuselage								
Pilot (same for co-pilot)	Serious or lethal wound; other pilot not incapacitated	1.5	1.5	2.5	2.0	2.5	10.0	Both pilot and co-pilot must be incapacitated
Each of two sets of elevator controls	Severing of cables	0.3	0.3	0.8	0.8	1.2	2.2	Elevator controls completely duplicated; both sets must be severed
Rest of frontal area		Not vulnerable						
Total plane from head on	Unconditionally lethal	0	6.0	6.0	21.4	25.8	58.0	
	Conditionally lethal	28.0	25.0	72.2	51.2	94.4	119.8	
	Grand Total	28.0	31.0	78.2	72.6	120.2	177.8	

11.5 APPLICATIONS OF FRAGMENTATION AND DAMAGE CALCULATIONS TO FLAK ANALYSIS

Fragmentation and damage studies involve a great deal of tedious computation and, hence, are usually undertaken only for some serious reason. The problems that should be studied are raised by two classes of people, the shooters and the dodgers. The broad questions are these:

1. What can be done to gun, shell, or director to improve the effectiveness of AA guns? Another formulation of this question would be: What change in effectiveness will result if some proposed change in AA equipment and tactics is made? The decision to produce proximity fuzes required some sort of answer to such a question and led to the studies issued as OSRD Report No. 738,[16] and AMP Note No. 19.[18]

2. What can be done by equipment and tactics to reduce the effectiveness of a given AA defense which cannot be avoided?

The manual *Flak Analysis*[18] and the report AMP 185.1R[11] were motivated by this latter question. Naturally, the whole problem of effectiveness of bombing operations is more than a problem of reducing losses. It is an intricate problem of balancing bombing results against losses from flak, fighters, and operational difficulties.

The principal methods of reducing the total risk from AA fire may be roughly classified as follows:

1. Minimizing the number of shells fired per bomber by

   (a) Reducing the time of exposure to AA fire through increased ground speed, reduced bombing runs, and carefully planned approach and withdrawal.

   (b) Attacking AA batteries in advance of bombing missions.

   (c) Devising formations and planning the spacing between formations so that fewer AA guns, stationary and mobile, per attacking plane can be effectively brought into action.

2. Decreasing the number of aircraft required for

bombing attacks by an increase in bombing efficiency. Note that this might involve tactical recommendations at variance with other methods of reducing flak risk. A judicious balancing of conflicting principles is most important.

3. Reducing risk of damage per aircraft per shell by
   (a) Measures intended to decrease the accuracy of AA fire (evasive flying, night attacks, high altitudes, radio countermeasures, attacks on AA batteries).
   (b) Improvements in bomber design (distribution of vulnerable parts, protective shielding, etc.).

Fragmentation and damage calculations do not appear directly in many of these considerations, but all of the measures intended to reduce the accuracy of enemy fire influence the values of any parameters used in the calculations. Hence in any such computations it is necessary to know the circumstances for which the computations are to be made. Let us discuss some cases when these computations are useful in furnishing basic information.

11.5.1 Flak Charts

To find the safest direction of approach and withdrawal from a gun-defended target requires some measure of the total risk accumulated by a group of aircraft during its run over the target area. This requires a means of estimating the risk on each shot and the probable rate of fire, and accumulating the risk along the given bomb-run.

For a single aircraft at the point of aim, the probability P_0 of damage to that aircraft can be computed by means of the formulas of Section 11.3 in any tactical situation where the accuracy of fire is known. If the dependence of accuracy on slant range, altitude, and other variables can be determined, then it is possible to compute P_0 for a given target aircraft in terms of its position relative to the gun. From the path of the target and the rate of fire of the gun, it is then possible to find the probable location of successive points of aim along that path, and compute the risk for each shot from its position. Then the sum of these risks gives the total risk for the course.

If it is assumed that the risk is negligible except on the bomb run, the computation can be simplified somewhat and the results shown on charts (called flak charts) which can be used to compute fairly rapidly the risks from a given gun defense. One chart is to be made for each of a selection of altitudes, say every 5,000 ft.

1. For a given altitude and gun draw a circle whose radius is the horizontal range within which the given gun can fire shells to the given altitude.

2. A number of parallel crossing courses, say at intervals of 500 or 1,000 yd, are drawn across this circle; each of these courses is divided into intervals of some convenient length, say L yards.

3. From the speed of the aircraft, the time between shots, and the time-of-flight curves for the given gun, the expected number of bursts in each interval can be computed. The formula is

$$\text{expected number of bursts} = \frac{L}{D} + \frac{\Delta T}{n},$$

where L is the length of the interval;
n is the number of seconds between shots;
D is the distance the aircraft travels in n seconds (D and L must both be measured in the same units);
ΔT is the time of flight (in seconds) of the shell from the gun to the beginning point of the interval minus the time of flight to the end point of the interval.

Since the distance D in the formula is related to the speed v of the aircraft by the equation $D = nv$, we see that when L/nv is large compared with $\Delta T/n$, i.e., when the aircraft is not traveling too rapidly so L/v is large compared with ΔT, the first term contributes most to the expected number of shots in the interval and that number of shots is, therefore, approximately inversely proportional to the speed. A flak chart is drawn for some fixed speed and the values of the risks computed for it are then corrected for true speed by multiplying by the ratio

$$\frac{\text{speed used in constructing flak charts}}{\text{true ground speed}}.$$

This correction is not precise for large changes of speed due, for example, to 100-mph winds at the target.

4. The average probability of damage to the aircraft from a burst in each interval is estimated by calculating P_0 at the center of the interval.

5. The total risk in the entire interval is the product of the expected number of bursts times the average risk per burst.

6. These risks are accumulated along each crossing course to show how the total risk from the gun increases as the aircraft flies along that part of the crossing course within range of the gun.

7. Contour curves for this risk are drawn to com-

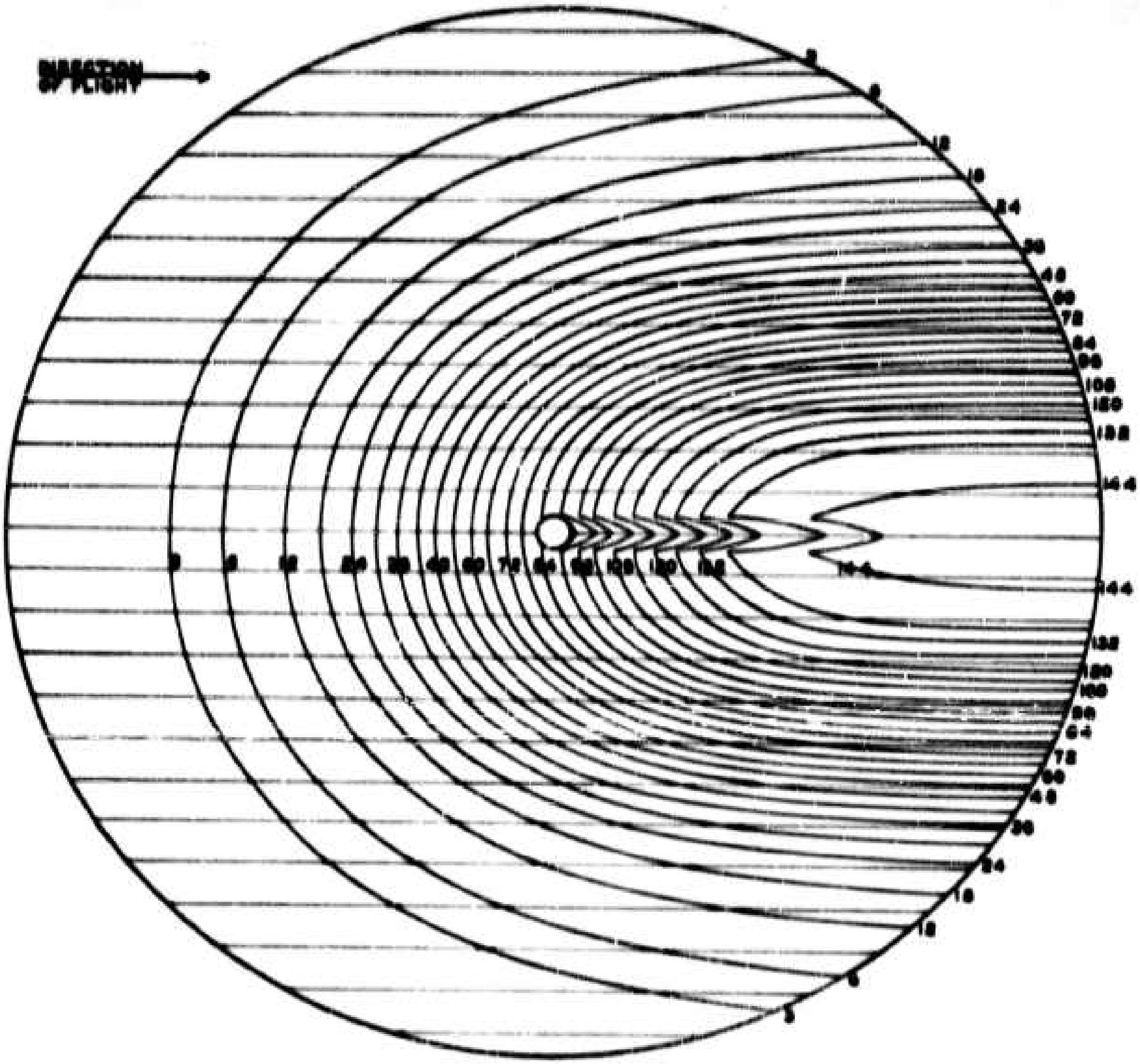


FIGURE 13. Flak chart for a single aircraft at altitude 8,000 yards with ground speed 260 miles per hour under service conditions (described in Table 1) with continuously pointed fire at the rate of 12 rounds per minute. The unit of risk is the risk to a single aircraft at 8,000 yards altitude and 10,000 yards slant range.

plete the flak chart. The number on each contour curve represents the risk (in some convenient units) due to flying along a crossing course in the given direction of flight up to that contour curve. Figure 13 shows a flak chart prepared in this way from the values of P_0 given in AMP report 185.1R.[9] The gun is assumed to be the U.S. 90-mm AA gun firing time-fuzed shells. The heading "service conditions" denotes the standard of accuracy used. The errors along and across the trajectory are given as functions of slant range by Table 1 of Section 11.4.1. The unit of risk is the value computed for P_0 under service conditions at 10,000 yd slant range and 8,000 yd altitude. For more details see reference 11.

To use a flak chart to select the safest course for a bombing run, the chart is used with a map drawn to the same scale showing the target to be bombed and the batteries defending it. The flak chart itself is made on a transparent film so that the map can be read through it. For any proposed bombing run, drawn on the map, the center of the flak chart is set on each battery in turn with the direction of flight

arrows pointing along the bombing run. The risk at the beginning of the run is subtracted from that at the end (these are read from the contours of the flak chart) and this difference multiplied by the number of guns in the battery to give the risk to the aircraft on that run from that battery. The process is repeated for all the batteries defending the target and the values from all batteries added together. The result is multiplied by the correction factor for ground speed to get the risk for that bombing run. (A refinement of this technique estimates the position of first accurate burst and uses that point instead of the beginning of the bomb run in computing the risk.) Fully detailed explanation can be found in a reference report.[13]

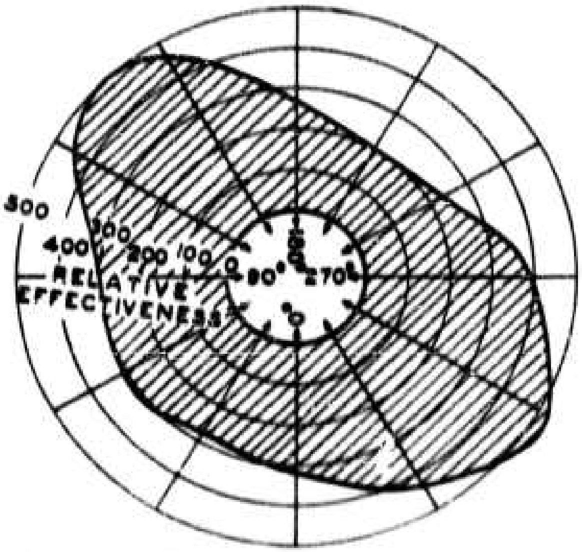


FIGURE 14. Relative hit expectancy as a function of the direction of attack.

The expected wind over the target has two effects on the computation; it moves the bomb release point and changes the speed so that the bombing run on a given target with wind is in a different position and of different length (on the ground) than it would be without wind. Hence the computation with wind uses a different bombing run and corrects by a different speed factor at the end of the process than the computation without wind. The general effect is that the increase of speed from going downwind into a well-balanced AA defense makes it much safer than going upwind into the same defense; if the wind speed is high, the wind direction is often the most important factor in determining the safest route into and out of the target area.

This computation is performed for each of a number of directions from the target, say every 30°, and the risks for bombing runs from these directions are displayed graphically in some manner to show safest route in and out. Two sample methods of representation are shown in Figures 14 and 15. The first is a polar coordinate diagram showing the relative risk for attack in each direction as a distance in that direction. The second, a "flak clock," shows only preferred directions of entry and withdrawal with the order of this preference.

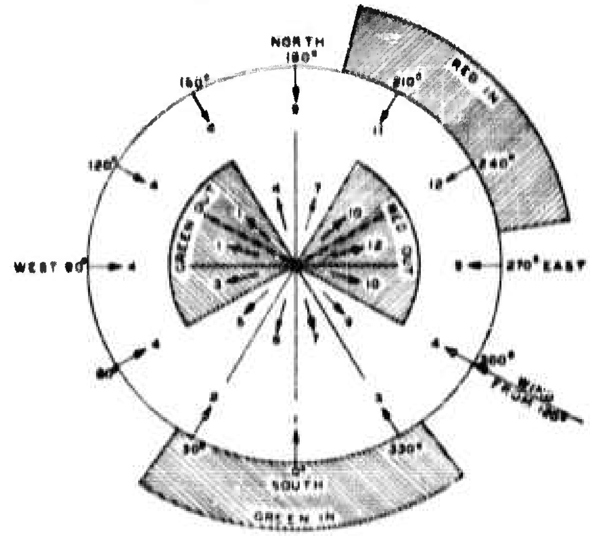


FIGURE 15. "Flak clock" showing preferred directions of attack and withdrawal with order of preference.

11.5.2 Flakometers

From flak charts it is possible to estimate the dependence of risk during a bombing run on the direction and altitude of the run. To isolate the effect of altitude we construct a figure, called a flakometer, showing the typical accumulation of risk at different altitudes.

We start with a given bombing unit. In preparing flak charts for that bombing unit we include the effect of altitude on the true airspeed. We use again the X,Y,H coordinate system of the preceding sections to locate the position of the target aircraft relative to the gun. To estimate the risk at a given altitude we take the following steps:

1. On the flak chart for the given altitude draw crossing courses parallel to the Y axis spaced 1,000 yd apart.

2. If the bombing unit flies along one of these

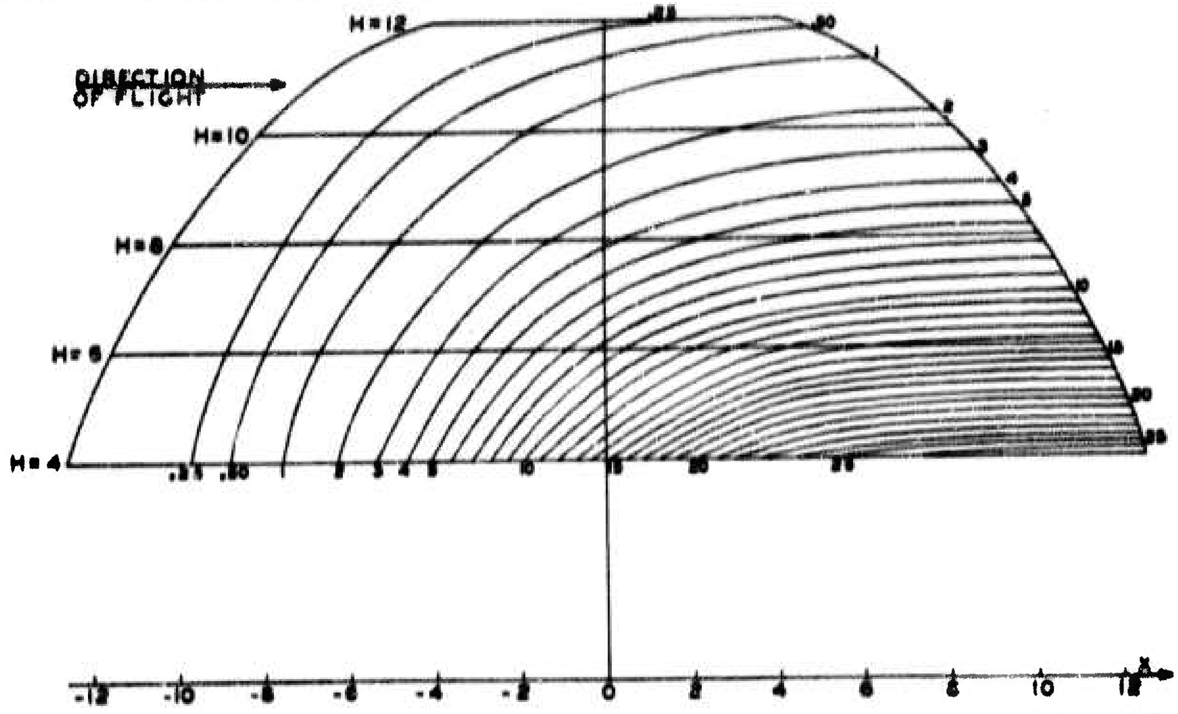


FIGURE 16. Flakometer. Single aircraft, service conditions. This figure shows the risk to a single aircraft from a line of guns spaced evenly one gun per 1,000 yards along a line at right angles to the line of flight. The flakometer also shows the relative risk in passing over a gun or battery whose distance from the course of the aircraft is not known.

crossing courses to the point (X,Y,H), the risk accumulated since the unit came within range of the gun can be read from the flak chart; call this risk $F(X,Y,H)$.

3. The value of Y determines which crossing course is involved; the crossing courses selected in step (1) are determined by the values $Y_m = 1{,}000m$ where $m = 0, \pm 1, \pm 2, \cdots$. If we wish to suppress the effect of the particular crossing course involved, for each X and H we add all the values $F(X,Y_m,H)$ to get a quantity which we will call $\Sigma(X,H)$.

4. A flakometer is a set of contour curves for the function $\Sigma(X,H)$.

This sum $\Sigma(X,H)$ of the risks on a collection of crossing courses can be regarded rather simply as the *risk to the bombing unit from a line of guns spaced 1,000 yd indefinitely far in either direction along the Y axis.* As the altitude H decreases and X is held fixed, $\Sigma(X,H)$ increases from a combination of three effects. First, as H decreases more crossing courses come within range of the gun so there are more terms in the sum making up $\Sigma(X,H)$. Second, for each X slant range decreases and hence $F(X,Y,H)$ increases because larger values of P go into it. Third, speed decreases with H so the unit has been in range longer at low altitudes, so again for each X and Y, $F(X,Y,H)$ tends to increase when H decreases.

Figure 16 shows a flakometer constructed under the same assumption about accuracy of fire as was the flak chart of Figure 13. In constructing the flakometer, the variation of ground speed with altitude was also prescribed.[11]

11.5.3 Group Probability Factor

When a number of aircraft are flying together, a shell aimed at one aircraft may damage another; hence the risk to a group of aircraft in a given position from the gun is greater than the risk to a single aircraft. However, the risks to the various aircraft in the group are not all the same; presumably the farther an aircraft is from the point of aim, the safer it is, so generally, the risk to a group of n aircraft is not just n times the risk to any particular one of them. To compute the risk to the whole group it is necessary to estimate the risk to each aircraft in the group and sum these risks.

For time-fuzed shells this is a relatively simple matter. Given the parameters σ_1 and σ_2 the computation of risk to a given aircraft of the group can be carried out when its position relative to the point of aim is given. This is done from the formula for $P(a,b,c)$, equation (16), or by some approximate method. Then summing the risks to the individual aircraft gives, approximately, the risk to the whole formation; the only error is due to shielding of one aircraft by another and this can probably be neglected.

For proximity fuzes the risk to a group of aircraft is not a simple combination of the risks to the individual aircraft. The reason for this is clear. For a given shell there is a surface in front of each aircraft on which the shell would burst if that target were alone and the shell happened to pass through that surface. When the aircraft are grouped together the risk to a single aircraft in the group is no longer the risk from a burst on the surface surrounding the target but is the risk from a burst on any burst surface surrounding any aircraft. That risk, at least for tightly packed formations, may be much smaller or much larger than the risk when that aircraft is isolated and has only its own burst surface to operate the fuze. Shielding will have a very strong effect with proximity fuzes, which it does not have with time fuzes. This shielding is not really a shielding of fragments but of bursts; it will be due to the fact that the bursts will occur before the shell reaches the first aircraft near its path and hence the bursts will be farther from those aircraft away from the gun. (With time fuzes, the bursts are as likely as not to be among the aircraft farther from the gun.) If, on the other hand, no aircraft happen to be shielded, the risk to each aircraft in the group is greater than if it were alone.

If we divide the risk to the whole group by the risk P_0 to a single aircraft in the same tactical situation, we get a number, called the group probability factor of the group, which depends on the accuracy of fire, the type of fuze, the number and arrangement of aircraft in the group, and (for proximity fuzes, especially) on the angle between the direction of flight of the target and the trajectory of the shell.

If a risk chart is to be built for a group of aircraft instead of a single aircraft, then the steps of Section 11.5.1 can be carried out with only one change. The probability P_0 in step (4) is to be multiplied by the group probability factor of the group, computed at that same point in space, to get the risk to the group.

11.6 CONCLUDING REMARKS

The preceding sections have discussed methods developed during World War II for comparing the effectiveness of different tactics and equipment in increasing or reducing risk to aircraft from antiaircraft fire. These methods, of necessity, involve many idealizations of more or less doubtful validity. The degree to which the results conform to actual, practical situations is subject to improvement in two basic, interrelated ways: (1) by modifying the hypotheses in the direction of greater realism and (2) by accumulating a larger body of reliable experimental data bearing on all phases of the problem. Insofar as available experimental data are inadequate, there is comparatively little immediate practical value in refinements of the theory.

One important question for combined theoretical and experimental investigation is the distribution in space of shell bursts from an AA battery, as determined by the random and systematic errors of the director-computer-gun systems. The assumption of a Gaussian distribution about the target as mean, with independent errors in range, elevation, and azimuth is of questionable validity.

The practical desirability of any investigation of the sort just suggested, involving specific pieces of equipment, can be assessed in terms of (1) probable abandonment or modification of such equipment and (2) possible development, in the course of the analysis, of methods applicable to other problems. From the latter viewpoint, it was considered desirable to stress methods rather than details in the present chapter; all the more so since unusually rapid changes in equipment and tactics can be expected to accompany further development of atomic power.

PART IV

GENERAL

Chapter 12

COMMENTS ON A GENERAL THEORY OF AIR WARFARE

12.1 INTRODUCTORY REMARKS

FROM THE FIRST, it must be clearly emphasized that it is not the purpose of this chapter to *present* a general theory of air warfare; but rather to give some tentative, and very incomplete and preliminary, ideas of what such a theory could and should comprise; to give some of the arguments for and against attempting to construct and use such a theory; and to indicate how certain activities of the Applied Mathematics Panel and of other agencies relate to a scheme for a broader and more inclusive analytical approach to the problems of air warfare (and of warfare in general).

The ideas herewith presented as to the scope, nature, and procedures of a general theory of air warfare must of necessity be wholly tentative. For such an overall analysis has not, so far as known, ever been attempted. We are dealing here with a tremendously complex situation, which not only has a bewildering number of technical elements, but is also basically affected by a host of practical nontechnical considerations of a very general character, such as Army-Navy-Air Force organization and policy, attitude of high command toward civilian technical assistance, attitude of field command toward innovations in procedure, conditions which attract and hold scientific personnel of high caliber, public support of effective measures for military preparedness. The precise character of such studies as are discussed herewith would also be profoundly affected by new weapons and the new tactics and strategy which they permit and demand. These factors — political, organizational, and technical — are obviously impossible to analyze or anticipate at this time, and accordingly the ideas presented herewith must necessarily be vague and general when they attempt any accuracy or permanency, and tentative and illustrative only, when they attempt to be specific.

The author of these comments, moreover, makes no pretense to any professional knowledge of military science. Thus he runs the risk of using phraseology which may seem naive or even ridiculous to the professional soldier. But we cannot get forward with this statement unless that risk is taken. All of what follows is thus to be considered as protected by a blanket apology for blunders of terminology, of understanding, or of fact.

12.2 OFFENSIVE AND DEFENSIVE AIR WARFARE

Offensive air warfare consists primarily of the unforced application of *bombs* (HE, incendiary, atomic, etc.), *guided missiles* (jet or rocket propelled or gliding; target seeking or externally controlled), *ordinary projectiles* (ranging in caliber from 0.50 in. to 75 mm), or *rockets* for the purpose of interfering with or destroying:

1. Uniformed enemy combatants.
2. Enemy military matériel (planes, tanks, guns, radar installations, ships, subs, fuel and ammunition stores, etc.).
3. Enemy communication, and transport in enemy countries.
4. Production facilities of enemy.
5. Enemy morale.

It also has the further purpose of destroying any and all enemy defenses (fighter planes, antiaircraft guns, radar, controlled missile sites, etc.) against such offensive attacks.

Defensive air warfare consists of forced applications of the weapons of the air arm against enemy agencies which seek to carry out direct attacks on us. Among such agencies are: enemy aircraft, controlled missiles and their launching sites, ships approaching our shores, etc.

It will be noted that offensive warfare has been here defined as the use of bombs, rockets, etc., there being no explicit mention of own aircraft. Except for the possible suicidal (or remotely controlled) use of aircraft as a defensive ramming device, the aircraft itself is primarily a means of transportation which,

with great speed, over long distances, and often unseen, brings to the effective place the bombsight and bomb rack, the rocket launcher, the gun, etc., which then attacks the enemy.

There has been no mention of the distinction between strategic and tactical offensive air action. The above list of numbered items starts with tactical actions, and ends with strategic actions. The distinctions between tactical and strategic action refer primarily to the *time* factor and to *specific* operations as contrasted with general preparation and direction — the actions which *at once* affect the military strength of the enemy of some particular place being tactical; those which affect the military strength of the enemy in general and after some time delay being strategic.

12.3 INTERRELATIONS

The brief remarks made in the preceding section concerning offensive and defensive air warfare were made for the single purpose of emphasizing that there is, in general, no such thing as isolated warfare. A modern war involves the productive capacity of a whole people, its ability to transport materials of war, its total capacity to strike by land, sea, and air. The part of this whole struggle which comes under the label "air warfare" is enormously larger and incalculably more important than in the past. But the part called air warfare is in most cases (and in all large and important cases) inextricably interrelated with *all* the other *parts* of the total effort. It is as unreasonable and self-defeating to attempt an exclusive theory of air warfare as it would be for a physician to look through a peephole at a patient's forehead and try to diagnose the cause of his headache.

Thus, the first main point is:

There cannot be any such thing as a theory of air warfare, unless one is in reality prepared to study warfare in general.

That naval forces, infantry, artillery, air forces, coastal defense, production, food, manpower, critical materials, training, transportation, time, etc., are all interrelatedly involved in any broad strategic plan is fairly obvious. Consider, for example, a strategic plan to reduce the level of machine tool production in an enemy country. What proportion of our own productive strength, money, manpower, etc., can we afford to expend in order to effect various levels of reduction of enemy machine tool production within certain stated times? The ramifications of this question, the cyclic intertwining of the manifold military and economic factors, are obvious.

But consider some much more detailed and specific questions — ones which might easily face a colonel at Wright Field, for example. A new bombsight is under development, intended for high-altitude radar bombing. Suppose the research and development agency reports that:

1. The mil error can be reduced from 50 mils to 40 mils by adding 30 lb to the weight; or to 20 mils by adding 100 lb; or to 10 mils by adding 500 lb.

2. The mil error which can be achieved through essentially perfect operation can be reduced by a factor of two by increasing the complexity of the sight and the complexity of operation (hence also increasing the cost, time to produce, time to train, maintenance difficulties, etc.).

3. The bombsight can be made to operate without the necessity of a straight bombing run by further increasing its weight by 300 lb.

What factors should enter into the analysis on which the colonel bases his recommendations? He cannot possibly himself know all the necessary things, but somehow he should certainly bring to bear on these questions a wide and precise knowledge of the probabilities of bombing accuracies; the logistics of the theatres in which these sights are to be used; the nature of the enemy targets; all the vast field of terminal ballistics; the importance of the time factor (which means war plans, among other things); the psychology and physiology of operation of bombsights; the selection and training of bombardiers; accessibility of qualified personnel; the basic cost of accomplishing the same objectives otherwise; the present and potential future effectiveness of the enemy's fighter attack against our bombers; etc.

Thus an apparently detailed question concerning the design of one specific piece of equipment cannot be answered logically or conclusively except through an *analysis* which takes into account an almost frightening array of factors.

The "practical" man will, at this point, wish to get in an emphatic disclaimer. He will wish to point out that these questions have to be answered in a reasonable time, that it is not possible to furnish each Wright Field colonel with a board of military Einsteins, that "*judgment*," "*experience*," "*common sense*," perhaps even "*horse sense*" are more important here than a lot of fine-spun theory.

The practical man is asked to exert an almost super-

human restraint, and read on. For it is hoped that his fears and objections will receive something of an answer before this chapter is concluded. To match his restraint, the author agrees to restrain himself and to cite, from his experience of the last five years, only allegorical and anonymous examples of the kind of decisions which, in such problems, sometimes result from "common sense" and "experience," divorced from technical knowledge and analysis.

12.4 NATURE OF A GENERAL THEORY

In order to expose even the rudimentary ideas of a general theory it is unfortunately necessary to face a certain amount of complication. That is unavoidable, since the complication is an inherent and characteristic aspect of such problems. And to enable us subsequently to talk without the constant repetition of long qualifying clauses, we must introduce just a little notation.

Suppose that a certain operation O is under consideration, the plan P for this operation becoming specific and definite only after a number n of decisions D_n have become definitely settled.

Thus the operation might relate to some specific detail in the design of a piece of equipment — say the maximum slewing speed of an aircraft turret. In this case the plan P is determined when one single decision D_1 is made definite, that decision consisting of assigning a definite number of mils per second to the maximum slewing speed. To take another very simple illustration, if the operation O involved the single question as to whether or not the turret controls utilize so-called aided tracking, then the plan P consists of the single decision D_1 of deciding "Yes" or "No." In any such case as this last one, the decision, although consisting of a choice between yes and no, can be given a numerical form (and thus made similar to the previous case) by arbitrarily calling 1 the equivalent of yes, and 0 the equivalent of no. Thus in both the illustrations given so far, the plan becomes definite when numerical values are assigned to the decision variables D_n.

In a case of slightly greater complexity, the operation O might have a plan P which involves two decisions, namely: D_1, at what altitude shall we carry out bombing attacks on a particular target; and D_2, shall we carry these out by day? It is assumed here that many of the other variables of the situation (types of planes, types of bombs, etc.) are fixed by considerations not under our control.

Here the plan P becomes definite when a numerical value is established for D_1 (say 30,000 ft), and when yes or no (that is, 1 or 0) is determined for D_2, so that day or night bombing is settled upon.

In a case of much more substantial complexity, the operation O might involve the optimum use of a given sized fleet of B-29 bombers operating out of the Marianas against Japan. But even now the problem is not nearly as broad as it might be, for it presupposes aircraft of certain definite characteristics, operating from a given set of bases; whereas a broader problem would include a consideration as to whether the attack should be made by inhabited aircraft (and what types of aircraft), remotely controlled aircraft, guided missiles, etc. But even in this somewhat restricted problem it is clear that the plan P for the operation now includes a large number of decision variables D_n whose values must be determined.

Thus, to indicate only a few, one would have to assign priorities to a set of targets (thus involving numerical values for a set of decision variables, one of which corresponds to each target under consideration). One would have to settle on a flight plan, this in turn breaking down into a large number of individual decisions, each of which must be definitely determined. This flight plan, among many other items, involves decisions as to the formations which will be flown en route, when fighter opposition appears, when flak is present, for the bombing itself, and on the return trip. All of these matters can be systematically set down in such a way that they are definitely settled by assigning numerical values to various decision variables D. Evasive tactics must be included in this flight plan, and in considering various formations and tactics it is clear that one is forced into a detailed consideration of the accuracy and effectiveness of the defensive fire of the bombers, their vulnerability to enemy attack by fighter planes, flak, rockets, air-to-air bombing, self-inflicted damage, etc. Questions of fuel consumption, wear on aircraft, crew morale, maintenance, training, psychological and physiological limitations on gunners, bombardiers, pilots, etc., must be considered. Weather, radar navigation aids, and radar bombing aids are intimately involved. Probable damage to own aircraft from flak, rockets, fighter attack, etc., involve complex and subtle studies of the vulnerability of aircraft components and personnel, fragmentation characteristics of enemy shells, fire control accuracies, etc. Effectiveness of the bombing involves

all the difficult probability and statistics of bombing accuracies, and the vast array of factors which enter into terminal ballistics. The enemy defenses of all sorts and against all types of attack must be estimated as best one can on the basis of past experience and intelligence (the past experience, incidentally, not being worth much if it consists merely of the understandably emotional reaction of crew members: provision must be made for first hand analysis in the field and at central locations by technically competent personnel such as the Operations Analysis Sections,[a] and at central correlating and analysis bureaus such as did not really exist during World War II.

Thus in this third illustration of an operation O, the plan P clearly involves a very large number of decision variables D_n which must be determined in order to decide the plan. And the brief description given above indicates another feature of such problems — a feature which we have not mentioned as yet, but which is essential.

It is, in fact, clear that the decision variables D_n themselves depend upon a large number of further and more basic variables, such as the fuel consumption of a B-29 at given load, speed, and altitude; the percentage of fragments larger than, say, one ounce in weight, which leave a certain bursting AA shell in certain angular directions; the probability that two .50-caliber slugs in a certain area of a fighter plane cause a fire; the probable value in mils of the harmonization error of certain guns of a B-29 when flying in bumpy air; the probable number of days per month (in a given season of the year) that a certain enemy target will be visible from 14,000 ft and not visible from 30,000 ft; etc. These basic variables, which we will call V_N (thus indicating that there are N of them) affect the decision variables D_n, but are distinct from them.

Certain of the basic variables V are at our disposal, in that we can control their values. Others are under the control of the enemy (such as, for example, the probability that enemy fighters attack at a certain place and time). Others, such as weather, are under the control, so to speak, of nature.

It is clear that the relations between the decision variables D and the basic variables V are very complicated. These two sets of variables are all tied together in all sorts of complicated ways. Some of the decision variables D are not logically independent, for assigning a value to one may involve restricting certain of the basic variables to values, or ranges of value, which in turn may effectively limit and may even specifically determine the value of other decision variables. This is, of course, merely the formal mathematical equivalent of the obvious fact of experience that often one decision commits a person to certain further decisions.

Among the variables which play important roles are the quantities, say N_{ij}, which designate the numbers of certain types of weapons which we or the enemy may have available at certain times and places. These variables may be decision variables in some problems, and basic variables in others. In strategic calculations one is interested in the values of N_{ij} over fairly long time intervals — one or two or even more years. In tactical calculations, one is interested in the values of N_{ij} at specific places, and over a shorter future interval of, say, a few weeks or months.

We have progressed far enough in a preliminary discussion so that we can now describe the essential and central feature of a general theory of air warfare. The central feature is obvious and simple. It consists merely of saying that the plan P should, in fact, be characterized by that particular set of decisions D which is *worth most to us*, namely the set which gives us the largest margin of profit — the largest excess of return over cost.

The words *profit*, *return*, and *cost* do not, of course, refer to worth as measured in dollars. They refer rather to what may be called *military worth*. On the profit side of the ledger one takes account of the destruction or other sort of harm which is imposed on the enemy. If the concept of military worth is developed along relatively decent lines, destruction and harm to the enemy will be counted as the more valuable, the more definitely and promptly this destruction and harm contribute toward the successful termination of the conflict. Longer range considerations involving possible future conflicts, however, cannot safely be entirely neglected, as is well illustrated by the decision to use the atomic bomb on Hiroshima and Nagasaki. On the cost side of the ledger one must take account of our necessary expenditures of labor, time, critical materials, money, matériel, personnel, etc. In some simple cases it may be possible to treat military worth as closely analogous to man-hours of labor. In other cases nothing so simple will suffice.

Military worth, as the phrase is here used, is

[a] See, for example, reference 42.

closely related to the general concept of *utility* in economic theory. And the reader is warmly urged to read the discussion of a numerical theory of utility given (on pages 15 to 20 and elsewhere) in "Theory of Games and Economic Behavior" by John von Neumann and Oskar Morgenstern.[1] This pioneering and brilliant book is, it should be pointed out, connected in a most important way with the viewpoint here being presented, for it develops a large part of the mathematics necessary for theories of competitive processes.

But assuming that there has been developed a satisfactory concept of military worth (for which we will use the notation MW), then *the essential procedure of a general theory of warfare is to determine, for any operation O a plan P, whose values of the decision variables D_n maximize the military worth MW.*

Let us try to make this situation somewhat more clear and definite. Suppose for the moment that a satisfactory concept of military worth has been developed, that techniques have been elaborated for computing, relative to a given operation O numerical[b] military worths of various plans P, that is to say, of various choices of the decision variables D_n. In fact, let us suppose that there has been constructed for this operation O a great *Tactical-Strategic Computer* [TSC]. This computer has one main output dial — the one which displays the military worth. It has a second set of dials, one for each of the decision variables D_n. And it has a third set of dials, one for each of the basic variables V's.

Suppose we seat ourselves, in imagination, before this tactical-strategic computer and play with its controls. For this process will illustrate, and perhaps make more clear, some things said above.

We start by setting in the appropriate values to those basic variables V which are under enemy control, and those basic variables V which are under nature's control. In some cases (as, for example, the setting time for a well-known enemy antiaircraft director) these may be inserted as definite fixed values. In other cases (as, for example, weather conditions over certain enemy targets) the computer must be equipped to accept parameters which characterize the statistical behavior of the quantity in question. In still other cases (as, for example, probable mil accuracy of enemy fire from flexible guns in aircraft) the computer must be equipped to accept a *pair* of values, setting lower and upper limits to the quantity in question; in the computing process which will take place presently, this particular basic variable will be allowed to vary over this estimated range. Other basic variables of this same type will similarly vary, but with constantly changing phase relations, so that within a short time the TSC has in fact taken into account all combinations of values of those basic variables for which it is necessary to estimate a range.

Having set in the "enemy" and "nature" basic variables (but leaving many of the other basic variable controls as yet untouched), we now begin to twiddle the decision variable dials. We may, for example, have in mind a certain set of the decision variables — one suggested, for example, by the Air Forces Board, or by Eglin, or by some general with an astronomical number of stars. We start to set these values of D_n in, one after another. As we do, the inner calculating mechanisms are set in operation, and values (or in some cases indicated ranges of values) begin to appear on these basic variable dials which we did not initially set. For example, if we set onto one of the decision variable dials a value which corresponds to the choice of a 5-in. rocket of a certain type, then the basic variable dials corresponding to fragment, angular, and mass distribution characteristics, move over to proper settings. If we set a decision variable dial to correspond to a certain sighting system, then the basic variable dials corresponding to probability of damage, etc., indicate ranges which correspond to this system.

It is perfectly possible that, after setting in a certain number of values of decision variables, the machine is found to be locked when one tries to set in another decision variable. This means that the decisions already made in fact determine the remaining decisions. The complex interrelationships, via all the basic variables, is such (and perhaps quite unexpectedly such) that the various decisions are not really independent. In this case, one pushes a special button, and all the remaining decision variable dials move automatically into their necessary positions, thus showing what further decisions we are perforce committed to.

In case, however, the decisions all prove to be independent, one continues until all their values have been set in. At this juncture (or very shortly thereafter, corresponding to some time delay required by

[b] Quantitative significance is not required of the scale of values of MW. Only linear order is important. Since both cost and profit have been taken into account, one set of decision variables is unambiguously better than a second set just provided the first set leads to a larger value of MW. How much larger is of no importance.

the computing mechanisms) the military worth dial lights up and displays the numerical value of MW. Only now do the really interesting things begin to happen. First of all, one watches the value of MW for a time during which the computer is trying out all combinations of those basic variables for which ranges, rather than specific values, were set in. If the MW varies widely during this period, one may possibly conclude that, before attempting to proceed, further research is necessary in order to delimit those basic variables to more narrow ranges. One may play with specific ones of these basic variable dials to determine which one or ones it is whose uncertainty contributes principally to the uncertainty in MW. Similarly for the basic variable dials on which have been set specific numbers (or statistical parameters) one can shift them one at a time to see whether the MW is sensitively or sluggishly increased thereby.

But of greater probable interest is the behavior of the MW dial when one alters the values set into the decision variable dials. *For then one is changing the military plan for the operation in question, and is observing directly whether the change is really for the better or for the worse.* To seek the optimum plan, one would set into operation a mechanism which (again with changing phase relation) shifts all the decision dials through cycles of accessible values, the resulting values of MW being recorded so that the maximum can be located and the corresponding set of optimum values of the decision variables D_n determined.

It should be admitted at once that so complete and so formally mechanized an analytical procedure doubtless lies far in the future. The speed and flexibility of modern digital and analogue computing devices (particularly those which operate at electronic computing speeds and have very flexible control systems), and the accuracy and stability of modern multistage servo systems (permitting great freedom in the use of feedback loops) effectively remove any practical difficulties due to the complexity and amount of the analysis involved. The factors which prevent the present construction of such a computer are not those of computing complexity, speed, etc. The real difficulties, and the limitations of such a procedure, will be discussed in the next section. But it seems wholly likely that these difficulties and limitations can, at least for many important problems, be overcome. And the present author would like to risk the prophecy that just as World War I saw the birth, and World War II the high development and effective use of fire control predictors which, so to speak, solve the immediate tactical problem for a single gun, so the next war (should there be one) may result in the development and use of general tactical computers for use in the field (as on the flagship of a fleet, for example), and even of strategic computers at the disposal of the high command.

12.5 CAUTIONS

The last few pages have described an imaginary machine. This description will, it is hoped, serve to make clear and vivid certain fundamental aspects of what a general theory of air warfare might sometime be. But this example, with its "Rube Goldberg" machine, runs the risk of giving certain wrong impressions. We want, therefore, to express here some warnings.

1. Will those who consider the possibility of such a machine to be wholly controversial or dubious just forget the last few pages, and consider the rest of the facts and arguments here presented?

2. No one can hope, at this early stage, to predict the actual form of the ultimate science of aerial warfare, so details are not to be taken too seriously.

3. It may very well be that, for a long time in the future, it will not be possible to carry out analyses of aerial warfare as "overall" in character as those indicated in the last few pages. But that would in no way reduce the importance of dealing with simpler component problems, and then working gradually up to the study of whole operations.

4. The concept of military worth is admittedly, at this stage, both vague and difficult. How can one measure, with one quantitative index, so many tanks and so many soldiers' lives, and so many dollars, and so many man-hours of labor? I am sure that I am not now prepared to answer. It may be that more than one index will be unavoidable (and hence several output dials on TSC). But there are deep and experienced military, economic, and analytical minds available to work on this problem, and one always returns to the stubborn fact that the overall comparison, whether of incommensurables or not, *has* to be made either by analysis, or magic, or blind guess. I am simply arguing for facing the complexity and the facts, and pushing analysis to its usable limit.

5. Remember that a great deal of progress has already been made. Some of these advances are referred to later in Section 12.6. But apart from the

encouraging progress made in the study of military problems, both in this war and earlier (as sketched in Section 12.0), there has been a great deal of more basic mathematical work which is relevant.

Reference has been made, in Section 12.4, to the powerful analysis of competitive processes carried out by von Neumann and Morgenstern.[1] There are many further important pieces of research to be found in the large literature of mathematical economics. In addition, the quantitative ecologists, aided by outstanding mathematicians, have developed extensive quantitative theories of the competition of two or more populations for food supplies, etc. The literature of competitive biological systems has been largely developed by the great Italian mathematician, V. Volterra,[2] the American statistician and biophysicist, A. J. Lotka,[3] the French biologist, Georges Teissier, and the Russian biologist, G. F. Gause.[4]

12.6 DIFFICULTIES, OBJECTIONS, AND COUNTER-ARGUMENTS

In the above descriptive remarks, all sorts of difficulties have been overlooked or blithely snow-plowed ahead; we will pause at this point to take some account of those matters.

To try to argue that a general theory of air warfare is easy to construct would be as ridiculous as it is unnecessary. The problems are of the gravest possible importance, and of staggering complexity. But the problems are also of compelling character. In many instances (and during war, in practically all instances) the problems are unlike many questions in pure science. The stark fact is that prompt decisions *have* to be made: a plan — excellent, good, mediocre, or disastrous — *has* to be adopted and followed. It is not a question of mere intellectual curiosity; there is not an indefinite amount of time during which we may philosophize; and it may be utterly disastrous to learn only by trial and error.

Under such circumstances it is really futile and irrelevant to keep remarking that the problems are complicated, subtle, and difficult. Of course they are! But the brutal fact is that they must, for better or worse, be solved. The relevant question is: how?

Is there any advantage to be gained from refusing to admit or face complexities and difficulties which are in fact inherent? How do "judgment" and "experience" and "common sense" cope with such problems? In some instances it must be confessed that they do not cope at all, and that these words are merely used as labels to cover up a prejudiced guess made by someone in authority. In other cases — in most cases — there is undoubtedly an eager and honest attempt to invoke experience and to exercise judgment and common sense. But judgment based on experience, to the extent that it is rationally utilized, must face the problem of deciding that certain features of the present do or do not correlate with the past — that is to say, must use the analytical method. And what, indeed, are such efforts other than disorganized and feebly intuitive shadows of a real analysis?

Any one who, with a background of scientific training, has watched the procedure of decision as it sometimes operates in military circles, has very likely ended up with a tremendous personal affection for the vast majority of sincere, intelligent, and patriotic officers, but with disappointment because of the frequent lack of technical competence at decision levels, and with distrust for what sometimes travels as "common sense."

One of the most paralyzing sets of circumstances in which to meet the difficulties of unsoundly based common sense is in connection with what one may call the "hero problem." A young and extremely capable pilot goes through a long and terrible siege of combat experience. He is, by this time, a lieutenant colonel or even a colonel. He is intelligent, and he obviously is experienced. He is brought back to Eglin, or Orlando, or Wright Field, or Washington, and he is suddenly put in a position of very considerable responsibility with jurisdiction over technical decisions. He may, for example, be concerned with sights and sighting mechanisms. An engineer, physicist, or mathematician who has been studying sights intensively and exclusively for several years comes to his desk for information, or for backing. The colonel thumbs through the material, turns a little pale at the mathematics, and then seizes on some one feature of the sight which he thinks is like some other sight he once tried and didn't like. At once his front-line vocabulary springs into action, and he makes it purply clear that he disapproves. If the technical expert says — "But Colonel, don't you think . . ." He is interrupted with, "Say, listen; I have been up there with those *** *** shooting at me, and I know what I am talking about!"

And you look at the triple row of ribbons on his chest, and you are devoutly thankful to him for his youth, skill, and bravery — and there is just no

decent way in which you, a theoretical stay-at-home, can tell him that he is wrong.

That is what was meant, above, by the "hero problem." The worst of it is that this energetic, patriotic, and altogether fine young man *is* often wrong. There are essential complexities and subtleties to the problem which he has never thought of, and for which his technical training does not at all equip him. His own ideas have been inflexibly set by his own experience under fire. He may have used a very bad sight and very bad tactics — but if they, in fact, carried him through alive, it is beyond reason or hope that he will ever be objective about them. He may even be the sort who thinks that the scientists are wasting their time interfering with soldiers in what is after all *their* business (as if it were not the business of all of us!). And he probably thinks that both for automatic weapon antiaircraft and plane-to-plane fire, "all you have to do is look at the tracers! Then, Doc, you see it all with your own eyes, and you don't need any of the goddam mathematics." This is what he calls "common sense."

It must be remembered that individual combat, as experienced in the Air Forces, is a highly variable experiment in probability, so that these individual experiences do not mean much apart from the total evidence (some of the most important of which would have to be given by dead men). Success is often achieved because of the personal characteristics of the man, and in spite of rather than because of his equipment. The testimony of outstanding individuals is therefore chiefly an argument for outstanding individuals, rather than for or against any particular pieces of equipment or any particular techniques.

There is no intention to imply that such well-intentioned and excellent, but misplaced officers constitute the majority. On the contrary, one of the strongest and happiest impressions of the scientists who have worked with the Services is the large number of really excellent officers — highly intelligent and energetic, and men of fine character. The point being made here is that:

Officers required to administer technical programs and to make recommendations on technical developments should be chosen for their technical training, not their battle experience. Such officers should be given credit for technical service, and should be maintained stably in such service long enough to make their technical competence effective.

One of the most outstanding characteristics of a technically competent man should be a knowledge of his own limitations, and a willingness to seek and accept more expert advice than he can himself furnish. He should be able to bring an unemotional logic to bear on the survey of problems, and he should have a capacity to recognize and appreciate the major points in a technical argument, even though he personally could not construct the argument. All of these comments are closely connected with the desirability of the briefing of officers at the decision level by a competent specialized staff.

There is good evidence that the points just emphasized are well recognized by certain highly placed officers — by General Arnold himself, for example. But unfortunately he plans to retire, and a new set of top brass must be convinced all over again.

But let us return to our main thesis. We have listed above a considerable number of the kinds of factors which enter into air warfare problems.

No one, presumably, would argue that such factors are unimportant. And if there are other, dominantly more important factors, then by all means let them be brought forward by their proponents, and it will at once be agreed that these factors (after their importance has indeed been established) should be included in such a scheme.

No one would presumably deny that military operations should be conducted in accordance with the plan which promises to give the greatest net return. And who is prepared to claim that, on the basis of experience and intuition and judgment, he can instantly and mysteriously produce solutions which he denies can be produced or improved by logic and analysis? The concept of military worth may very well be a most difficult one, hard to apply to specific cases. But if it is in fact a basic concept which plays an inevitable, even though often a hidden and unrecognized role, what virtue is there in disregarding it? Nothing is to be gained by the ostrich-like procedure of burying our heads in the sand to try to escape complication and difficulty. The complications have to be faced, and the sooner they are brought out into the open and subjected to logical, quantitative analysis, the better.

It will be urged by some that it is very difficult, if not impossible, to devise any quantitative measures of worth which can be applied alike to factories, ships, food, aircraft, men, etc. Admittedly and assuredly it is difficult, but it is paralyzing and self-defeating to admit that it is impossible. In fact, any judgment, including those produced on the mysterious basis of "judgment" or "experience," actually

and necessarily involves such evaluations. It merely involves them — which would not seem to be any specially proud advantage — in a concealed and unanalyzable way.

To urge that the problems are too complicated to solve is (not to repeat again the obvious rejoinder that, complicated or not, they *have* to be solved) to deny the whole history of man's conquest, through the imaginative power of the analytical mind, of complicated problems. Suppose that the atomic physicists or the cosmologists or the geneticists had been content to say, "No, the facts are too subtle and inaccessible, the problems too complicated!"

It is, moreover, important to recognize that useful results can come from a start which is admittedly incomplete and inaccurate. A precise analysis of *relationships* is sometimes possible even though the numerical values of the various parameters be wholly unknown. And then it may turn out that certain very important conclusions flow, even though one has to make wild estimates of certain difficult parameters.

This last point is important enough to deserve illustration. Suppose one wishes to compare the relative effectiveness of two alternative armaments for a fighter plane (say eight .50-caliber wing guns versus four such wing guns and two central cannon). This comparison depends on a considerable number of factors — fire control accuracies and vulnerabilities chiefly — which are very difficult to estimate with any accuracy. But if it turns out that one asks several persons to estimate the fire control accuracies and the vulnerabilities, and receives very disparate replies, and if, in spite of this disagreement, it turns out that for each and every combination of guesses, one armament turns out to be superior to the other, then clearly the analysis has produced a result that overrides the difficulties of estimation of parameters.[a]

Many problems of warfare involve probability considerations, and this is a field which is notoriously tricky, and within which "common sense" is often quite helpless. The author, for example, knows one little probability question which can very nicely be used as a basis for bets. Actually the odds for the person who understands the game are about three to one; but it is so deceptive, and seems to any common-sense person so fair, that the author has felt himself morally restrained from playing the game with any persons other than professional statisticians.

Examples of a similar sort can very readily be found in military problems. For example, what is the optimum mixture of armor-piercing [AP] and incendiary ammunition for the rear guns of a bomber? Specifications often designate such mixtures as five AP to two incendiary (we are neglecting tracers here). Why? The somewhat striking and by no means obvious fact is that given any fixed type of target it is better to have either *all* AP or *all* incendiary, depending on the nature of the target. The justification for any other intermediate mixture should be based on knowledge of the relative probability of encountering different targets, certain of which would be more vulnerable to AP, and others more vulnerable to incendiary.[b]

As another example, consider the effect of a weather forecast for a certain future military operation. To oversimplify the problem, suppose that one is only interested in whether the weather is good (G) or bad (B). Suppose that one can in some way assign the relative military worth (MW) for the four possible cases that:

1. The forecast is G and the weather actually turns out to be G.

2. The forecast is G and the weather actually turns out to be B.

3. The forecast is B and the weather actually turns out to be B.

4. The forecast is B and the weather actually turns out to be G.

In a given case, for example, the MW_1 of case 1 might be exceedingly large and positive, and the MW_2 of case 2 very large and negative; while MW_3 and MW_4 might both be moderate or small. (This would be the case of an operation which is very important if successful, disastrous if unsuccessful, but which could be postponed until later without great loss.) What should be the weather forecast if one wishes to maximize the expected worth of the operation?

Again the answer is striking, simple, and not obvious. Anyone who is interested can find a discussion of this problem in a reference.[c]

To conclude these two examples it may be worth while to point out that, from a mathematical point of view, they are not separate and different examples, but merely two illustrations of a single, simple, basic, analytical fact. This point is mentioned because it is by no means trivial. One of the important reasons

[a] P. M. S. Blackett has said: "No pregnant problem should be left unattempted for lack of *exact* numerical data, for often it is found on doing the analysis that *some* significant conclusions recommending concrete action can be drawn even with very rough data."

the analytical approach is powerful is that it strips off the confusing superficial aspects of problems, and exposes their inner logical structure. And often, when this inner structure is exposed, it turns out to be logically equivalent to some other situation already studied and solved.

In certain cases the analysis will reveal that the final result (say the military worth) is extremely insensible to the value of a certain parameter. That fact of itself may be of great importance. In other cases it may turn out to be impossible (or in any event impractical) to assign sufficiently definite values to the various basic variables as to permit anything like a unique determination of the military worth of a given plan. But it may nevertheless be of extreme importance to delimit the total range of possibilities. This is in a way analogous to the procedure of the cosmologist. He does not have enough nor precise enough information to construct a complete theory of the universe. But he can succeed in establishing the fact that, of all possible universes, the actual one has properties which lie within certain specifiable ranges. And then he has a definite framework of procedure, and can systematically refine his analysis and narrow its limits of uncertainty.

There is, moreover, further impressive and practical evidence in favor of attempting the quantitative analysis of complex situations involving elements which seem to elude quantitative specification. For this, in fact, is precisely what was done with such outstanding success by certain operational research groups during the war. Professor P. M. S. Blackett, to whom is due so much of the success of the British Operational Research activity, has set forth the essence of this matter in a brilliant and deceptively obvious, but really profound paper entitled, "A Note on Certain Aspects of the Methodology of Operational Research."[7] And for a classic example of success in dealing with a very complex situation, the reader should study the Admiralty Operational Research reports on the convoy problem, or the classic and critically important paper, "Air Offensive Against U-Boats in the Bay of Biscay."

It is recognized that practically all of the really broad military problems are characterized by great complexity and by the lack of a great deal of accurate numerical data. The problems are, as has been emphasized by Blackett, more like the problems of biology and economics[d] than the problems of physics. And therefore one cannot reasonably hope that, at least for some time to come, we will have complete logical theories of the a priori sort which have been so successful in physics or astronomy. One must usually be content, as Blackett states, with variational theories which, like estimates of marginal profit, study the response of the military worth to changes in the decision variables, often using statistical as well as analytical methods of estimating differential coefficients, and making maximum use of operational constants as well as operational functions.

The operational constants and the operational functions constitute some of the most precious and dearly-bought results of combat experience. These results should be distilled out of the every day experience at the front by the Operations Research personnel, and should be importantly supplemented by overall comparisons of the effectiveness of operations in various theatres.

This latter type of evaluation has, granted the lack of adequate technical and analytical assistance, been carried out during World War II with skill and objectivity by various groups and for various areas. A good example, for instance, is the report of the AAF Evaluation Board, Pacific Area.[18] A detailed examination of this honest and able report would convince anyone that errors had been made which could have been anticipated and prevented had sufficiently broad studies, of the sort urged herewith, been carried out and *the results utilized*.

Throughout this chapter the atmosphere of the discussion is the atmosphere of war. Many of the remarks imply that studies were being made *while a war is going on*. These studies are absolutely essential, for the complexity and other characteristics of the problems are such that progress cannot very well be made without knowledge of the operational constants and operational functions mentioned just above.

But even more emphasis should be given to the necessity that *such subjects be studied during the years of peace*. Precisely because the problems are so difficult, we have little chance of making important progress and of being in a strong and well-prepared position unless a substantial and a sustained research program in this field be carried out after the war.

Between wars it is difficult if not impossible to give attention effectively to detailed (especially operational) problems of the sort referred to later in this chapter as micro-problems. But just for that reason we should, between wars, seek to develop the frame-

[d] Remember the concluding remarks of Section 12.4.

work and technique for the solution of the broad (the macro-) problems.

12.7 PROGRESS IN WORLD WAR II

As has been indicated by the remarks of the last section, there was substantial progress during this war in the quantitative analysis of military problems. Indeed, the utilization of technical experts is by no means new or recent. The Trojan Horse was presumably designed by some ancient Omicron Sigma Rho Delta. Archimedes was the V. Bush of Syracuse. Napoleon had a board of technical advisers. Those who urge large extension of the use of analytical methods will, of course, be confronted by two extreme groups: one will be quite certain that the whole idea is visionary, unnecessary, and impossible; the other will be equally certain that propaganda for the idea is quite unnecessary because the notion is so old and so obvious that everyone has accepted it.

The present author's association with research and development problems in World War I, though intimate, was on a very junior level, so that he is not in a position to prove what he nevertheless is reasonably sure is true. Namely, that there was at that time very little effective use of quantitative analytical thinking in connection with the instrument development, and probably none in connection with tactical or strategic plans. It is, however, interesting to note that Frederick William Lanchester wrote, in 1914, a series of papers entitled, "Aircraft in Warfare: The Dawn of the Fourth Arm." [8] Paper No. V of this series contains the paragraph:

> There are many who will be inclined to cavil at any mathematical or semi-mathematical treatment of the present subject, on the ground that with so many unknown factors, such as the morale or leadership of the men, the unaccounted merits or demerits of the weapons, and the still more unknown 'chances of war,' it is ridiculous to pretend to calculate anything. The answer to this is simple: the direct numerical comparison of the forces engaging in conflict or available in the event of war is almost universal. It is a factor always carefully reckoned with by the various military authorities; it is discussed *ad nauseam* in the Press. Yet such direct counting of forces is in itself a tacit acceptance of the applicability of mathematical principles, but confined to a special case. To accept without reserve the mere 'counting of the pieces' as of value, and to deny the more extended application of mathematical theory, is as illogical and unintelligent as to accept broadly and indiscriminately the balance and the weighing-machine as instruments of precision, but to decline to permit in the latter case any allowance for its known inequality of leverage.

In this paper Lanchester develops, by mathematical reasoning, the principle that:

> The fighting strengths of two sources are equal when the squares of the *numerical strength* multiplied by the *fighting value of the individual units* are equal.

During World War II, and not knowing of this earlier work of Lanchester, a mathematician associated with AMP wrote a paper [9] which reproduced and extended the previous results of Lanchester, giving estimates of times required and victorious forces remaining when one military or naval force meets a second. The author also investigated the quantitative aspects of division of forces, showing, for example, that a first force need be only 71 per cent as strong as a second to liquidate the latter totally if the former be clever enough to maneuver the latter into a position where his forces are temporarily divided in two. Other papers in this field were written by a Tufts College group working under the auspices of the Special Devices Division of the Navy.

Reference has been made above to the brilliant and critically useful analyses carried out during this war by the British Antiaircraft, Coastal Command, and Admiralty Operations Research groups. Similar, and also highly successful, groups operated in connection with our forces, the outstanding examples being Research Group M which furnished the central analysis for the antisubmarine campaign, and the operational groups assigned to the various theatre commands by the AAF. This is no place, nor is there any need, to rehearse the accomplishments of these groups. Looking at our own effort from the outside, but from the vantage point of fairly close relationship from the time of the first organization of the British AA group onward, the author would, however, risk two recommendations. First:

> *That every important arm of the Service develop Operational Research groups; and that in addition to the absolutely essential groups in the theatres, there be one or more central correlating groups.*

The absence of such central study and correlation seems to this author to be a striking weakness in the AAF setup, and an evident strength in the Navy antisubmarine setup. Second, and in spite of the outstanding personal qualities of the personnel which recruited and organized the AAF groups:

> *That the recruitment of Operational Research personnel, the organization of the system, and the*

central direction of it be carried out by technically trained personnel, including mathematicians, physicists, engineers, psychologists, physiologists, etc., but with a minimum of lawyers and architects.

In England, in addition to the Operational Research groups, a large amount of general analysis of air warfare was carried out by the extensive group working under the direction of Dr. L. B. C. Cunningham. Certain of these studies relate to rather specific details; but others are grave and useful attempts to tackle rather broad problems, such as Cunningham's own extensive analysis of air duels.*

Many bureaus and divisions of our Army and Navy recruited and utilized, often with great effectiveness, both civilian and uniformed analysts. Aberdeen Proving Ground (particularly the Ballistic Research Laboratory), the Ordnance Department, the Naval Bureau of Ordnance and the Bureau of Aeronautics (particularly at the Naval Research Laboratory, Inyokern, Patuxent, etc.), the Army Air Forces (particularly at Eglin, Orlando, Wright Field, and Laredo) are only a few scattered but important instances with which the author happens to be familiar. Many divisions of the OSRD had large and excellent groups of analysts of various types, Division 14 (Radar), Division 6 (Subsurface Warfare), Division 7 (Fire Control), Section T, and the Applied Mathematics Panel, being perhaps outstanding instances.

In many — perhaps in almost all — of the instances mentioned in the scattered and fragmentary list just given, the analyses were related to what might be called the micro-theory of war, rather than the macro-theory. These two expressions can be given more specific definition after certain discussions of the next and concluding sections of this chapter. But by macro-theory is meant the general and broad analytical theories discussed in Section 12.4, whereas by micro-theory is meant the analytical study of much more detailed problems which can be isolated from general tactical and strategic considerations, such as, for example, a study of how much the *theoretical accuracy* of a bombsight can be improved by certain changes in design. If, in contrast, one wishes to study not the *theoretical accuracy* but the *practical effectiveness* of the modified bombsight, then he must take into account all sorts of general considerations (training, operational simplicity, logistics, time delays in changing equipment and techniques, value of increased accuracy, operating conditions in theatre, nature of target, etc.); and then one is perforce faced with a macro-problem.

The great progress made during World War II by many groups and agencies in the solution of micro-problems is of real significance for a macro-theory; for the structure of the macro-theory requires, so to speak, bricks to be laid up into the walls, and general blueprints for the design. The individual bricks are, in many instances, the solutions of micro-problems. If these solutions were not available, and if we had not thus learned how to make bricks, then there surely would be little sense in trying to draw up elaborate blueprints of general plans.

A very large fraction of the activities of AMP was directed toward solutions of military micro-problems. Various of the more important fields of micro-problems are reviewed in other portions of AMP's final report and many micro-problems in air warfare are referred to in other chapters of this volume. It is, however, a specific duty of this chapter to report briefly on two studies, carried out under AMP auspices, which, while not actually macro-theoretic in character, at least come closer to having general tactical or strategic scope than do most other AMP studies.

The former of these studies began, before AMP was created, under the auspices of the analytical section of the Fire Control Division of the NDRC, that is, under Section 5 of Division 7. Since the Chief of AMP is also Chief of Section 5, Division 7, the later transition to AMP auspices was hardly noticeable. The latter study to be reported here came rather late in the history of AMP. Thus AMP, whatever it did in between, started and more or less ended with fairly general studies of air warfare. We will now briefly describe these two studies.

12.7.1 Study of Alternative Fighter-Plane Armaments

This study arose out of the enthusiasm which a few of us had for two powerful and pioneering papers concerning a mathematical theory of air combat.[10,11] We showed and praised these papers to various officers of the Army Air Forces and of the Naval Bureau of Aeronautics until, in self defense, they suggested that we try to digest and simplify these papers, interpreting them in terms not so formidably mathematical. Then, when they had been aided to

* See Section 12.7.1.

an understanding of what had been done, they proposed that we apply the same or similar methods to a problem they would set.

The first step was accomplished by two reports, the first of which [12] gave an outline of the two British papers, and the second of which [13] gave a nonmathematical exposition of the two papers. These reports were made the basis for an extended conference with AAF, Bureau of Aeronautics, and RAF personnel; we were then asked, as the second step, to apply these ideas to the problem of comparing the relative effectiveness of four 20-mm guns versus eight 0.50-in. guns as armament for a fighter making a stern attack on a defended twin-engine bomber.

The methodology of the analysis of this problem was rather carefully set down, and a list was made of the kinds and pieces of information (or of estimates, or of guesses) which would have to be made before the answer would be forthcoming. These questions (nature of combat, bomber and fighter armament, value and variations of accuracies, ammunition, vulnerabilities, etc.), were then discussed at very considerable length with the experienced officers. As a result estimates were arrived at which every one agreed were almost certain to bracket the true values, although in many instances the true values were admittedly unknown. Thus the analysis was based on five alternative assumptions of vulnerability, three assumptions concerning bomber firepower index (which takes into account rate and accuracy of fire) and four assumptions concerning fighter firepower index. For four of the five assumptions concerning vulnerability the eight 0.50-in. were found to have an advantage (ranging from 27 percent to 75 percent) over the four 20-mm guns. For the fifth vulnerability assumption, the 20-mm armament was preferable, having about 20 percent advantage.[14]

The study was thus necessarily inconclusive. It did, however, make clear just what sort of information was necessary to obtain a conclusive answer, and it furnished the necessary analytical methods. Furthermore the study rather strongly suggested the superiority of the 0.50-in. armament especially since it was discovered that the use of an optimum ammunition mixture increased the superiority of the eight 0.50-in. guns for the first four vulnerability assumptions and reduced the advantage of the 20-mm guns to 10 percent in the case of the last vulnerability assumption.

As by-products, this study also issued important memos on the subject of optimum ammunition mixtures,[8,15] on the optimum interrelation of aiming and dispersion errors.[16,17]

12.7.2 B-29 Studies

The fighter armament study was largely carried out in 1942 and 1943, although not formally concluded until 1944. The B-29 studies, however, did not originate until mid-1944. The initiation of AAF Project AC-92, which resulted in these B-29 studies by AMP, was effected by a communication, dated June 14, 1944, from Headquarters, AAF, asking that the NDRC collaborate with the AAF "in determining the most effective tactical application of the B-29 airplane." This communication was the direct result of a conference held at Orlando, Florida, under the auspices of the Army Air Forces Board. The conference report, which was later made an official part of the AC-92 directive, stated in part:

1. That no scientifically controlled or scientifically evaluated investigation of various defensive formations of bomber aircraft had ever been conducted by the AAF.

2. That it was of the utmost importance to conduct such investigations.

3. That no scientifically controlled or evaluated investigation had ever been made of the B-29 defensive armament.

4. That no standard central fire control gunnery doctrine had been developed, no standard manual of B-29 gunnery developed, nor any detailed training standards developed for B-29 gunnery.

The NDRC was therefore requested, as Project AC-92, to collaborate with the AAF in planning, organizing, carrying out and evaluating an investigation of the most efficient B-29 formations. Account was to be taken of:

1. Self protection of bomber aircraft from fighter by bomber's own gun.
2. Self-inflicted damage.
3. Support fire.
4. Maneuverability and evasive action.
5. Effectiveness of bombing.
6. Formation control.
7. Flak.
8. Rockets.
9. Air-to-air bombing.
10. Fighter escort.

In addition, Project AC-92 was specifically asked to evaluate the defensive armament of the B-29 (a task which, in fact, was also required by item (1) just

above), to study the ability of the average gunner to learn the effective use of the system, and to develop a standard procedure and gunnery doctrine for the B-29.

If anyone is surprised and shocked that such a request needed to be made in June 1944 relative to the B-29 airplane, which was then being produced in large numbers, there are two replies which should be made to him. The first is that it *was* both surprising and shocking. But the second reply is that this airplane was, with almost incredible imagination and energy, forced through to combat use on a schedule which made it difficult if not impossible to follow an orderly, or even a rational procedure. And one should also remember how magnificently the actual result justified all the unorthodox means.

The directive for Project AC-92 referred to one specific aircraft (B-29), but was in other respects very broad. Extensive flying experiments were obviously going to be necessary, and to carry out these essential flight tests, it was specified that the facilities of the 2nd Air Force would be available. General U. G. Ent, then in command of the 2nd Air Force, had in fact been one of the leaders in initiating and planning the project.

After some considerable negotiation, this project was assigned to AMP, which accepted it reluctantly. Although we had by that time developed a sizeable personnel expert in the analytical aspects of bombing, air gunnery, flak analysis, etc., it was clear that this project involved a huge experimental program and also clear that organizational difficulties were bound to occur. For civilian organization would, in fact, be trying to do many things (such as operating military aircraft from a military field) that could be done properly only under military organization, and this was bound to lead to confusion if not to trouble. In view, however, of the fact that there seemed to be no reasonable alternative, AMP agreed to do the best it could. From that time until AC-92 was closed out, the Chief of AMP spent essentially all his time on this one job.

To meet the objective of Project AC-92, AMP set up a broad program which included:

A. Experimental tests at Alamogordo, New Mexico.

B. Model experiments at Pasadena.

C. Dynamic testing of the Central Fire Control at Austin, Texas (under Division 7, NDRC).

D. Gun camera tests at Eglin Field, Florida (AAF Board, with collaboration of Division 7 and AMP).

E. Collaboration with General Electric in tests at Brownsville, Texas.

F. Psychological studies (under Applied Psychology Panel).

G. Analytical studies of special features of the problem such as vulnerability, flak, etc. (carried out by AMP under other studies reported in Part III of this volume).

H. Broad analytical studies, such as those which relate to bombing effectiveness, general "economic" theory of bombing, etc.

As is briefly indicated by this list, items C, D, E, F, and G could be taken over by existing AMP facilities or by other parts of the NDRC. It was necessary to provide for A, B, and H, and to provide general direction and correlation for the whole program.

As to A, the Alamogordo (and Albuquerque) portion of the program, chiefly involving experimental tests and their interpretation, was carried out through a contract with the University of New Mexico and in very active collaboration with the Second Air Force. This work will be further described below.

As to B, the optical model experiments and their interpretation were carried out through a contract with the Mt. Wilson Solar Observatory (Pasadena) of the Carnegie Institution of Washington [CIW]. This work will also be further described in Section 12.7.3

As to H, and the problem of central supervision and correlation of so complicated and far-flung a program, a contract was made with Princeton University which made available a group including physicists and engineers as well as mathematicians, which had had long experience with fire control problems.

In addition a Steering Committee was set up, consisting of two representatives of AMP; one of Division 7 (Fire Control); one from Operations Commitments and Requirements, AAF Headquarters; one from the AAF Board; one from the 20th Air Force; and one from the 2nd Air Force. A highly experienced and expert RAF officer also attended all Steering Committee meetings, and took an active part in the Alamogordo experiments, and a representative of the War Department NDRC Liaison Office attended most meetings.

Although the writer, who was responsible for the direction of the project as a whole, steadily and stubbornly took the position that the directive (with its broad inclusiveness and its emphasis on scientific

planning and evaluation of experiments) could not possibly be met unless *all* the items A to H inclusive progressed simultaneously, it soon developed that the Service personnel were preponderantly interested in A, eventually almost equally interested in B, rather uninterested (with one exception) in C, D, and E, only very mildly interested in F and G, and totally (and in most cases emphatically) uninterested in H.

What we could accomplish in this project depended essentially on AAF support and assistance, and we did, after all, consider it to be our duty to come as close to doing what the AAF wanted done as our scientific and personal consciences would permit. Thus it is natural and in fact inevitable that Project AC-92 faded out of the picture when items A and B had produced their major contributions. Items D, F, and G eventually produced results which are certainly of large and lasting value.[f] But item H, which should appear as the crowning jewel in this chapter on general studies in air warfare, was a total flop. It was a flop, this author is convinced, because it never was given a chance of being anything else. Some persons in the 20th Air Force were interested, in principle at least, but they considered that such studies should be carried out within their own organization, say by their own Operations Analysis personnel. The rest of the Service personnel, without whose complete backing this part of the work could not go on at all, had apparently used up all their interest in broad studies in the process of asking for them in the directive.

To make one of those sage observations which are so easy to make in retrospect, it is quite clear that the AAF should never have allowed itself to get into such a position that a very few months before the aircraft in question was to enter combat on a large scale, they would have to ask a civilian organization to assume responsibility for planning, conducting, and interpreting tests and studies aimed to discover whether the defensive armament of this great bomber was or was not any good; whether it might be better to throw the armament away and trust to altitude and speed; how to operate the fire control system; how to choose and train personnel; what kind of formations to fly and how to fly them; etc.

Such questions obviously should, in any reasonable and rational development, have been settled long before, by test establishments, analysis groups, and decision groups maintained by and within the AAF.

* * *

Having now sketched the technical and organization background for the B-29 studies, it remains only to describe the work itself.

In the early spring of 1944 a group at the University of New Mexico had collaborated with the 2nd Air Force in carrying out some studies of the K-3 sight in a Sperry upper local turret and of an N-6 sight in a Martin upper turret, these being camera tracking tests with the turrets mounted on movable platforms. Previous studies by the same group of the stability of the B-17 in flight indicated the types and amounts of movements which should be given the platforms.

These studies stimulated the interest of the New Mexico group which, together with General Ent, played an important role in the Orlando conference mentioned above, which resulted in Project AC-92. It was thus natural that AMP turned at once to the University of New Mexico, offering them an NDRC contract under which they would organize a group at Albuquerque (and at the large B-29 field at Alamogordo) to carry out experimental tests on the B-29 airplane. This New Mexico program, as should be clear from the explanation above, actually constituted a large portion of AMP's activity under Directive AC-92. The remainder (except for the Pasadena optical studies which will be described in Section 12.7.3) consisted primarily of the labors of the Princeton group in correlating and otherwise assisting[g] the large and widely distributed pattern of activities in AC-92 by frequent visits, by continual assistance in obtaining necessary equipment and information, especially from the AAF, by needling the various organizations into action on our work, by extending specially valuable aid to the Applied Psychology Panel in their part of the program, by carrying out analyses of various special problems, and by trying to keep all parts of this program mutually informed through a series of frequent "AC-92 Bulletins."[10] It had originally been planned that the Princeton group would also gather together all the results of the separate tests and analytical studies, and synthesize them into some more general

[f] As the project developed, one important item was added — the problem of devising workable laboratory and field methods of harmonizing the B-29 guns. This was worked out cooperatively with Section 1 of Division 16, and it is the writer's impression that very important results were achieved.

[g] A great deal of such assistance was also furnished by the Technical Aides in the central office of AMP.

attack on the original objective of determining the most effective tactical use of the B-29 airplane. This synthesis, as has been stated above, was not attempted for the very simple reason that it could not go forward without the complete backing of the AAF officials, and, as explained above, it turned out that in spite of the wording of the official directive, they actually just were not interested.

A brief description of the New Mexico program will now be carried out under the four headings: Problem, Facilities, Techniques, Results.

Problem

Two main problems were attacked:

1. In what way, how frequently, and how effectively can fighter planes attack a single[h] standard or stripped B-29?

This involved a determination, at various speeds and altitudes, of the types of possible fighter passes, the frequency with which the passes could be carried out, and the accuracy and duration of the fighter's fire during these passes.[i]

2. What is the defensive strength of a single B-29 bomber against fighter attack?

This involved a determination of the accuracy and duration of fire from bombers' guns. Air tests of these accuracies were to be supplemented with air-to-ground firings, and ground tests of the accuracy of the central fire control system.

In addition to these two main problems, certain other problems were studied, either because they were necessary and/or inevitable concomitants of the two studies just mentioned, or for the practical reason that the New Mexico group developed a facility which was able to take on and push through to necessarily approximate but remarkably rapid conclusion a test of almost any aerial gunnery device. Since the AAF did not itself have such a facility, it was natural that there was a tendency to forget the directive (and indeed to forget the NDRC itself), and when any sort of a rush problem arose, just telephone the boys down in Albuquerque and ask them to do a quick job.

[h] The New Mexico experiments actually dealt almost exclusively with the problem of single B-29 bombers. This was, in any event, the sensible and almost the necessary way to start. Some aspects of the problem of bomber formation were studied at Pasadena, and in other auxiliary studies such as those reported in Part III of this volume.

[i] Such a study should, of course, include all possible types of evasive action on the part of the bomber. Time did not permit this.

Among auxiliary problems that were closely connected with the original job should be mentioned an extensive amount of time spent in determining the performance characteristics of the B-29 airplane at different altitudes, speeds, loads, and armor and armaments. There were also studies of dispersion from the various B-29 guns, harmonisation, etc.

Among the disconnected studies should be mentioned:

1. Test of APG-5 with K-3 sight in B-17.
2. Speed reduction of B-29 due to radar domes and open bomb doors.
3. Turret dispersion tests, A-26 aircraft.
4. Performance of APG-15 (radar versus optical range).
5. Test of APG-5 with Mark 18 (K-15) sight.
6. Test of APG-15 with special GE computer.

Facilities

At Albuquerque and Alamogordo, the civilian personnel on this project included 161 persons, as follows: 11 major technical persons, 16 research assistants, 14 skilled field assistants, 26 administrative assistants, 70 technicians (including a large staff of computers), 18 machinists, guards, Service personnel, etc.

At Alamogordo, a special unit of the 2nd Air Force (AAF Base Unit 200) was assigned to this project, and extremely cordial cooperation was furnished by the 2nd Air Force Command. BU 200 included a total of approximately 100 Service personnel, of which something over 40 were officers.

Thus taking into account the Princeton group and the central AMP office, there were about 350 persons involved in all in this study.

The assigned aircraft totaled 43, as follows:

8	B-29 (6 standard, 2 stripped)	16	P-47
		6	P-63
5	B-17	8	Other

A total of 1,448 separate flights was carried out (without accident), including 348 flights of B-29 craft, 106 of B-17, and 744 of P-47. A total of 138,050 ft of 16-mm film was exposed in gun cameras. A large computing and analysis section was developed at Albuquerque.

Techniques[10–14]

The basic reports must be consulted for details, but the main results of this study were obtained by carrying out actual flights of bombers and actual

attack by fighter planes (with simulation of battle conditions for interception, etc.); the results of all "firings" of guns of bomber and fighter being recorded photographically by gun cameras; this great mass of film then being measured and analysed. The gun camera method was essentially a single camera technique, making use of distant background[39] points, such as clouds or mountains, as reference points. Thus both the terrain and the climate of southern New Mexico were essential to the procedure. The method is admittedly not as accurate, particularly for flexible guns, as deflectometer methods, but probably sufficed for the purposes in hand. In any event, it was several cuts better than nothing, and no other known method was really possible, taking into account the mass of data which was necessary, and the time available.

A great many other methods were skillfully and energetically developed by the New Mexico group to handle other details of the program.

Results

In this program was experimentally demonstrated, and for the first time, as far as we know, the ability of the stripped B-29 to carry great loads over great distances at great speeds and high altitudes. In particular, this project carried out successful advance simulations of the Saipan-Tokyo and return bombing missions.

The character of the fighter plane attacks which can be carried out against a B-29 was experimentally determined, for altitudes ranging from 20,000 ft to 32,000 ft, and the errors of various types of computing sights determined for these actual courses.

The accuracy of fighter plane fire against the B-29 was experimentally determined for altitudes ranging from 14,000 ft to 32,000 ft, and the frequency of hits analysed for different approaches, etc. These tests were all made with fighter pilots who had had combat experience.

Fighter hits on bomber per pass were obtained as a function of angle and range, and converted through a study of pass frequency to fighter hits per hour as a function of angle and range.

Gun and turret dispersions were measured for the B-29, as well as harmonization errors on the ground and (approximately) in flight. Tracking errors were determined carefully for all positions, and actual gun errors determined approximately.

Certain inherent limitations of the B-29 central fire control system were determined in ground tests which measured the output of the whole fire control system for known and controlled inputs.

The ability of the P-47 fighter and of certain faster and more modern fighters to attack a stripped B-29, flying at high speed and great altitude, was determined.

All these jobs could be done better now, or could have been done better then if more time had been available. But it should be mentioned that the experimental program did not begin until early July 1944, and the main results were reported to the AAF in accordance with the requested schedule,[39] namely, on November 15, 1944, less than five months later.

It also seems almost unnecessary to add that this study demonstrated that, in a reasonable length of time, it is possible to obtain the kinds of experimental data necessary for a more general, more inclusive, and more penetrating study of the employment of large bombers.

12.7.3 Optical Studies at Pasadena

Just as the experimental program at New Mexico was largely based on the previous experience and accumulated enthusiasm of the New Mexico group, so the model experiment program at Pasadena was primarily based on the previous experience and accumulated enthusiasm of a member of the staff of the war research division of the Mt. Wilson Solar Observatory, who had had special experience with many sorts of optical devices and procedures, and who had more recently become well informed concerning aerial gunnery. Shortly after Project AC-92 was accepted by AMP, a contract with the Carnegie Institution of Washington was recommended, under which was made available the very unusual facilities, both of personnel and of shop equipment and experience, of the Mt. Wilson Solar Observatory of the CIW at Pasadena, California.

We will now briefly describe these Pasadena studies under the four headings: Problem, Facilities, Techniques, Results.

Problem

To analyse the intensity and distribution of firepower about variously sized and designed squadrons of (B-29) aircraft, and

To furnish a method whereby these complex relationships can be visualized.

The complexity of the situation can be appreciated when one notes that a single B-29 has 5 turrets and

5 sighting stations, with 36 independent arrangements of the controls of the various turrets by the various sighting stations.

Facilities

Employed under the contract were four senior scientific men and six assistants. In addition, Mt. Wilson Observatory donated the full or part-time services of 19 of their own staff; so that nearly 30 persons were involved in toto. The program was carried out under a local Directive Council headed by the Director of the Mt. Wilson Observatory.

Shop and other facilities (particularly computational) were made available by the Observatory, and the gymnasium of a local school was rented in order to have a room large and high enough to accommodate the experimental setup.

Techniques[36-38]

This study was based upon the idea of making small (actually 1 to 72 scale) model planes, and installing a small light source at the position of each gun (and of each sighting station). These lights were equipped with shields which restricted the field of illumination in accordance with whatever restrictions there might be on the field of fire of the gun in question (or the field of visibility of the sighting station in question). A switching system permitted the simulation of all the possible methods of control of the various guns by the various sights. Then since the effectiveness of fire varies roughly with the inverse square of distance, as does the intensity of illumination from a point source, the total intensity of illumination of any point in space was approximately proportional to the total firepower which can be brought to bear at this point.

Thus one constructs a set of models, installs the lights and shields, adjusts individual intensities in a calibrating operation, arranges the models in space to conform to some formation design to be studied, and then determines the intensity and distribution of firepower for various briefing systems by measuring the corresponding light intensity pattern.

The model squadrons were set up at the center of a 25-ft radius spherical shell, on the white inside surface of which one could actually observe the light intensity pattern. Some 18 model planes were constructed for use at Pasadena, and 12 more for use in the Marianas.

The light intensities could be measured, point by point, with photocells. This method was developed to an accuracy of about 1 per cent. Or one could photographically record the light intensity pattern over some area, and then carry out photometric measurements. Or, still more simply, one could simply count the lights visible from a certain point, i.e., the guns which could be brought to bear on this point, tape distances, and rapidly compute firepowers. Although this last method does not make full use of the optical scheme (and permits, for example, any law of decay of firepower with distance), it actually proved to be the easiest and quickest to use in many instances.

Results

Using the system described above, an exhaustive but preliminary study was made of the firepower pattern for a single B-29 with all of the various types of control.[39] Then similar studies were made for a squadron of 4 planes,[30-32] for the 11-plane squadron as used by the 21st Bomber Command,[33] and for the 12-plane squadron as used by the 20th Bomber Command.[34]

In February 1945, the 20th Air Force held a large conference[35] at Pasadena, primarily to observe and consider the results thus far obtained in this study. As a result of this conference certain changes were recommended in the stacking and briefing of the 12-plane squadron; and this modified 12-plane squadron was then thoroughly analyzed by the Mt. Wilson group.[36]

Early in this project it became obvious that one of its greatest contributions was that it furnished a method whereby one could *see*, simply and convincingly, a pattern of relationships otherwise so complicated and subtle as to provide topics for perpetual arguments. Two officers, urging different patterns for a formation, could set them up on this optical model, and could together *directly examine* the resulting pattern of firepower.

Thus it was clear that the device had important applications for training, and in the field as well as in a scientific laboratory. Moving pictures were made, showing the firepower variation of formations as one circles about them. Concerning such pictures the President of the Army Air Forces Board remarked that he "believed these motion pictures gave the best idea to airmen as to the relative effect of firepower about a formation yet presented." Certain of these pictures were flown to the Marianas and viewed by General LeMay and by many gunnery officers and men at the front. And as was indicated above, a

model setup was constructed and flown to Guam for use there in analyzing firepower problems.

In addition to the successful achievement of their primary aim, the personnel of this project carried out and reported on certain auxiliary studies. A considerable analysis, for example, was made of fighter attacks,[87–89] and it was shown that for a wide variety of attacks, all the breakaways pass through a limited region some 20° in angular width and some 400 yd off the stern. Studies were also made of the problem of offset guns in fighters,[90] and of the duration of strafing attacks.

Again the author of this chapter cannot resist emphasizing that these studies, although much narrower in scope than the "general" studies here being advocated, are excellent examples to show that the information necessary for the general studies can in fact be obtained.

12.8 MATHEMATICAL ANALYSES AND WAR

The topic of this chapter is "General Theory of Air Warfare." But it has been a central purpose of this chapter to point out that modern war is essentially and inevitably a combined operation. Thus it seems not inappropriate to close with some general remarks concerning the contribution that mathematics can make to the broad field of national defense. These remarks will involve some topics which have not been mentioned before in this chapter, and should serve to furnish a setting for the general activities which are the special concern of this chapter.

To give any adequately complete discussion of the possible contributions of mathematics to war and to national defense would require far more space than is available here, and quite other authorship. All that is intended here is to give that minimum description which will serve to indicate the relationship of the general analytical studies of warfare, here urged as vital, to other mathematical activities of military significance.

We will discuss mathematical activities of military importance under the following headings: Basic, Service, Design and Use of Individual Devices, Operational Analysis, General Theories.

This list of headings is poor for several reasons. The subheadings are neither exclusive nor self-explanatory, and in some cases they are worse than vague, in that they appear to say something less than or more than is intended. But since some effort has failed to produce a better list, let us take these topics, one at a time, and explain briefly what is meant.

12.8.1 Basic

The most important point is that *all* developments of mathematics, from the most abstract concept of the purest of pure mathematics to an isolated item of new technique in applied mathematics, are basic contributions to national technical strength. Engineering, physics, chemistry — yes and biology and medicine as well — use for their central core of quantitative precise thinking the analytical tools provided for them by the mathematician. In these days of the atomic bomb, one hardly needs to emphasize the close relationship between military might in the field, and the abstract pencil theories of the analytical expert. Thus the stimulation and support of the field *as a whole*, especially through fellowship and other training aids, is both an essential and a sure way to provide the mathematical talent required in time of emergency, and is also an essential basis to a proper development of other elements of the technical strength of the nation.

Volume 1 of the Summary Technical Report of the Applied Mathematics Panel, together with Volume 3 which is devoted to probability and statistics, although primarily concerned with describing some of the intermediary steps in the military utilization of basic mathematics, does thereby indicate some of the classical fields of pure and applied mathematics that connect closely and obviously with military problems. It is intended to include here, under the heading of "Basic," all such activity. But it is also intended — and this is the main point — to include here the whole of mathematics as a vitally necessary element of scientific strength, basic to all developments of the other sciences and technologies, and hence basic to the national welfare and the national defense.

12.8.2 Service

This title is an egregious misnomer, for from one valid point of view it covers all of mathematics in direct relation to military problems. The mathematician who is trying to make his discipline serve the national need should, in one sense, *have nothing but service in mind.*

But one can serve in many ways. Sometimes one can serve best precisely by *not* doing what the other

fellow wants; or by doing something that he has never, and would never, think of. And the word *service* is here used in the restricted and limiting sense of "doing what the other fellow wants." We should not lose sight of the fact, however, that in the process of so doing, one's major opportunity often lies in gently leading the other fellow to see what he *ought* to want.

There is a great deal of useful mathematical service that can be done on the level of isolated requests. Thus individuals, or military agencies, or nonmilitary or government agencies, or NDRC groups, or university and commercial laboratories working on national problems, may come and say:

Please evaluate this integral.

Please solve this differential equation.

Please compute the value of this determinant.

Please interpolate, using third order differences, nine values between every two values of this table.

These requests may arise out of secret projects, and it may therefore be necessary or wise (which, by the way, are not equivalent) to keep the mathematician completely in the dark as to the nature of the problem. In such cases, these are items of *pure service*, so to speak, where the mathematician (provided he can) does precisely what he is asked, and no more.

It is easy to see, however, that isolated requests are extremely likely to lead to another and more important grade of service. For if the problem is, for example, not to evaluate a single determinant, but rather to evaluate many determinants, then the mathematician may well inquire as to the original formulation of the problem. And he may be able to suggest alternative equivalent procedures which lead to simpler expressions, easier to compute. Or he may refer the client to devices which have been constructed for the solution of systems of linear algebraic equations. Or he may even uncover errors in the formulation of the problem which make it undesirable to do any computing at all.

As another example, suppose the specific question to be: Study these two records of firing trials; and tell us which gun and fire control system is better, by how much, and how certainly.

Here there are almost surely two stages of services involved. First (and it is unwise to omit or delay this stage), the service mathematician should accept the situation, however bad it is, and do the utmost that can be done to answer the questions asked. Second, he should point out to the client why the problem was a bad one, how the trial could have been improved. This may well open up, to the client, the whole field of statistical design of experiments, and the second stage of service may well be of indefinitely greater significance than the answer to the original problem.[j]

A great many groups and agencies and establishments, recognizing the value of mathematical service of this sort, obtained for themselves what the German physicists used to call "house mathematicians," i.e., locally available service mathematicians. The AMP, in addition to the service which it was prepared to furnish through the personnel of the contractual groups in New York, Princeton, Providence, Chicago, Cambridge, etc., also farmed out individual mathematicians to various groups on either short- or long-term loans. One of the conspicuously successful of these ventures involved the placement of an AMP mathematician with the Army Air Forces Board at Orlando, where he served for over twenty months as a mathematical consultant. As a result of this experience, he has written a set of comments and recommendations which seem so important that they are included, at the end of this chapter, as Appendix A. Since the views of this consultant are here reproduced, it seems relevant to add that his services were so appreciated that he was given, by Brigadier General E. L. Eubank, President of the AAF Board, a special letter of commendation:

> ". . . for his splendid record of mathematical assistance to the Army Air Forces Board . . . he has been actively engaged in various projects the successful completion of which was, in no small part, the direct result of his efforts . . . in each of the following . . . he was a major contributor . . . the . . . selection was made since each of the projects was widely recognized as having a measurable effect upon the war effort and its successful termination."

12.6.3 Design and Use of Individual Devices

A large fraction of the mathematical work in this war has related to the design and use of individual devices — antiaircraft gun directors, tracking mechanisms, bombsights, proximity fuses, radar sets, plane-to-plane gunsights and sighting systems, rocket sights, guided missiles, torpedo sights, etc. In con-

[j] This is essentially the way in which AMP got into the field of sequential analysis as applied to acceptance testing, quality control, etc.

nection with such activities there are several very important types of questions to consider, such as:

1. What performance is this device capable of when operated perfectly?

2. What performance is this device capable of when operated by a random untrained GI, by a selected untrained GI, by a selected trained GI?

3. What is the optimum selection and training process? (This involves close collaboration with psychologists and physiologists.)

4. What modifications in design would improve (whether they improve or impair the *theoretically possible performance*) the *actual performance* with the type of operating personnel likely to be available in the field?

5. What is this device supposed to accomplish, and to what extent does it do so (that is to say, an analysis of the *problem the device is supposed to meet, rather than of the device*)?

These particular questions are intended to refer to a device which is not available for actual test. They involve studies which should be made in time to influence design.[k] The first of the above questions refers, in a fairly strict sense, to what has previously been called *a micro-problem*. The character of the request is such that an analyst could be handed essentially nothing more than a complete description of the design, and could be locked in an otherwise empty room until the results were completed. By the time one gets down to the fifth question, however, it is clear that he has left behind the field of micro-problems, where one can study one isolated problem without reference to a whole large set of practical considerations, and has entered the field of macro-problems. The fourth and fifth questions are bound to involve tactical considerations, and may involve strategic considerations.

If one supposes that operable examples of the device exist, then a new set of problems enter — those of *testing*. One should consider *bench testing*, both of components and of whole devices; *use testing* under controlled conditions which to a lesser or greater extent attempt to reproduce actual field or combat conditions; *extended field trials*, which are carried out on a small scale but under field conditions; and finally *combat trials*.

One of the great advances of this war, at least in those fields with which the author is acquainted, consisted of eventual recognition by military authorities of the *necessity for quantitative testing under controlled conditions, together with analytical (often statistical) assistance in the planning and the interpretation of the tests*.

The author was present at some of the demonstrations of military equipment given in the past (about four or five years ago). These tests were wholly inadequate from the standpoint of controlled scientific testing, and certainly should not have constituted a basis for acceptance or rejection.

The advances in objective, quantitative, and controlled testing of fire control equipment, for example, have been great and, it is hoped, irreversible. The development of dynamic testers and associated computers for antiaircraft artillery equipment, and the development of great testing engines for plane-to-plane fire control equipment constitute two great contributions to this field, both carried out by Division 7 of the NDRC and doubtless fully covered in their reports. The development of camera testing procedures and the design and assessment of firing test procedures for flexible aircraft gunnery, the analysis of frangible bullet trials, and a large number of other studies and developments were aided by AMP, and are covered in Part I of this volume.[l]

One of the important practical results of such analysis of device performance, such studies of device purpose, and such testing of devices as is here envisaged is the emergence of some rational basis for setting *military* requirements and also for setting the actual *production specifications* which must be met by the manufacturer. Too often some military board has in the past put out, with presumably straight faces, a set of military requirements which come ludicrously close to demanding a device of perfect accuracy, simplicity, and reliability, small in dimensions and light in weight, demanding no skill from the operator, and being producible out of noncritical materials in very little time. Too often the tolerances set in the manufacturing specifications seem to be based not so much on any rational requirement, but rather on the notion that the manufacturer is having too easy a time of it if any reasonable fraction of his output passes.

All these questions of device analysis and device testing, military requirements, field trials, etc., are interrelated in such a way that sensible progress cannot be made unless the organizational structure of

[k] This sounds ridiculously obvious, but as a matter of experience, this condition was by no means always met in the cases with which we had experience.

[l] See also the comments of the previous section of the present chapter.

the military group involved is such as to permit the requisite interlocking of activities. At the request of Dr. Edward L. Bowles, Expert Consultant to the Secretary of War and Special Consultant to the Commanding General, Army Air Forces, the author prepared a memo concerning the development of assessment facilities for air gunnery in the Army Air Forces. Because of its direct bearing on the issues just mentioned, this document is reproduced as Appendix B of this chapter. It is gratifying to be able to add that when Dr. Bowles issued, on April 12, 1945, a letter of instruction to the man who had just been given an advisory appointment in the Secretary of War's office, with special responsibilities in the field of aerial gunnery, he outlined the War Department's plans for developing an assessment agency, and concluded this section of his letter with the direction that the remainder of the pattern was to be completed along substantially the lines set forth by the memorandum which had been furnished him by AMP and which forms Appendix B of this chapter.

12.8.4 Operational Analysis

This type of activity has been referred to previously in this chapter, and it lies outside both the purpose and the competence of the present author to add here any detailed description or critique of operational analysis. It must, however, be included as a major item in the list of important ways in which mathematics serves the war effort.

Operational Research Sections, working in the various theatres of war, are concerned with such questions as:

1. How well are we using a certain device?
2. How can we use it more effectively?
3. What minor (including field) modification of this device would increase its effectiveness?
4. What devices do we need that we do not have?
5. What are the causes of our successes and failures?
6. What results are we getting against the enemy?
7. What should we do on the next mission? For example:
   a. If we attack on the deck will our wings slice through barrage balloon cables without amputation?
   b. If we meet jet fighters, what rule of thumb should our waist gunners use?
   c. How many bombs should we drop to achieve a probability of 0.95 of destroying a target?

It is clear that the success of such groups depends essentially on: (1) the personal characteristics of the members, particularly their ability to meet field conditions, their attitude of unselfish service, their willingness to tolerate inevitable limitations, and their capacity to get along with military command and with operating personnel; (2) the spread and excellence of the scientific techniques which they can make available (first-rate pure and applied mathematicians, statisticians, physicists, engineers, psychologists, physiologists, etc.); and (3) the outstanding excellence of their technical leadership. Furthermore, they cannot work to anything like full advantage unless they are backed up by a central home agency which correlates and compares their various findings, points out duplications of effort and weak points in the program of certain groups, and carries out those more basic and long-term studies which cannot possibly be handled by field groups.

12.8.5 General Theories

This topic like the preceding one has been included here primarily so that the present list be complete. Since this chapter has been chiefly concerned with indicating the nature and scope of broad theories of air warfare, or of warfare in general, it does not seem appropriate or necessary to repeat any of the description or argument here.

It is however hoped that the more detailed comments of the present section on "Mathematical Analyses and War" will serve to indicate that the general studies — the general plan for the structure of defense and war — have available as necessary building bricks a vast number of dependable and essential individual or micro-studies.

12.8.6 Additional Comments

Although we have now concluded the remarks forecast in the outline at the beginning of this section, there is one topic which merits some final comments. Every field of mathematics, including some of the most abstract and erudite, has important contributions to make. It would seem unnecessary and perhaps even unfortunate to single out some special field for special comment. But the fact remains that a great many mathematicians — especially some so-called pure mathematicians — have in the past

been relatively uninterested in the subject of mathematical statistics, and the fact also remains that mathematical statistics has proved to be an exceedingly powerful and useful tool in connection with many war problems. Of the three volumes of final reports of AMP, one is devoted directly and exclusively to statistics, while the present volume leans on that subject heavily. If AMP had done nothing at all except what it did in applying mathematical statistics, its budget (to put the matter on a very practical basis) would have been justified many times. One particular application out of very many, for example, would have effected money savings, if the war had continued until the end of 1945, at a rate in excess of one million dollars a month, this money saving being less important than the fact that the improved method could produce, at the fighting fronts, more, and certainly a more acceptable quality of ammunition.[9]

* * *

There is one further postscript remark. One has known of instances in which our military leaders were skeptical or even contemptuous of the importance of certain weapons or techniques — until we learned something about them the hard way, from the enemy. And then we suddenly accept the wisdom or necessity of these weapons or techniques, and get feverishly to work on them.

It may therefore be desirable to point out that when, after the war, the leading aeroballistician of Germany was interrogated, he remarked that he considered it as good as certain that, by statistical study, efficiency factors for battles between formations can be devised which permit one to measure the worth of the armament and of the fire control. This same German expert, after describing their studies of vulnerability, pursuit and nonpursuit attacks of fighters on bombers, combat film analysis, etc., remarked that the basic purposes were *to analyze air warfare and to lay objective foundations for theoretical studies of tactico-technical questions.*

To indicate what almost incredible delays are sometimes involved before an essential idea is absorbed into military practice, this enemy expert mentioned, concerning the use of gun cameras to obtain necessary quantitative data on aerial gunnery, that in Germany:

1. The idea was proposed during the Spanish War, 1936/7, but turned down by the Service.

2. The first film strip of a Spitfire kill was assessed in November 1940.

3. The first lecture on this subject was given before the General Staff of the GAF seven months later (June 10, 1941).

4. The first analysis section was set up nearly ten months later (April 1, 1943).

5. Gunnery assessment was "recognized" by military units sometime in 1944.

6. A central efficiency assessment bureau was finally set up about seven years (October 1, 1944) after the idea was first proposed.

The present author is glad to be able to make this point without the necessity of using local data. Indeed our record, in the decadent and nonmilitary democracy, is a good deal better. But if we want to exclude a completely disastrous "next time," it must be much better still.

APPENDIX A

General comments of Dr. S. S. Cairns based on his experiences as mathematical consultant to the Army Air Forces Board, Orlando.

The following comments are based on over twenty months' experience with the Army Air Forces Board, Orlando, Florida, as a mathematical consultant. The writer served in this capacity from February 1944, through October 1945, as a member of the Statistical Research Group — Princeton, under direction of the Applied Mathematics Panel of the NDRC. These comments are offered in the hope that they may prove useful in connection with any future plans for similar consulting services, especially on the part of military organizations, during either peace or war.

A great need existed at the Board for the services of a general mathematical consultant. This need, which will presumably continue during peace time, arises from the fact that the Board is charged with testing and reporting on new equipment, methods of employment, and tactical questions, with a view to developing official Army Air Forces policies. These duties of the Board call for a combination of personnel having combat experience with those having sufficient scientific training. During wartime, up-to-date combat experience can be provided by a system of assignments of officers fresh from overseas duties. This was adequately accomplished at the Board, though the officers assigned there were not systematically selected on the basis of any peculiar fitness for the work of the Board. The empirical knowledge provided by active military service needs to be supplemented by the abilities of experienced, scientifically trained personnel. There was a serious shortage of such personnel at the Board. It may also be noted that there was no widespread appreciation of this lack or of the potentialities of analytic methods applied to many of the Board's problems.

The writer's position as consultant was somewhat informally established. In the future, it would be well if, in the formal statements defining the Board (or any similar agency), specific provision were made for a mathematical consultant, with a definition of his duties, his privileges, and his status. The following suggestions are based on the writer's experiences in Orlando, where these questions were settled, more or less satisfactorily, by an evolutionary process. Many of the suggestions are afterthoughts. As such, they are not to be interpreted as adversely critical of anyone concerned. In all probability, favorable action would have been taken, whenever practicable, on any of these suggestions, had they been offered during the period of hostilities:

1. It is desirable for a consultant to be a civilian, so that he may be free from the irrelevant aspects of military protocol. As a civilian, he can freely associate with and express himself to officers and enlisted men of all ranks and grades. Conversely, he can be approached with equal freedom by all categories of military personnel. He should, of course, recognize the authority of the commanding officer of the post to which he is assigned.

2. The privileges of a consultant to the Board should include the opportunity to comment on all official projects, not merely on those regarding which he is formally approached by the project officer. Too frequently, a project officer does not realize his own need for technical assistance or else does not appreciate the potential value of such help. Many Board projects were carried out, both before and during the writer's stay at the Board, which could have been improved with the aid of mathematical assistance. In the case of some projects, advice was sought too late, after tests had been designed and performed in a manner inappropriate to the required analysis. Consultation on design of tests is of especial importance.

3. The preceding item implies that the Board consultant should participate in the staff meetings at which projects are discussed. He should be expected to submit suggestions for new projects when they seem appropriate. The writer attended a few staff meetings, but only when his presence was especially requested in connection with a particular project on which he was officially giving assistance. He was always free to offer suggestions of any sort through any officers on the Board or directly to the President of the Board.

4. Some reliable system should be set up whereby all circulating documents of possible interest to the consultant should cross his desk. The responsibility of filling out routing slips at the Board frequently devolved on persons with strangely limited notions of the breadth of the consultant's legitimate interests, so that many documents of importance to him either never reached him or else came to his attention only by accident.

5. The value of a consultant to the Board would be enhanced by the services of an assistant capable of dealing with routine matters involving straightforward computations, questions of trigonometry, and so on.

6. A consultant requires (and the writer received) every consideration in the matter of secretarial services, office supplies, official mailing privileges, and everything else of the sort necessary to the performance of his duties.

7. A technical consultant should have an office of his own, appropriately labeled. The writer came to be associated, in the minds of most officers at the Board, with the Armament Division, so that members of other divisions occasionally felt they were borrowing his services when they consulted him. This situation arose partly because the writer shared an office with members of the Armament Division, and partly because Colonel V. C. Huffsmith, Chief of that division, was, of all officers on the Board, the quickest to appreciate and avail himself of the services of the consultant. This situation had certain advantages, since the consultant was able to secure Colonel Huffsmith's advice and backing in assigning his own personal

priorities to different tasks — a very ticklish problem when several projects were simultaneously calling for urgent attention. If the recommendations implied in this memorandum were followed, a mathematical consultant to the Board would be greatly overburdened during wartime, unless he was in a position (as was the writer) to refer problems promptly to such agencies as the various AMP groups.

8. It should be understood in advance that a technical consultant is entitled to military orders when traveling on the business of the agency to which he is assigned. The writer succeeded in making arrangements for the regular use of such orders, authorizing travel by government aircraft and granting priorities on commercial aircraft, after he had served about a year in his post at the Board.

9. In general, a definite arrangement should be made, perhaps by an AGO pass, or other suitable official document, whereby a consultant would be assured, wherever he went, of all appropriate rights and privileges of a field grade officer. These affect assignments to sleeping quarters, Officers' Club privileges, eating in officers' messes and so on. The writer received such considerations, after a reasonable period of "breaking the ice," but they should be recognized as prerogatives, rather than as personal courtesies.

10. During wartime, a consultant to the Board could occasionally benefit by visits to active theaters. Such visits would almost necessarily entail the availability of more than one consultant, so that duties would not suffer during prolonged trips. There were occasions when the writer's work required his presence in Orlando, even though certain experiences elsewhere would probably have increased his value to the Board.

APPENDIX B

AIRCRAFT FIRE CONTROL DEVELOPMENTS IN THE AAF[a]

February 28, 1945

1. *Scope of Memo.*

1.1. This memo has been written in response to a request, from Dr. E. L. Bowles, for suggestions concerning the organization, program, and staff of an aerial gunnery research unit at Eglin Field.

The moment, however, one attempts to analyze this individual situation he becomes inescapably involved in the broader problem of the way in which the AAF as a whole provides for the development of new devices and techniques of aerial warfare.

The body of the present memo is therefore devoted to an analysis of the broad problem; and the recommendations relating specifically to gunnery research at Eglin are to be found in 5.

2. *The Functional Relation between the Steps Involved in Plane-to-Plane Control Problems.*

2.1. On the Flow Chart [given in Figure 1] is schematically indicated the main sequence of the steps involved in the genesis and solution of the equipment problems in plane-to-plane fire control. The diagram is intended to illustrate the fact that the military problems arise from combat needs, and pass through a series of steps which culminate in new equipment in combat use.

2.2. *Air Warfare Analysis* [AWA] occupies a central place in the diagram; and should in fact exercise a central and correlating function in the actual process.

The AWA group should develop:

a. Knowledge of the quantitative theory of the tactical and strategic problems of air warfare.

b. Knowledge of the analytical theory of aircraft fire control equipment (sights, sighting mechanisms, bombsights, rocket sights, turret controls, etc.).

c. Knowledge of the theory (probability and statistics) of obtaining hits in aerial gunnery; and of the procedures for estimating the effectiveness of such hits (vulnerability theory).

d. Knowledge of the psychological-physiological aspects of the man-machine coordinations in aerial gunnery (tracking, training, etc.).

e. Correlated knowledge of all ORS activities. In fact, AWA should serve as a central group which unifies the activities of the separate Operation Research Sections; and should carry out comparative analyses of their separate findings.

f. In addition to this large body of scientific and technical knowledge, the AWA group should also have an understanding of the practical problems involved in the introduction of new equipment (logistics, spare part and maintenance service, training, etc.).

The purpose of AWA should be to provide the Chief of Air Staff with expert scientific advice on the employment of air power; to maintain current information on the

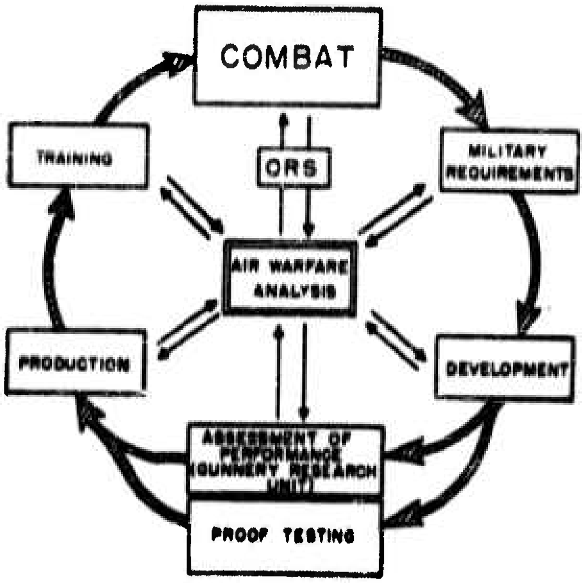


FIGURE 1. Flow chart for aerial gunnery fire control devices.

This chart is drawn for the case of aerial gunnery developments. A closely similar chart exists for other types of items required in aerial warfare (rockets, guided missiles, bombs, etc.); and the AWA group should occupy the same position in all such charts.

There is no intention to imply that at present, when AWA does not exist, there are no useful liaison cross-connections between the main functions in the outside circle; nor would all of these liaison cross-connections necessarily function through AWA if this group existed.

tactics and techniques of aerial gunnery, aerial bombing, etc.; to investigate the mathematical principles which underlie the conduct of aerial warfare; and to reduce those principles to a form which will assist the staff of a combat air force in its planning of operations. This may be more specifically stated as follows:

A. To assist the Chief of Air Staff by

a. Developing a scientific and quantitative basis for the operation of combat aircraft.

[a] Except for minor changes in form, which do not affect the subject matter, this is a copy of the original memorandum.

b. Determining the relative desirability of

(1) different categories and types of combat aircraft,

(2) different equipment for a given combat aircraft,

(3) different methods of operating combat aircraft, by the use of so much of this scientific and quantitative basis as has been developed.

c. Making recommendations consequent to these determinations concerning

(1) military characteristics of future military aircraft,

(2) changes in characteristics of present aircraft,

(3) changes in training,

(4) changes in the operational use of aircraft.

B. To determine what tests and studies are required to accomplish A, and to recommend where these should be carried out.

C. To study the basic problems of bombardment aviation and to analyze the results of tests and studies made elsewhere.

D. To conduct such tests and studies required to accomplish A as the Chief of Air Staff may direct.

2.3. The diagram recognizes a distinction between *Proof Testing* and *Assessment of Performance*, although the diagram also emphasizes that these two are closely interlocked.

The primary function of *Proof Testing* is to determine whether the equipment is operable under conditions of Service use, it being assumed that the quality of performance is assured by the design. The primary function of *Assessment* is to measure performance by quantitative tests, and to interpret the practical effectiveness of this performance.

2.4. It is particularly important that the two-way communication, via the AWA group, be actively maintained between *Training*, *Assessment*, and *Combat*.

3. *The Required Administrative Organization.*

The Flow Chart discussed above takes no account of the command function; nor of the practical problem of cognizances. But if the administrative set-up is to serve usefully, it must be such that the various interchanges and main currents of flow of the diagram can proceed promptly.

Examination of the diagram will serve to indicate some of the weaknesses of the present situation. The chief comments appear to be:

3.1. The central correlating AWA group does not exist within the present structure of the AAF.

3.2. The *Development* function is largely fulfilled by commercial groups operating under contract to Wright Field; and their links with the other functions of the diagrams are weak.

3.3. The *Assessment* function cannot be carried out properly unless a Gunnery Research Unit (GRU) is established and suitably supported. A fine nucleus for such a unit now exists at Eglin.

3.4. The research aspects of the *Training* function appear to be well provided for at Laredo.

3.5. The *Proof Testing function* appears to be well provided for at Eglin.

3.6. The problem of cognizances is a serious one, and does not appear to be suitably solved at present. Thus certain functions listed on the chart come under M and S, others under O C and R; and the inter-relations are not clear, nor do they always appear to be well defined.

3.7. The Assessment function cannot be carried out and utilized effectively unless tactical and strategic considerations are allowed to enter. This means one of two things: either the Gunnery Research Unit at Eglin must itself be permitted to cut across the preconceived command functions, or these broader relationships must be assumed by some other group, working at a higher echelon.

The Chart assumes the second of these two solutions. That is, it assumes that the AWA group would cut across command functions, and would appear on the AAF organization chart at the same level as Management Control. A close tie-up between Air Warfare Analysis and the Gunnery Research Unit would then assure that the latter had access to necessary information and to decision levels.

4. *Recommendations.*

4.1. It is recommended that an Air Warfare Analysis Section, as briefly described above, be set up. This group should be located in Washington, and should report at the same level as Management Control. More detailed suggestions as to staff and program can be furnished if desired.

4.2. It is recommended that the existing program of gunnery research at Eglin Field be expanded, and organized into a Gunnery Research Unit, which is discussed in more detail in 5.

5. *Gunnery Research Unit.*

5.1. *Function and Program.*

The primary function of this unit should be to carry out such ground and air tests of aerial gunnery equipment and devices as will result in reliable quantitative measures of performance; and to carry out such analyses of these performance data as will result in the assessment of this performance in terms of practical end results such as hits on enemy aircraft, probable number of enemy planes destroyed, etc.

It is recognized that, in order to carry out this function, GRU must maintain a program of fundamental research and development, both in analytical methods and in the instrumental aspects of the testing.

The Gunnery Research Unit, occupying as it does a place just in advance of production, should furnish the quantitative and objective evidence on the basis of which the Command can make correct decisions.

5.2. *Staff and Materiel.*

It is clear that any estimates of staff and materiel will be subject to frequent modification. The following outline, although made on a unit basis, is subject to the likelihood of frequent changes. There are five units: military administration, planning and analysis section,

flight test section, bench test section, and experimental squadron (pretest).

a. *Military Administration.*
The size of the staff required here is a military decision. The function of the military administration is to provide housekeeping and administrative services for the civilian and officer scientific and technical personnel.

b. *Planning and Analysis Section.*
This group would be responsible for the planning, direction and analysis of all the projects of the Gunnery Research Unit. The personnel and qualifications required are:

1 Director 1 Associate Director	Experience in fire control of Armament work, understanding of statistical principles, ability to plan and direct broad programs of research.
2 Fire Control Statisticians	At least one with experience in hit-probability work, at least one with experience in experimental design.
2 Aerial Gunnery Mathematicians	At least one with experience on lead computing sights, at least one with experience in ballistics.
2 Instrument Engineers	One with mechanical, the other with electrical, background.
1 Aerodynamical Engineer	Attack curve theory — aerodynamical limitations on maneuverability, etc.
1 Radar Engineer	
1 Psychologist-Physiologist	
4 Project Technical Representatives	Understanding of the scientific techniques used in gunnery research, ability to direct projects to obtain the right data.
4 Computers	
Secretaries, Stenographers, Clerks, etc.	

c. *Flight Test Section.*
This includes director, pilots, aircraft, cameras, gunners, engineers, computers, photographers, and shop facilities.

d. *Bench Test Section.*
This would include:
1 "Texas Engine" with maintenance crew and shop facilities.
2 Experimenters (at least one trained in Physics, at least one trained in Psychology, at least one trained in Statistics).
3 Clerks
4–6 Gunners.

e. *Experimental Squadron (Pretest).*
This unit would be concerned with preliminary tests of suggestions, gunnery equipment and gunnery tactics. It should include something like:
2 heavy bombers (B-17 or -24 and B-29),
2 medium bombers (B-25 or -26 and A-26),
4–6 fighters (P-40, P-47, P-51 and P-61, possibly a P-80A and a "piggyback" P-38 or P-63).
Air crews (The members of these air crews having
(1) recent combat experience,
(2) a willingness and liking for experiment, and
(3) technical background and belief in gunnery research.)
Ground crews.
Intermediate shop facilities.
Adequate photographic equipment.
The members of these aircrews should be slowly rotated as adequately qualified personnel become available after combat tours of duty.

5.3. *Comments.*

a. The GRU should be given the broad responsibility for developing and carrying out a basic program in aerial gunnery research. It must be given the authority to plan and develop this program; and although the unit would of course be expected to carry out needed tests, it must be protected from the necessity of accepting every directive that any one can think up.

This is an essential point. What is needed here is an establishment carrying on a real scientific program, not a mail-order (or telephone) service station. The Unit should be assigned broad problems and fields of activities; but should not be required to accept, from non-scientific sources, directives on specific jobs.

b. The organization and program must be maintained sufficiently flexible to meet new conditions as they inevitably arise.

c. The Unit should be an independent Branch of the Proving Ground Command, and should not be subsidiary to some other Branch.

d. The Office personnel should remain with this Unit for extended assignments; and should receive recognition for this type of service.

e. Such compensation and status must be available for civilian scientists and engineers as will attract and hold men of high caliber. This definitely requires status and privileges equivalent to those enjoyed by officers.

f. The Unit must have (preferably through such an organization as AWA) close liaison with the policy and decision levels of the AAF.

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Numbers such as AMP-502.1-M5 indicate that the document listed has been microfilmed and that its title appears in the microfilm index printed in a separate volume. For access to the index volume and to the microfilm, consult the Army or Navy agency listed on the reverse of the half-title page.

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PART I

Chapters 1–8

In references 53 through 110 the *Study* number is the AMP Study number and the *Paper* number is the contractor's Paper number.

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OSRD APPOINTEES

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CONTRACT NUMBERS, CONTRACTORS, AND SUBJECTS OF CONTRACTS FOR THE APPLIED MATHEMATICS PANEL

Contract Number	*Name and Address of Contractor*	*Subject*
OEMsr-444	The Franklin Institute Philadelphia, Pa. Technical Representative, H. B. Allen	Computations.
OEMsr-618	Columbia University New York, N. Y. Official Investigator, H. Hotelling Director: W. Allen Wallis	Statistical methods applied to air combat analysis, torpedo tactics, acceptance inspection, research and development, and related problems.
OEMsr-817	University of California Berkeley, California Technical Representative, J. Neyman	Statistical analysis applied to bombing research concerned with problems of land mine clearance, the theory of pattern bombing and the bombing of maneuvering ships, and the theory of bomb damage.
OEMsr-818	Columbia University New York, N. Y. Technical Representative, J. Schilt	Mathematical and statistical studies of bombing problems; the application of IBM computing techniques to statistical problems in warfare analysis.
OEMsr-860	Princeton University Princeton, N. J. Technical Representative, S. S. Wilks	Statistical methods applied to miscellaneous problems in warfare analysis and to (1) verification of various long-range weather forecasting systems, (2) a study of fire effect tables and diagrams for warships; (3) bombing accuracy studies, analysis of guided missiles, and the performance of certain heat-homing devices; and (4) the clearance of mine fields by explosive devices.
OEMsr-944	New York University New York, N. Y. Technical Representative, R. Courant	Investigations in shock wave theory.
OEMsr-945	New York University New York, N. Y. Technical Representative, R. Courant	Research in problems of the dynamics of compressible gases, hydrodynamics, thermodynamics, acoustics, and related problems.
OEMsr-1007	Columbia University New York, N. Y. Technical Representatives, E. J. Moulton S. MacLane A. Sard	Miscellaneous studies in mathematics applied to warfare analysis with emphasis upon aerial gunnery, studies of fire control equipment, and rocketry and toss bombing.
OEMsr-1066	Brown University Providence, R. I. Technical Representative, R. G. D. Richardson	Problems in classical dynamics and the mechanics of deformable media.
OEMsr-1111	Institute for Advanced Study Princeton, N. J. Technical Representative, John von Neumann	Studies of the potentialities of general-purpose computing equipment, and research in shock wave theory, with emphasis upon the use of machine computation.
OEMsr-1305	Princeton University Princeton, N. J. Technical Representative, Merrill M. Flood	Coordination of activities under Project AC-92 at the University of New Mexico, Carnegie Institution of Washington at Pasadena, Columbia University, and Brown University.
OEMsr-1379	Northwestern University Evanston, Ill. Technical Representatives E. J. Moulton Walter Leighton	Studies in aerial gunnery, particularly the camera assessment of the performance of sights and of airplanes.

Contract Number	*Name and Address of Contractor*	*Subject*
OEMsr-1381	Carnegie Institution of Washington Pasadena, Calif. Technical Representative, Walter S. Adams	Studies and experimental investigations in connection with the defensive fire power of various bomber formations by means of model planes with their guns replaced by suitable light sources, the total fire power being estimates of the light intensity.
OEMsr-1384	Harvard University Cambridge, Mass. Technical Representative, Garrett Birkhoff	Studies of the principles which determine the dynamic behavior of a projectile entering water and the application of these principles quantitatively to the prediction of underwater trajectories and ricochet.
OEMsr-1300	The University of New Mexico Albuquerque, N. M. Technical Representative, E. J. Workman	Studies and experimental investigations in collaboration with the Army Air Forces of the most effective formations and flight procedures for the B-29 airplane. Emphasis, originally upon the tactical use of the B-29, was later changed to a study of the defense of the B-29.
Transfer of Funds	National Bureau of Standards	Computations by the Mathematical Tables Project for various agencies concerned with war research.

SERVICE PROJECT NUMBERS

The projects listed below were transmitted to the Office of the Executive Secretary, OSRD, from the War or Navy Department through either the War Department Liaison Officer for NDRC or the Office of Research and Inventions (formerly the Coordinator of Research and Development), Navy Department.

ARMY PROJECTS

Service Project Number	*Subject*
AC-27	Design data for bombardier's calculator.
AC-91	Statistical problems of combat bombing accuracy.
AC-92	Collaboration of the NDRC with the AAF in determining the most effective tactical application of the B-29 airplane (continuing under AAF Proving Ground Command, Fire Power Analysis Project).
AC-95	Analysis of Waller trainer film.
AC-109	Textbook on flexible gunnery.
AC-115	Study of data accumulated in sight evaluation tests.
AC-122	Study of gun camera film scoring in order to devise a scoring computer.
AN-23	Studies of HE-1B attack on precision target.
CE-33	Checking of hydraulic tables.
OD-143	Study of fuze dead-time correction in AA director.
OD-179	Statistical assistance in rocket propellant tests and specifications.
OD-181	Study of relative destructive effect of machine gun fire against airplane structures.
QMC-35	Food storage data statistics.
QMC-38	Studies of various statistical problems encountered at the Climatic Research Laboratory.
QMC-43	Statistical consultation for Quartermaster Corps inspection service.
SC-89	Rapid solution of linear equations with up to twenty-six unknowns.
SC-100	Binomial distribution calculations.
SOS-2	Probability theory of balloon barrages.

NAVY PROJECTS

Service Project Number	*Subject*
N-110	Mathematical studies of lead-computing sights for use with gunnery training.
N-112	Study and evaluation of sighting methods of instruction used in U. S. Naval Aviation free gunnery training.
N-120	Preparation of instruction course for quality control and statistically based sampling procedures.
NA-107	Study of nozzle design for jet motors.
NA-177	An analytical method of determining ships' speeds in turns from photographs of ships' wakes.
NA-195	Study of jet propulsion devices operating at subsonic and supersonic velocities [continuing under Contract NOa(s)-7370].
ND-2	Assistance to the Air Technical Division — studies of aircraft weapon effectiveness.
NO-108	Probability and statistical study of plane-to-plane fire.
NO-130	Air testing of Mark 15 bombsight.
NO-131	Probability studies desired in connection with estimating hits made by close-range AA gun fire at head-on airplane targets.
NO-139	Mathematical studies of dive-bomber and bomb trajectories in connection with Aiken dive-bombsight.
NO-145	Mathematical studies of bombing.
NO-145 Ext.	Train probability calculations for bombing, November 1944.
NO-145 Ext.	Probability curves for use in connection with gunnery salvo fire, June 1945.
NO-158	Antitorpedo harbor defense nets.
NO-161	Theoretical studies of water entry phenomena [continuing under Contract NOa(s)-7370 with New York University and under Navy Contract with Harvard University].
NO-188	Study of torpedo spreads and their use against maneuvering targets.
NO-200	Studies of acceptance tests on ordnance material.
NO-237	Determination of depth of underwater explosions from surface observations [continuing under Contract NOa(s)-7370].
NO-261	Statistical analysis of the data on thermal characteristics of targets and the relative performance of candidate heat-homing equipment.

SERVICE PROJECT NUMBERS (*Continued*)

Service Project Number	*Subject*
NO-264	Gun equilibrators.
NO-269	B-scan radar plotting device.
NO-270	Computation services (continuing under a transfer of funds from the Office of Research and Inventions to the Bureau of Standards).
NO-272	Computation of dynamic performance of AA computer (continuing under Contract NOrd-917?).
NO-280	Statistical assistance in rocket propellant tests and specifications.
NO-294	Study of tactical utilization of offset guns in fighter aircraft.
NR-101	Probability study of a proposed type of antiaircraft projectile.
NR-105	Fire effect tables (continuing under Contract NOrd-9240).
NS-165	Nonlinear mechanics.
NS-166	Gas globe phenomena in underwater explosions.
NS-302	High-temperature metals.
NS-304	Investigation of wave patterns created by surface vessels (continuing under Contract NOa(s)-7370).

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